THE IMPACT OF VIBRATION MECHANISM’ INSTALLATION PLACE ON THE PROCESS OF RETREIVING STUCK DRILL PIPE

V. Moisyshyn¹, K. Levchuk²*
¹Higher Mathematics Department, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine
²Oil and Gas Equipment Department, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine
*Corresponding author: e-mail kgl.imp.nan@gmail.com, tel. +380933106549

ABSTRACT

Purpose. The present paper studies the dependence of the dynamic oscillatory movements in the area of drill string sticking on the parameters of vibrating mechanism and place of its installation in the heavy weight drill pipe. The authors focus specifically on conditions responsible for resonant vibrations excitation in the drill string, which facilitates the release of stuck drill pipe. The aim is to use the results of the research in the development of the method for vibrator application, to work out recommendations concerning the choice of its installation place in the heavy weight drill pipe, its tuning to the resonant frequency, the choice of induced force amplitude.

Methods. The authors proposed a mathematical model of the drill string functioning with vibration mechanism installed into the heavy weight drill pipe. The vibrator is needed to eliminate sticking of the drilling tool. The suggested discrete-continuum model which takes wave processes into account allows to conduct grounded experimental research.

Findings. The developed mathematical model served as the basis for designing a computer program aimed to visualize oscillatory processes taking place in the string of pipes, and to calculate basic dynamic and kinematic characteristics of the analyzed system. Frequency oscillation spectrum was calculated for the selected layout of the drill string and the resonant amplitudes at these frequencies.

Originality. The proposed technique allows to significantly determine strains, stresses and safety margins in the arbitrary drill string section more accurately and to predict its sticking while drilling oil and gas boreholes.

Practical implications. A specific method of selecting a place for the vibrator installation was elaborated. Recommendations concerning the choice of installation site, amplitude of excitation force, resonant frequencies for elimination of pipes’ sticking and prevention of drill strings destruction are provided.

Keywords: drilling, drill string, elastic waves, mathematical model, vibrations, sticking, vibrator, stress, energy of the stuck drill string

1. INTRODUCTION

Drilling engineers pay great attention to the trouble-free drilling of oil and gas wells. It is explained by costly and long-term maintenance works aimed to liquidate breakdowns especially those caused by sticking of drilling tool, which occur far more often than other accidents (Wen, Chen, & Dong, 2014).

Stuck-pipe problems are still considered (Bodine, 1994) as potentially dangerous, despite a great number of publications, developed modern drilling technologies and methods for prevention of drill string (DS) sticking. Thereby, one cannot underestimate the importance of elaborating new technological solutions and appropriate techniques for the stuck DS retrieval (Houlbrook & Lyon, 2006).

Current methods of solving stuck-pipe problems are based on the use of physical, mechanical and hydraulic impact on the sticking zone (Liu, Wang, Li, & Xu, 2011). It leads to weakening of wall rock pressure forces because of rock dissolution, and, consequently, reduction of friction. Hydraulic methods are based on the alteration of
hydraulic pressure in the accident zone by way of regulating hydro-statistic pressure component, or forming hydraulic waves of the drilling mud. Mechanical methods are based on creation of vibration disturbance, a shock or explosive loading, applied in the zone of DS sticking (Abraham & Marsden, 2008).

A considerable amount of experimental researches and engineering tests have proved that the mechanical methods are especially effective. One of the most advanced methods of retrieving stuck DS is the use of vibration mechanisms, given simplicity of their design. These devices allow to realize a big variety of operational modes, beginning from excitation of disturbing forces to selection of vibration mechanisms' parameters (Bailey & Gupta, 2008). Moreover, vibrators are very durable.

2. MATHEMATICAL MODEL OF ELASTIC WAVES EMMITED BY A DRILL PIPE WITH A VIBRATOR

Mechanical scheme of DS with a vibrating mechanism can be represented by a discrete-continuous system comprising six sections (Fig. 1).

![Figure 1. Drill string with built-in vibrator](image)

It will examine the dynamic characteristics of DS vibrator, built in a heavy weight drill pipe (HWDP). The first section is composed of cylindrical steel pipes made of steel with density \( \rho_1 = 7869.5 \text{ kg/m}^3 \), outer diameter \( D_1 = 127 \text{ mm} \), inside diameter \( d_1 = 107 \text{ mm} \) and the total length \( l_1 = 1400 \text{ m} \). The second section is a part of HWDP located above the vibrator, the third section is located below the vibrator. The stuck part of DS is also divided into three sections: the fourth – loose area above the sticking zone, fifth – the stuck part of DS, sixth – loose area located below the sticking zone.

The length of the heavy weight drill pipe is \( l_2 + l_3 = 130 \text{ m} \), and the length of the stuck drill pipe \( l_4 + l_5 + l_6 = 40 \text{ m} \). These pipes are made of steel with density \( \rho_2 = 7759 \text{ kg/m}^3 \), outside diameter \( D_2 = 177.8 \text{ mm} \) and inside diameter \( d_2 = 71.4 \text{ mm} \), \( i = 2 – 6 \). The vibrator installed in the heavy weight drill pipe. Its location is determined by the length of the second section \( 0 < l_2 < 130 \text{ m} \).

The mathematical model for the proposed scheme of the drill string describes dynamic processes in DS. According to the theory of elasticity (Giovine, 2012), wave differential equations for the cross-cuts of each section can be written as:

\[
\dddot{u}_i(x,t) + 2\mu_1 \ddot{u}_i(x,t) - \alpha_i^2 u_i(x,t) = g;
\]

\[
\dddot{u}_2(x,t) + 2\mu_2 \ddot{u}_2(x,t) - \alpha_2^2 u_2(x,t) = g;
\]

\[
\dddot{u}_3(x,t) + 2\mu_3 \ddot{u}_3(x,t) - \alpha_3^2 u_3(x,t) = g;
\]

\[
\dddot{u}_4(x,t) + 2\mu_4 \ddot{u}_4(x,t) - \alpha_4^2 u_4(x,t) = g;
\]

\[
\dddot{u}_5(x,t) + 2\mu_5 \ddot{u}_5(x,t) - \alpha_5^2 u_5(x,t) = g;
\]

\[
\dddot{u}_6(x,t) + 2\mu_6 \ddot{u}_6(x,t) - \alpha_6^2 u_6(x,t) = g;
\]

