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V.S. KHILOV

THEORETICAL FUNDAMENTALS OF ELECTRICAL ENGINEERING

Literary editor, Cand. Sc. (Philol)
Mariia Isakova

Recommended by the Academic Council of the State Higher Educational Institution
“National Mining University”
as the textbook for students majoring in “Electromechanics” and “Electrical engineering” of higher educational institutions

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Reviewers:

Rezinkina M.M. – Doctor of Technical Science, Professor, head of department Theoretical Fundamentals of Electrical Engineering of National Technical University "Kharkiv Polytechnic Institute"
Stakhiv P.G. – Doctor of Technical Science, Professor, head of department Theoretical and General Electrical Engineering of National University "Lviv Polytechnic".

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The Theory of Electrical Engineering is presented in three parts: the Basic Theories of Steady-State and Transients in Electrical Circuits and the Basic Theory of Electromagnetic Field.

For students of electrotechnical specialties of higher educational establishments, as well as for scientific and technical specialists dealing with modern problems in the theory and practice of electric power engineering and electromechanics.

Викладено теоретичні основи електротехніки в трьох частинах: теорія стаціонарних процесів в електричних колах, теорія перехідних процесів в електричних колах і теорія електромагнітного поля.

Для студентів електротехнічних спеціальностей вищих навчальних закладів, а також для науково-технічних фахівців, що займаються сучасними проблемами в теорії і практиці електроенергетики та електромеханіки.

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FOREWORD

In 1903 the theoretical lecture course of "Electrotechnics" was for the first time given to the students of technological specialities at the Ekaterinoslav Higher Mining Technical School (EHMS, 1899) in connection with the extensive introduction of electrification into the mining industry of the Donetsk Carboniferous and Krivoi Rog Iron Ore Basins. After reorganization, the School obtained the status of the Ekaterinoslav Mining Institute (EMI, 1912), and then the State Higher Educational Institution "National Mining University" (SHEI "NMU", 2009). This course received further development in 1921 when future mining engineer-electricians started to be trained in Dnepropetrovsk Mining Institute (DMI, 1926). Professors G.E. Evreinov, V.P. Nikitin, S.I. Telny, A.O. Spivakovsky were the first mentors for mining engineer-electricians. In essence, they laid the foundation for the development of mining electrical engineering as an independent scientific discipline. The founder of this school V.P. Nikitin carried out a number of important studies on electrical engineering, and elaborated the theory of electric arc welding. The follower of this school – Prof. V.B. Umansky – created the foundations for the design of electrical mine hoist installations and solved the problems of automation of mine hoisting machines with an asynchronous drive motor. University scientists were pioneers in the development and creation of national flameproof mining electrical equipment.

The increase in the number of electric power and electromechanical specialties led to the creation of the Electrotechnical Faculty at the Dnepropetrovsk Mining Institute in 1962. Since that time, the course "Theoretical Foundations of Electrical Engineering" is given in full within three academic semesters. Further development of this course was obtained after the Institute of Electric Power Industry in 2002 was created at the Mining University, and students' participation in academic mobility programs.

The integration processes taking place in the world community in all spheres of human activity have also affected the system of higher education. At present, a single world educational space is being formed, which manifests itself, first of all, in the unification of educational standards, approaches, curricula, specialties in different countries around the world.

The participation of Ukrainian universities in academic mobility programs (eg Vesby, Erasmus Mundus, DAAD, Tempus, etc.) expands students' access to European higher education. This makes it possible to improve the quality and attractiveness of higher education, broadens the mobility of students, and also ensures the successful employment of university graduates due to the fact that all academic degrees and other qualifications are oriented to the all-European labour market.

Under the conditions of modern market economy, the mobility of education provides for the continued possibility of training students in one of the European universities. For students' preparation in European universities training, best practice is to deliver fundamental courses in English, therefore starting from 2010 the course "Theoretical Fundamentals of Electrical Engineering" is taught in English for students of electromechanical speciality.

The present textbook is the result of generalization of the course of Theoretical Fundamentals of Electrical Engineering given by Professors P.Pirotsky, N.G. Polyakov, Associate Professors N.E. Kuvaev, A.A. Podolsky, V. Mashkovsky, and then Professor V.S. Khilov at the Dnepropetrovsk Mining Institute and the State Higher Educational Institution "National Mining University" for students of Electrical Engineering and Electromechanical specialities.

PREFACE

The development of electrical engineering as the science began in early 19th century, when the basic laws of electromagnetic and electrical phenomena were discovered and the first attempts were made in practical use of electrical energy. Today one cannot name any technology branch that does not use electricity. Modern achievements in science technology in such industries as electronics, information technology, radio, cybernetics, robotics, medicine, as well as household use, acoustics, domestic consumption, etc. are based on using electric energy.