(1)

where:

\[
h_i = \frac{\alpha_i}{2m_i}, \quad i = 1 – 6 \quad \text{– multiple coefficient of viscous resistance;}
\]

\[
\alpha_1 = 48.4 \text{ kg/c}, \quad \alpha_2 = 380.4 \text{ kg/c}, \quad i = 2 – 6 \quad \text{– coefficients drilling fluid interaction with DS sections;}
\]

\[
m_1 = \rho_1 d_1^2 l_1 \quad \text{– mass of the first section;}
\]

\[
m_2 = \rho_2 d_2^2 l_2 \quad \text{– mass of the second and the third sections of HWDP accordingly;}
\]

\[
m_3 = \rho_3 d_3^2 l_3 \quad \text{– mass of the fourth, fifth and sixth sections of the stuck drill pipe accordingly;}
\]

\[
F_i, \quad i = 1 – 3 \quad \text{– cross sectional areas of drill pipes;}
\]

\[
a_i = \sqrt{\frac{E_i}{\rho_i}}, \quad i = 1 – 3 \quad \text{– speed of elastic waves propagation;}
\]

\[
E_1 = E_2 = E_3 = 210 \text{ GPa} \quad \text{– modulus of elasticity or Young’s modulus;}
\]

\[
a_i = \frac{\rho_1}{\rho_2}, \quad i = 1 – 6 \quad \text{– current coordinate of DS } i \text{-th section cross-cut;}
\]

\[
t \quad \text{– current time.}
\]

Moving parts of the hoisting system are presented by multiple mass \( m_0 = 9855 \text{ kg} \) and rigidity of drilling lines \( c_0 = 53 \text{ MN/m} \). We assume that the vibrator produces harmonic disturbing force \( P(t) \).

Next, we have to attach boundary conditions at the ends and docking sections of DS to the dynamic equations of the drill string motion (Han, Kim & Karkoub, 2014):

– at the top of DS is hoisting system the first section of the drill string:

\[
E_1 F_1 u_1(0,t) + m_0 u_1(0,t) + m_0 a_1^2 u_1(0,t) = 0;
\]

(2)
\[ u_1(t_1, t) = u_2(0, t), \]
\[ E_1 F_1 u_1(t_1, t) = E_2 F_2 u_2(t, 0) + q(F_2 - F_1) \gamma_l; \] (3)

\[ u_2(t_2, t) = u_3(0, t), \]
\[ E_2 F_2 \left[ u_3(t, 0) - u_2^s(t_2, t) \right] = -P(t); \] (4)

\[ u_3(t_3, t) = u_4(0, t), \]
\[ E_2 F_2 u_3^s(t_3, t) = E_3 F_3 u_4(0, t) + q(F_3 - F_2) g \sum_{j=1}^{3} l_j; \] (5)

\[ u_1(x_1, 0) = \frac{m_0}{c_0} g + \left( a_{iS} - \frac{P_0}{g} l_5 \right) \left( \frac{1}{c_0} + \frac{x_1}{E_1 F_1} \right) g - \frac{\rho_f}{2E_1} x_1^2; \] (8)

\[ u_2(x_2, 0) = \frac{m_0}{c_0} g + \left( a_{iS} - \frac{P_0}{g} l_5 \right) \left( \frac{1}{c_0} + \frac{l_1}{E_1 F_1} \right) g + \left( a_{2S} - \frac{P_0}{g} l_5 \right) \frac{g x_2}{E_2 F_2} - \frac{g}{2} \sum_{j=1}^{2} \rho_f E_j x_j^2; \] (9)

\[ u_3(x_3, 0) = \left( m_0 + a_{iS} - \frac{P_0}{g} l_5 \right) \frac{g}{c_0} + \frac{g}{2} \sum_{j=1}^{2} \left( a_{jS} - \frac{P_0}{g} l_5 - \frac{m_j}{2} \right) \frac{l_j}{E_j F_j} + \left( a_{3S} - \frac{P_0}{g} l_5 - \frac{m_3}{2} \right) \frac{g l_3}{E_3 F_3}; \] (10)

\[ u_4(x_4, 0) = \left( m_0 + a_{iS} - \frac{P_0}{g} l_5 \right) \frac{g}{c_0} + \frac{g}{2} \sum_{j=1}^{2} \left( a_{jS} - \frac{P_0}{g} l_5 - \frac{m_j}{2} \right) \frac{l_j}{E_j F_j} + \left( a_{4S} - \frac{P_0}{g} l_5 - \frac{m_4}{2} \right) \frac{g l_4}{E_4 F_4}; \] (11)

\[ u_5(x_5, 0) = \left( m_0 + a_{iS} - \frac{P_0}{g} l_5 \right) \frac{g}{c_0} + \left( a_{iS} - \frac{P_0}{g} l_5 - \frac{m_i}{2} \right) \frac{g l_i}{E_i F_i} + \frac{g}{2} \sum_{j=2}^{3} \left( a_{jS} - \frac{P_0}{g} l_5 - \frac{m_j}{2} \right) \frac{l_j}{E_j F_j}; \] (12)

\[ u_6(x_6, 0) = \left( m_0 + a_{iS} - \frac{P_0}{g} l_5 \right) \frac{g}{c_0} + \frac{g}{2} \sum_{j=4}^{6} \left( a_{jS} - \frac{P_0}{g} l_5 - \frac{m_j}{2} \right) \frac{l_j}{E_j F_j} + \frac{g}{2} \sum_{j=1}^{6} \left( a_{jS} - \frac{P_0}{g} l_5 - \frac{m_j}{2} \right) \frac{l_j}{E_j F_j}; \] (13)

where:
\[ P_0 \] – evenly distributed load intensity of DS stick part;
\[ a_{iS} = (\rho_1 - q) F_1 l_i + (\rho_2 - q) F_2 \sum_{j=2}^{3} l_j + (\rho_3 - q) F_3 \sum_{j=4}^{6} l_j; \]
\[ a_{2S} = \left( \rho_2 \sum_{j=2}^{3} l_j - q \sum_{j=1}^{3} l_j \right) F_2 + (\rho_3 - q) F_3 \sum_{j=4}^{6} l_j; \]
\[ a_{3S} = \left( \rho_3 \sum_{j=2}^{3} l_j - q \sum_{j=1}^{3} l_j \right) F_3; \]
\[ a_{4S} = \left( \rho_4 \sum_{j=2}^{3} l_j - q \sum_{j=1}^{3} l_j \right) F_4; \]
\[ a_{5S} = \left( \rho_5 \sum_{j=2}^{3} l_j - q \sum_{j=1}^{3} l_j \right) F_5; \]
\[ a_{6S} = \left( \rho_6 \sum_{j=2}^{3} l_j - q \sum_{j=1}^{3} l_j \right) F_6. \]

At the beginning of the motion in the position of static equilibrium speeds in the current DS cross-cuts are:
\[ \dot{u}_i(x_i, 0) = 0, \quad i = 1 - 6. \] (14)

Since at the initial time \((t = 0)\) all cross-cuts of the DS stick part are motionless \(u_5(x_5, 0) = 0\), from equation (12) we determine the place \(l_4\) and length \(l_5\) of the sticking, and intensity of the load \(P_0\).
3. DYNAMICS OF CROSS SECTION DRILLING STRING

Since the differential equations (1) are constantly inhomogeneous equations of the second kind (Kelly, 2008), the stationary solutions have the form of a quadratic expression. Static deformations of the current DS cross-cuts, satisfying the system of equations (1) and boundary conditions (2) – (7) are of the form:

\[ u_1(x_1) = \frac{m_0}{c_0} g + (a_{1S} - \mu m_5) \left( \frac{1}{c_0} + \frac{x_1}{E_1 F_1} \right) g - \frac{\rho_2}{2E_1} x_1^2; \]
\[ u_2(x_2) = \frac{m_0}{c_0} g - \frac{\rho_2}{2E_1} g l_1^2 + (a_{1S} - \mu m_5) \left( \frac{1}{c_0} + \frac{l_1}{E_1 F_1} \right) g + (a_{2S} - \mu m_5) \frac{g x_2}{E_2 F_2} - \frac{\rho_2}{2E_2} x_2^2; \]
\[ u_3(x_3) = (m_0 + a_{1S} - \mu m_5) \frac{g}{c_0} + g \sum_{j=1}^{2} \left[ a_{JS} - \mu m_5 - \frac{m_j}{2} \right] \frac{l_j}{E_j F_j} + (a_{3S} - \mu m_5) \frac{g x_3}{E_3 F_3} - \frac{\rho_2}{2E_3} x_3^2; \]
\[ u_4(x_4) = (m_0 + a_{1S} - \mu m_5) \frac{g}{c_0} + g \sum_{j=1}^{2} \left[ a_{JS} - \mu m_5 - \frac{m_j}{2} \right] \frac{l_j}{E_j F_j} + (a_{3S} - \mu m_5) \frac{g x_4}{E_4 F_4} + \frac{\rho_2}{2E_4} x_4^2; \]
\[ u_5(x_5) = (m_0 + a_{1S} - \mu m_5) \frac{g}{c_0} + \left( a_{1S} - \mu m_5 - \frac{m_1}{2} \right) \frac{g l_1}{E_1 F_1} + \sum_{j=2}^{3} \left[ a_{JS} - \mu m_5 - \frac{m_j}{2} \right] \frac{l_j}{E_j F_j} + \frac{\rho_2}{2E_3} (1-\mu) x_3^2; \]
\[ u_6(x_6) = (m_0 + a_{1S} - \mu m_5) \frac{g}{c_0} + g \left( a_{1S} - \mu m_5 - \frac{m_1}{2} \right) \frac{l_1}{E_1 F_1} + \sum_{j=2}^{3} \left[ a_{JS} - \mu m_5 - \frac{m_j}{2} \right] \frac{l_j}{E_j F_j} + \frac{\rho_2}{2E_3} (1-\mu) x_3^2; \]
\[ \begin{align*}
    &u_{id}(x_i,0) = u_{i}(x_i,0) - u_{id}(x_i); \\
    &\dot{u}_{id}(x_i,0) = \dot{u}_{i}(x_i,0), \quad i = 1 - 6
\end{align*} \]

and initial conditions that satisfy the following equalities:

\[ \begin{align*}
    &u_{id}(x_i,0) = u_{i}(x_i,0) - u_{id}(x_i); \\
    &\dot{u}_{id}(x_i,0) = \dot{u}_{i}(x_i,0), \quad i = 1 - 6
\end{align*} \]

and according to the conditions obtained above (8) – (15) take the form:

\[ \begin{align*}
    &u_{id}(x_1,0) = \frac{1}{c_0} + \frac{x_1}{E_1 F_1} a_0; \\
    &u_{id}(x_1,0) = \frac{1}{c_0} + \frac{l_1}{E_1 F_1} + \frac{x_1}{E_2 F_2} a_0; \\
    &u_{id}(x_3,0) = \frac{1}{c_0} + \sum_{j=1}^{2} \frac{l_j}{E_j F_j} + \frac{x_1}{E_3 F_3} a_0; \\
    &u_{id}(x_4,0) = \frac{1}{c_0} + \sum_{j=1}^{2} \frac{l_j}{E_j F_j} + \frac{x_1}{E_4 F_4} + \frac{x_4}{E_5 F_5} a_0; \\
    &u_{id}(x_5,0) = \frac{1}{c_0} + \frac{l_1}{E_1 F_1} + \frac{1}{E_2 F_2} \sum_{j=2}^{3} \frac{l_j}{E_j F_j} + \frac{x_2}{E_3 F_3} + \frac{x_3}{2E_4 F_4} a_0;
\end{align*} \]

and

\[ \begin{align*}
    &\dot{u}_{id}(x_6,0) = \frac{1}{c_0} + \dot{u}_{i}(x_6,0); \\
    &\dot{u}_{id}(x_i,0) = \dot{u}_{i}(x_i,0), \quad i = 1,6
\end{align*} \]

where:

\[ a_0 = \mu m_5 g - P_{0S}. \]

The solution of the equation system (16) is complicated by the fact that the boundary conditions (17) are inhomogeneous because they contain a non-stationary component – a disturbing force \( P(t) \). First it is necessary to find such special function \( \phi_i(x_i, t) \), \( i = 1 - 6 \), so as to ensure homogeneous boundary conditions for new unknown functions \( w_i(x_i, t) \). After selecting special functions, the sought-for dynamic movements of DS cross-cuts \( u_{id}(x_i, t) \) must be shifted to \( \phi_i(x_i, t) \).
Functions $\varphi_i(x_1, t)$ describe additional components of DS forced oscillations (Maugin, 2007). They ensure homogeneous boundary conditions for functions $w_i(x_1, t)$ and are completely defined by inhomogeneous boundary conditions (17):

$$
\begin{align*}
\varphi_1 (x_1, t) &= \varphi_2 (x_2, t) = 0; \\
\varphi_3 (x_3, t) &= \frac{p(t)}{E_3 F_3} x_3 \left( x_3 - l_3 \right); \\
\varphi_4 (x_1, t) &= (-1)^i \frac{p(t)}{E_3 F_3} x_1 \left( l_1 - x_1 \right); i = 4, 5; \\
\varphi_6 (x_6, t) &= \frac{p(t)}{E_3 F_3} x_6 \left( 2l_6 - x_6 \right).
\end{align*}
$$

The obtained laws of motion indicate that the disturbing force $P(t)$ excites vibrations below the vibrator installation zone. Equations (21) describe forced components of drill pipes cross-cuts oscillations, excited by vibration

where:

$$
\begin{align*}
q_5 (x_1, t) &= \frac{A}{E_2 F_2} \left[ \left( x_3 \left( l_3 - x_3 \right) \alpha^2 + 2x_3 \right) \sin (\alpha t + \gamma) - 2l_3 x_3 \left( l_3 - x_3 \right) \omega \cos (\alpha t + \gamma) \right]; \\
q_6 (x_1, t) &= (-1)^i \frac{A}{E_2 F_2} \left[ \left( x_1 \left( x_1 - l_1 \right) \alpha^2 + 2x_1 \right) \sin (\alpha t + \gamma) - 2l_1 x_1 \left( x_1 - l_1 \right) \omega \cos (\alpha t + \gamma) \right], i = 4, 5; \\
q_6 (x_6, t) &= \frac{A}{2E_3 F_3} \left[ \left( x_6 \left( 2l_6 - x_6 \right) \alpha^2 + 2x_6 \right) \sin (\alpha t + \gamma) - 2l_6 x_6 \left( 2l_6 - x_6 \right) \omega \cos (\alpha t + \gamma) \right].
\end{align*}
$$

Taking into account (19), (20) and (21) initial conditions for the DS current cross-cuts to the sticking zone took the form:

$$
\begin{align*}
w_1 (x_1, 0) &= \left( \frac{1}{c_0} + \frac{x_1}{E_1 F_1} \right) a_0; \\
w_2 (x_2, 0) &= \left( \frac{1}{c_0} + \frac{l_1}{E_1 F_1} + \frac{x_2}{E_2 F_2} \right) a_0; \\
w_3 (x_3, 0) &= \left( \frac{1}{c_0} + \sum_{i=1}^{2} \frac{l_1}{E_1 F_1} + \frac{x_3}{E_1 F_1} \right) a_0 - \frac{A \sin \gamma}{E_2 F_2} x_3 \left( l_3 - x_3 \right) (24) \\
w_i (x_i, 0) &= 0, i = 1, 2; \\
w_3 (x_3, 0) &= A \omega l_3 x_3 \cos \gamma; \\
w_4 (x_4, 0) &= \omega l_4 x_4 \cos \gamma.
\end{align*}
$$

To find a non-trivial solution to the system of inhomogeneous equations (16) related to system (23) We will use the Fourier’s method according to which the DS cross-cuts moving will be expressed by the product:

$$
\begin{align*}
w_i (x_i, t) &= X_i (x_i) \cdot T_i (t), i = 1 - 6,
\end{align*}
$$

where:

$$
\begin{align*}
X_i (x_i) &- \text{the function of the current cross-cut;} \\
T_i (t) &- \text{the function of the current time.}
\end{align*}
$$

After substituting relationships (25) into homogeneous dynamic equations which relate to (23), we received a differential equations system of the current cross-cut functions:

$$
\begin{align*}
X_1 (x_1) + A_2 l_2^2 X_1 (x_1) &= 0; \\
X_i (x_i) + A_2 l_2^2 X_i (x_i) &= 0, i = 2, 3; \\
X_i (x_i) + A_2 l_2^2 X_i (x_i) &= 0, i = 4, 5, 6.
\end{align*}
$$

Thus, we have obtained the Sturm-Liouville problem with the following non-trivial solutions to $X_i (x_i)$:

$$
\begin{align*}
X_1 (x_1) &= A_1 l_1 \sin \left( \frac{P_k}{a_1} x_1 \right) + A_2 \cos \left( \frac{P_k}{a_1} x_1 \right); \\
X_i (x_i) &= A_1 l_1 \sin \left( \frac{P_k}{a_1} x_i \right) + A_2 \cos \left( \frac{P_k}{a_1} x_i \right), i = 2, 3; \\
X_i (x_i) &= A_1 l_1 \sin \left( \frac{P_k}{a_1} x_i \right) + A_2 \cos \left( \frac{P_k}{a_1} x_i \right), i = 4, 5, 6.
\end{align*}
$$

After substituting (27) into boundary conditions (22), we obtain a frequency equation for vibrations of the drill string:

$$
\begin{align*}
\left[ \left( \mu_2 - \mu_3 \tan \frac{P_l}{a_3} l_3 \right) \frac{M_3}{\mu_3} - \left( \tan \frac{P_k}{a_3} l_4 + \tan \frac{P_k}{a_3} l_5 \right) \frac{M_4}{\mu_2} \right] M_1 - \left[ \left( 1 + \tan \frac{P_k}{a_3} l_4 \tan \frac{P_k}{a_3} l_5 \right) \frac{M_3}{\mu_2} - \tan \frac{P_k}{a_3} l_4 \times \right. \\
\left. - \tan \frac{P_k}{a_3} l_5 \right] M_2 = \left[ \left( 1 - \tan \frac{P_k}{a_3} l_4 \tan \frac{P_k}{a_3} l_5 \right) M_4 + \left( \mu_1 \tan \frac{P_k}{a_3} l_4 + \mu_2 \tan \frac{P_k}{a_3} l_5 \right) \frac{M_3}{\mu_3} \right] M_1 + \left[ \left( \tan \frac{P_k}{a_3} l_4 \times \right. \\
\left. \times \tan \frac{P_k}{a_3} l_5 + 1 \right) M_3 + \left( \tan \frac{P_k}{a_3} l_4 - \tan \frac{P_k}{a_3} l_5 \right) \frac{M_4}{\mu_2} \right] M_2 \left. \times \frac{P_k}{a_3} l_6, \right]
\end{align*}
$$

force $P(t)$. For the purpose of the research we assumed vibration force to be harmonic $P(t) = A \sin (\omega t + \gamma)$.

Thus, the general boundary problem for functions $u_0 (x_1, t)$ is reduced to the problem with homogeneous boundary conditions:

$$
\begin{align*}
E_1 F_1 w_1 (0, t) &= c_0 w_1 (0, t) + m_1 \alpha^2 w_1 (0, t); \\
E_1 F_1 w_1 (l_1, t) &= E_2 F_2 w_2 (0, t); \\
E_2 F_3 w_2 (l_3, t) &= E_3 F_3 w_3 (0, t); \\
E_3 F_3 w_3 (l_3, t) &= E_1 F_1 w_1 (0, t), i = 1, 2, 5; \\
w_1 (l_1, t) &= w_{i+1} (0, t), i = 4, 5; \ w_6 (l_6, t) = 0.
\end{align*}
$$

For functions $w_i (x, t)$ which are solutions to inhomogeneous differential equations obtained in (20) and (21).

$$
\begin{align*}
w_1 (l_1, t) + 2h_1 w_1 (x, t) - \alpha^2 w_1 (x, t) &= 0; \\
w_3 (x, t) + 2h_3 w_3 (x, t) - \alpha^2 w_3 (x, t) &= m_3 (x, t); \\
w_i (x, t) &= 2h_i w_i (x, t) - \alpha^2 w_i (x, t) = q_i (x, t).
\end{align*}
$$

Aver. 10(3), 65-76
where:
\[ M_1 = \left( 1 - \frac{\mu_k p_k}{c_0 - m_0 p_k} \frac{p_k}{a_1} \right) \frac{p_k}{\mu_2} ; \]
\[ M_2 = \frac{\mu_k p_k}{c_0 - m_0 p_k} + \frac{p_k}{a_1} ; \]
\[ M_3 = 1 - \frac{p_k}{a_2} l_2 + \frac{p_k}{a_2} l_3 ; \]
\[ M_4 = \frac{p_k}{a_2} l_2 + \frac{p_k}{a_2} l_3 ; \]
\[ \mu_i = F_i \frac{1}{E_i} p_i^1 i = 1 - 3. \]

Frequency equation (28) is transcendental and has an unlimited number of solutions, so the frequency spectrum \( p_i \) \((k = 1, \ldots , \infty)\) of DS sections natural oscillations is aliquant in character. It should also be noted that the DS frequency value depends on the parameters of all sections of the drill string and the multiple mass and rigidity of the hoisting system. The mathematical model of DS with a vibrating device built in HWDP was compiled to conduct parametric research (Fitzgibbon, Kuznetsov, Neitammentäki, & Pironneau, 2014). This model allowed to calculate with reasonable accuracy movement and speed of DS cross-cuts, amplitude-frequency, phase-frequency and dynamic characteristics of the drill string, excited by the vibrating mechanism. For the selected DS layout we calculated the frequency spectrum of free oscillations, the first ten of which are presented in Table 1 (Heinz, 2011).

| Table 1. Natural frequencies of the drill string |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Frequency        | \( p_1 \)         | \( p_2 \)         | \( p_3 \)         | \( p_4 \)         | \( p_5 \)         | \( p_6 \)         | \( p_7 \)         | \( p_8 \)         | \( p_9 \)         | \( p_{10} \)       |
| Hz               | 3.45             | 12.70            | 23.34            | 34.50            | 45.50            | 56.20            | 65.97            | 74.30            | 82.87            | 91.61            |

In general, the frequency spectrum of DS natural oscillations is very wide, and the generated wave propagates along the pipe string and transferred to the place of drill pipes’ sticking.

\[ X_{ik} (x_i) = \sin \frac{p_k}{a_1} x_i + \frac{\mu_k p_k}{a_0 - m_0 p_k^2} \cos \frac{p_k}{a_1} x_i ; \quad X_{ik} (x_i) = \left( M_1 \sin \frac{p_k}{a_2} x_2 + M_2 \cos \frac{p_k}{a_2} x_2 \right) \cos \frac{p_k}{a_1} l_1 ; \]

\[ X_{ik} (x_i) = \left( M_1 \sin \frac{p_k}{a_2} (x_3 + l_2) + M_2 \cos \frac{p_k}{a_2} (x_3 + l_2) \right) \cos \frac{p_k}{a_1} l_1 ; \]

\[ X_{ik} (x_i) = \left( \frac{p_k}{a_2} l_2 + M_2 \right) \frac{p_k}{a_2} l_3 - M_2 \frac{p_k}{a_2} l_2 + M_2 \frac{p_k}{a_2} l_3 + \left( M_1 - M_2 \frac{p_k}{a_2} l_2 + M_2 \frac{p_k}{a_2} l_3 \right) \sin \frac{p_k}{a_3} x_4 + \left( M_2 \frac{p_k}{a_2} l_2 + M_2 \frac{p_k}{a_2} l_3 \right) \sin \frac{p_k}{a_3} x_4 \]

\[ \left( M_1 - M_2 \frac{p_k}{a_2} l_2 + M_2 \frac{p_k}{a_2} l_3 \right) \sin \frac{p_k}{a_3} x_4 + \left( M_2 \frac{p_k}{a_2} l_2 + M_2 \frac{p_k}{a_2} l_3 \right) \cos \frac{p_k}{a_3} l_2 \cos \frac{p_k}{a_2} l_3. \]

Based on the discovered natural functions (29), solutions \( w_i (x_i, t) \) are written as series:

\[ w_i (x_i, t) = \sum_{k=1}^{\infty} X_{ik} (x_i) T_{ik} (t) , \quad i = 1 - 4. \]

Next, let us determine the time function \( T_{ik} (t) , i = 1 - 4, \) so that the series (30) satisfy the system of equations (23) and the initial conditions (24). Let us decompose the right parts of differential equations system (23) and the initial conditions (24) at intervals \((0, l)\) on their own functions (29) according to Strokes’ decomposition theorem:

\[ q_i (x_i, t) = \sum_{k=1}^{\infty} C_{ik} (t) X_{ik} (x_i) ; \]

\[ w_i (x_i, t) = \sum_{k=1}^{\infty} T_{ik} (0) X_{ik} (x_i) ; \]

\[ \dot{w}_i (x_i, t) = \sum_{k=1}^{\infty} \dot{T}_{ik} (0) X_{ik} (x_i) , \]

where:

\[ C_{ik} (t) = \int_{0}^{l} q_i (x_i, t) X_{ik} (x_i) dx_i / \int_{0}^{l} X_{ik} (x_i) dx_i ; \]

\[ T_{ik} (0) = \int_{0}^{l} w_{ik} (x_i, 0) X_{ik} (x_i) dx_i / \int_{0}^{l} X_{ik} (x_i) dx_i ; \]

\[ \dot{T}_{ik} (0) = \int_{0}^{l} \dot{w}_{ik} (x_i, 0) X_{ik} (x_i) dx_i / \int_{0}^{l} X_{ik} (x_i) dx_i . \]

According to (24) \( \psi_1 (x_1, 0) = \psi_2 (x_2, 0) = 0 , \) then \( \tilde{T}_{ik} (0) = \tilde{T}_{2k} (0) = 0 , \quad k = 1, \ldots , \infty. \)

As a result, the system of inhomogeneous dynamic equations allowed to obtain a system of an infinite number of time function equations which are a classical Cauchy problem:

\[ \ddot{T}_{ik} (t) + 2h_i \dot{T}_{ik} (t) + p_i^2 T_{ik} (t) = C_{ik} (t) ; \]

\[ \tilde{T}_{ik} (0), 0; i = 1, 4 , \quad k = 1, \infty. \]

Then the time function takes the form:

\[ T_{ik} (t) = e^{-h_i t} \left[ B_{ik} \sin \left( \sqrt{p_i^2 - h_i^2} t \right) + B_{2ik} \cos \left( \sqrt{p_i^2 - h_i^2} t \right) \right] + \frac{C_{ik} (t)}{\sqrt{p_i^2 - h_i^2}}, \]

\[ \sqrt{p_i^2 - h_i^2} \left( 4h_i^2 + 4h_i^2 \right) \]

and stable integration:

\[ B_{ik} = \frac{1}{\sqrt{p_i^2 - h_i^2} \left( 4h_i^2 + 4h_i^2 \right)} \left[ h_i T_{ik} (0) + T_{ik} (0) + \frac{C_{ik} (0) - h_i C_{ik} (0)}{\sqrt{p_i^2 - h_i^2} \left( 4h_i^2 + 4h_i^2 \right)} \right] ; \]

\[ B_{2ik} = \frac{1}{\sqrt{p_i^2 - h_i^2} \left( 4h_i^2 + 4h_i^2 \right)} \left[ \frac{C_{ik} (0)}{\sqrt{p_i^2 - h_i^2} \left( 4h_i^2 + 4h_i^2 \right)} \right]. \]

Thus, the solution of equation (1) for the third and fourth sections of the drill string is:
\[ u_3(x,t) = \left( m_0 + a_{1S} - \frac{\mu S}{c_0} \right) \frac{g}{c_0} + \frac{g}{E} \sum_{j=1}^{2} \left( a_{2j} - \frac{m_j}{2} \right) \frac{E}{F_j} + \left( a_{3S} - \frac{m_3}{2} \right) \frac{E}{F_3} + \frac{g_{x_3}}{E_3} - \frac{g_{p_2}}{E_2} 2 \frac{x_3}{E_2 F_2} l_j \]

\[ u_4(x,t) = \left( m_0 + a_{1S} - \frac{\mu S}{c_0} \right) \frac{g}{c_0} + \frac{g}{E} \sum_{j=1}^{2} \left( a_{2j} - \frac{m_j}{2} \right) \frac{E}{F_j} + \left( a_{3S} - \frac{m_3}{2} \right) \frac{E}{F_3} + \frac{g_{x_3}}{E_3} - \frac{g_{p_2}}{E_2} 2 \frac{x_3}{E_2 F_2} l_j \]

\[ u_4(x,t) = \left( m_0 + a_{1S} - \frac{\mu S}{c_0} \right) \frac{g}{c_0} + \frac{g}{E} \sum_{j=1}^{2} \left( a_{2j} - \frac{m_j}{2} \right) \frac{E}{F_j} + \left( a_{3S} - \frac{m_3}{2} \right) \frac{E}{F_3} + \frac{g_{x_3}}{E_3} - \frac{g_{p_2}}{E_2} 2 \frac{x_3}{E_2 F_2} l_j \]

So, moving of drill string cross-cuts (35) includes four components:

- stationary, which is determined by the parameters of the hoisting system, the drill string tension, pipes weight and drilling fluid density, viscous resistance power, and rock pressure on the DS stuck part. This component fully determines the location, length and strength of sticking;
- forced oscillations with the frequency of the exciting force and the amplitude, which depends on DS parameters, vibrator location in the heavy weight drill pipe and the amplitude of the disturbing force. Such fluctuations have the shape of standing waves and reach maximum amplitude the middle points of the sections. These waves do not transfer energy from the vibrator to the DS sticking place, but only convert kinetic energy into potential;
- inherent attenuating oscillations consisting of the sum of aliquant harmonics;
- forced oscillations with the frequency of the exciting force and the amplitude, which depends on DS parameters, vibrator location in the heavy weight drill pipe and natural DS frequencies, as well as the amplitude of the disturbing force. Such oscillations have the shape of traveling waves that spread energy in the DS. It is possible to enhance efficiency of energy transfer by tuning the vibrator to the resonance – one of the DS natural frequencies. (Bailey & Gupta, 2008).

4. MATERIALS UNDER ANALYSIS

Using mathematical modeling for the selected layout of the DS, we designed and built graphs for all components of drill pipes cross-cuts displacements (Kelly, 2008). Figure 2 shows elastic deformations of the drill pipe cross-cut above the DS sticking zone.

![Figure 2. Static movement of the loose part of the drilling pipe](image)

Figure 2 demonstrates that the extended cross-cut \( u_4(0, t) \) begins to shrink as it approaches the accident zone \( u_4(l_i, t) = u_4(0, t) \). Integral values of the sticking force and sticking length can be calculated from equations (15), taking into account the parameters of DS, hoisting system, and the physical properties of the drilling fluid and friction forces.

If the resistance is viscous, as in the chosen layout, simple oscillations attenuate exponentially (Fig. 3). The rate of decrease in the amplitude of these oscillations is determined by the logarithmic decrement of oscillations \( \Delta k \), which is determined by the viscosity of drilling fluids. It should be noted that logarithmic decrement of the first section oscillations \( \Delta_{k_{\text{max}}} = 0.0004 \) is small compared to the logarithmic decrement of the fourth section oscillations \( \Delta_{k_{\text{max}}} = 0.15 \).

![Figure 3. Standing waves when configuring the vibration device on the first frequency of the drill string](image)

Forced oscillations of the standing waves for the selected DS layout were calculated for harmonic disturbance \( P(t) = A \sin(\omega t + \gamma) \). With amplitude \( A = 1 \text{kN} \).

Figure 3 demonstrates standing waves when the vibrator is configured to the first natural DS frequency, and Figure 4 – to the fifth natural frequency. Thus, the frequency increases with decreasing oscillation period.

For the studied drill string, the oscillations of standing waves were low amplitude, and therefore their influence \( \phi(x, t) \) on the drill string vibration \( u(x, t) \) can be neglected.

Figure 4. Standing waves when configuring the vibration device to the fifth frequency of the drill string

Figure 5 shows free oscillations of the first DS section that slowly fade away as they approach the borehole being close to a harmonious in character.

The study into free oscillations has shown that they quickly fade in the course of approaching the accident zone.

Figure 6. Simple oscillations of the heavy weight drill pipe above the vibrator installation location

Figure 7. Simple oscillations of the heavy weight drill pipe below the vibrator location

Figure 6 and 7 show oscillations of HWDP that are beginning to slowly attenuate, and Figure 8 presents oscillations of the stuck drill pipe that attenuate rather quickly.

Forced oscillations of the heavy weight drill pipe (Fig. 9) are multi-harmonious, but the intensity of the stuck DS section oscillations decreases sharply with the increase in resonance frequency.

Figure 8. Simple oscillations of the drill string stuck section

Figure 9. Forced oscillations of the heavy weight drill pipe
Figure 10 shows resonance achieved by configuring the vibrator to the first natural DS frequency, and Figure 11 – to the fifth natural frequency.

To analyze the setting up of the vibration mechanism to natural DS frequency, we calculated the resonance amplitudes (Table 2) of the forced oscillations in the sticking place and constructed the amplitude-frequency characteristic (Fig. 12).

Table 2. Resonance amplitudes of oscillations in the sticking zone

<table>
<thead>
<tr>
<th>Resonance frequency</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$P_7$</th>
<th>$P_8$</th>
<th>$P_9$</th>
<th>$P_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude, cm</td>
<td>144.6</td>
<td>40.8</td>
<td>22.3</td>
<td>15.2</td>
<td>11.6</td>
<td>9.3</td>
<td>8.0</td>
<td>7.1</td>
<td>6.4</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Past studies have shown that the intensity of forced oscillations increases with the approach of the installation site vibrator in HWDP to the accident.

To analyze the vibration mechanism setting up on natural frequency DS calculated the resonance amplitude (Table 2) forced oscillations in the place and constructed stuck frequency response (Fig. 12).

Previous studies have shown that the intensity of forced oscillations increases as the vibrator installation place in HWDP approaches the accident zone (Table 3).

It is worth noting that along with the trend described above, coefficient of the forced oscillations amplitude transmission increases if the vibrator installation location is moved in HWDP in the direction of the borehole.

An important dynamic condition of drilling tool effective operation is instantaneous longitudinal forces that arise in cross-cuts of drill pipes:

$$P_i(x, t) = E_i F_i u'_i(x, t), i = 1, 4.$$  \hspace{1cm} (36)

Another important dynamic operational characteristics is strength testing of all cross-cuts of DS.

$$\sigma_i(x, t) = E_i u'_i(x, t), i = 1, 4.$$ \hspace{1cm} (37)

Research into the force variations in drill pipes cross-cuts showed that the vibrator installation locations $P_3(0, t)$ and $\sigma_3(0, t)$ are subjected to the greatest loads and stresses excited by external vibration.

Amplitude values of these forces and stresses (Fig. 13) for the selected DS layout grow when the vibrator is set up to a higher natural DS frequency and sharply fall when the vibrator is lowered along HWDP.
For effective elimination of sticking it was necessary to study DS kinetic and potential energy as well as its dependence upon the location of the vibrator:

\[
E_{\text{kin}}(t) = \sum_{i=1}^{4} \rho_i \int_0^t u_i^2(x_i,t) \, dx_i;
\]

\[
E_{\text{pot}}(t) = \sum_{i=1}^{4} E_i \int_0^t \frac{1}{2} k_i u_i^2(x_i,t) \, dx_i, \quad i = 1,4,
\]

where:

\( u_i(x_i,t) \) – speed of the current cross-cut of DS \( i \)-th section with coordinate \( x_i \) at an arbitrary time \( t \).

For the selected layout of the drill string, kinetic energy of the stuck DS is 1.34 MJ and potential – 2.18 MJ. Studies have shown that energy curves have clearly defined extremes (Fig. 14): the largest kinetic energy is accumulated by DS in case of placing the vibrator at a distance that is 0.36 HWDP length (Fig. 14), and maximum of potential energy – in the middle of HWDP. Thus kinetic energy constitutes only 5 – 10% of the potential energy (Fig. 15).

![Energy characteristics of DS oscillations excited by the vibrator on the first three resonant frequencies](image)

So, it is more feasible to eliminate accidents at the expense of DS elastic deformations that can be reinforced by the vibration device.

### 5. CONCLUSIONS

In this work we presented a discrete-continuum mathematical model of a drill string with vibration mechanism built into the HWDP. Based on the elaborated mathematical model we have developed a computer program which helped us to visualize oscillatory processes of the drill-string, as well as to carry out numerical calculations of power and energy performance.
The obtained results were analyzed. The developed model allows to promptly analyze and explain the choice of vibration mechanism location, so as to provide fast retrieving of the stuck drill string. Departing from the aforementioned results, we elaborated the following recommendations:

– the vibrator should be set up to one of its three natural frequencies;
– vibration device has to be situated at the distance which is 0.35 – 0.5 of the HWDP length, starting from its top. It will allow saving the biggest amount of potential energy – elastic deformation of the bottom of the drill-string loose part;
– in the case of setting up the vibration device to its natural frequencies (higher than the third), the vibration mechanism has to be located in HWDP as low as possible. This is because excited vibrations can cause destruction of drilling pipes;
– for every given layout of the drill-string we should carry out numerical calculations that will allow to predict any possible sticking, and also to choose location for the vibration mechanism;
– we can use the given results for further research and perfection of existing engineering methods of modeling and calculation of drill-strings on the stage of their designing and construction.

ACKNOWLEDGMENTS

The authors express their sincere gratitude to Dr. Ja. Kuntsya (Joint Stock Company “Scientific Design Bureau for Testing of Drilling Tools”, Kyiv, Ukraine) and Prof. V. Vekery (Ivano-Frankivsk National Technical University of Oil and Gas, Ukraine) for consultations on technical issues of borehole drilling.

Scientific advice was obtained from The National Academy of Sciences of Ukraine, Academic Society of Michal Baludyansky (Bratislava, Slovak Republic), National Lviv Polytechnic University (Lviv, Ukraine).

We greatly appreciate Prof. B. Kopec (Ivano-Frankivsk National Technical University of Oil and Gas, Ukraine) for his valuable improvements to this manuscript.

REFERENCES


ABSTRACT (IN UKRAINIAN)

Мета. Дослідити залежності динамічних характеристик коливальних рухів у місці застрявання бурільної колони від параметрів вібраційного механізму та місця його установки в обтяжній бурільний трубі. Окремо розглянути умови збурення резонансних коливань у бурільний колоні. За результатами проведених досліджень розробити методику застосування вібратора, рекомендації з вибору місця його встановлення в обтяжній бурільній трубі, на основі розглянути умови збурення резонансних коливань у бурільний колоні, настроївання на резонансну частоту, вибір амплітуди збурювальної сили.

Методика. Для проведення експериментальних досліджень отримана дискретно-континуальна модель, в якій враховано хвильові процеси. Запропонована математичну модель роботи бурільної колони з вібраційним механізмом, вміщеним у обтяжній бурільній трубі.

Результати. На основі математичної моделі складено комп’ютерну програму для візуалізації коливальних процесів, що відбуваються в колоні труб, та чисельного розрахунку основних кінематичних і динамічних характеристик досліджуваної системи. Розраховано частотний спектр власних коливань для вибраної компоновки бурільної колони, а також резонансні амплітуди на цих частотах.
Наукова новизна. Запропонована методика дозволяє забезпечити суттєве підвищення точності визначення зусиль, напружень і запасів міцності у довільному перерізі колони труб, а також прогнозувати прихоплення бурильних колон при бурінні кафтових і газових свердловин.

Практична значимість. Наведено методику вибору місця монтування вібратора. Розроблено рекомендації з вибору місця установки, амплітуди збуреної сили і резонансних частот для ліквідації прихоплень труб і запобігання руйнування бурильних колон.

Ключові слова: буріння, бурильна колона, пружні хвилі, математична модель, вібрація, прихоплення, вібраційний пристрій, напружения, енергія прихопленої колони

ABSTRACT (IN RUSSIAN)

Цель. Исследовать зависимости динамических характеристик колебательных движений в месте прихвата бурильной колонны от параметров вибрационного механизма и места его установки в утяжеленной бурильной трубе. Отдельно рассмотреть условия возбуждения резонансных колебаний в бурильной колоне. По результатам проведенных исследований разработать методику использования вибратора, рекомендации по выбору места его установки в утяжеленной бурильной трубе, настраивания на резонансную частоту, выбора амплитуд вынужденной силы.

Методика. Для проведения экспериментальных исследований получена дискретно-континуальная модель, в которой учтены волновые процессы. Предложенная математическая модель работы бурильной колонны с вибрационным механизмом, вмонтированным в утяжеленную бурильную трубу. Вибратор используется для ликвидации прихватов бурильного инструмента.

Результаты. На основании этой модели составлена компьютерная программа с целью визуализации колебательных процессов, происходящих в колонне труб, и числового расчета основных кинематических и динамических характеристик исследуемой системы. Произведен расчет частотного спектра собственных колебаний для выбранной компоновки бурильной колонны, а также резонансных амплитуд на этих частотах.

Научная новизна. Предложенная методика позволяет обеспечить существенное повышение точности определения усилий, напряжений и запасов прочности в произвольном сечении колонны труб и прогнозировать прихваты бурильных колонн при бурении нефтяных и газовых скважин.

Практическая значимость. Приведена методика выбора места монтирования вибратора. Даны рекомендации по выбору места установки, амплитуды возбуждающей силы и резонансных частот для ликвидации прихвата труб и предупреждения разрушения бурильных колон.

Ключевые слова: бурение, бурильная колонна, упругие волны, математическая модель, вибрация, прихват, вибрационное устройство, напряжение, энергия прихваченной колонны

ARTICLE INFO
Received: 11 June 2016
Accepted: 31 August 2016
Available online: 30 September 2016

ABOUT AUTHORS
Vasyl Moisyshyn, Doctor of Technical Sciences, Professor of the Higher Mathematics Department, Ivano-Frankivsk National Technical University of Oil and Gas, 15 Karpatska St, 0-407, 76019, Ivano-Frankivsk, Ukraine. E-mail: math@nung.edu.ua

Kateryna Levchuk, Candidate of Technical Sciences, Doctoral Candidate of the Oil and Gas Equipment Department, Ivano-Frankivsk National Technical University of Oil and Gas, 15 Karpatska St, 7-7301, 76019, Ivano-Frankivsk, Ukraine. E-mail: kgl.imp@gmail.com