The main advantage of electric energy over other forms of energy is that it is easily converted into other forms of energy, is easily transmitted over long distances with high efficiency, well distributed over different power loads, enables automation of manufacturing technology, and provides the best environmental, sanitary and hygienic working conditions in industry, agriculture and households. Consumers of electricity work without the release of harmful substances and is environmentally safe.

Electrical engineering is the branch of science and technology which deals with the theory and practical application of electromagnetic phenomena in the field of production, transmission, distribution and conversion of electromagnetic energy into other forms of energy: light radiation, thermal, mechanical, chemical ones.

All branches of electrical engineering are interconnected and based on identical regularities that are systematically studied in the fundamental course “Theoretical Fundamentals of Electrical Engineering” (TFEE). This course underlies special electrical engineering subjects.

The proposed TFEE course is a very important and integral part of the engineering training for a future specialist in electric power engineering and electromechanics. This course has always been the starting point for a beginner in electrical engineering education. The discipline of TFEE is based on the courses of general physics, higher mathematics and computing technics. The TFEE course is also valuable for students specializing in other branches of the physical sciences, since the developed models of electrical circuits and electromagnetic fields, the mathematical apparatus used, the analogies are also used in studying other physical phenomena arising in heating engineering, hydraulics, etc.

The main objectives of the TFEE course are the creation of adequate mathematical models of electromagnetic phenomena, experimental study of the features of electromagnetic phenomena that correspond to the essential aspects of physical processes, and the choice of methods for calculating the characteristic electromagnetic processes.

In the structure of the discipline, two separate parts are distinguished: the theory of electrical circuits and the theory of electromagnetic field. These are two fundamental theories that form the basis used in all applied fields of electrical engineering, namely in the electric power industry, electrical machines, automatic regulation and control systems, microcircuitry and power electronics, communication equipment and measuring devices, non-stationary processes in power supply systems and in all other electrotechnical applications.

In the course on the theory of electrical circuits, to facilitate the assimilation of the material, the following sections are singled out: direct current circuits, single-phase and three-phase harmonic current circuits, circuits of non-harmonic currents, circuits with lumped and distributed parameters. Circuit operation modes are analyzed both in stationary and non-stationary modes, when there are transient processes associated with a change in the accumulated electromagnetic energy. On the basis of the theory of electrical circuits, the frequency properties of electrical circuits are investigated in sections dealing with the theory of two-port circuits and the theory of passive filters.

In the theory of the electromagnetic field the following sections are distinguished: electrostatic fields in dielectric and conducting media, magnetic-static fields induced by direct currents and alternating electromagnetic fields in stationary media.

The content of book is based on the author’s experience in teaching the "Theoretical Fundamentals of Electrical Engineering" course to bachelor students majoring in Electrical engineering and Electromechanics at the State Higher Educational Institution “National Mining University” over a long period.

I express my gratitude to the reviewers M.M. Rezinkina, Dr. Sc. (Tech.), Professor, Head of the Department of Theoretical Foundations of Electrical Engineering of the National Technical University "Kharkov Polytechnic Institute"; S.G. Stakhiv, Dr. Sc. (Tech.), Professor, Head of the Department of Theoretical and General Electrical Engineering National University "Lviv Polytechnic", whose critical comments contributed to the improvement of the textbook content.

I express my deepest gratitude to Candidate of Philology, Associate Professor of the Department of Foreign Languages of the State Higher Educational Institution "National Mining University", Associate Professor M.L. Isakova, who kindly took the trouble of editing the manuscript and showed patience, which allowed successfully completing this work and bringing it to publication.

This textbook is recommended for students majoring in Electrical engineering and Electromechanics.

Professor of the Department of Metrology and
Information and Measurement Technologies
of the State Higher Educational Institution
"National Mining University"
V.S. Khilov

В.Хилов

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BRIEF HISTORY

Below is a brief overview of the outstanding scientists in the field of electrical engineering and their basic discoveries and inventions.

1. Thales of Miletus (625-547 years BC) is a Greek philosopher, physicist and mathematician, who first began to conduct electrical experiments. He found that amber being rubbed shows the ability to attract light-weight objects. He found that a magnet attracts iron objects.

2. Willam Gilbert (1544-1603) is an English physicist and physician. He wrote the first book on Electricity and Magnetism (1600), "About the magnetic phenomena, magnetic bodies and about the large magnetic field of the Earth." He constructed a sphere, magnetized it and used it as a model to study the Earth magnetic field. He found that steel is magnetized. He magnetized steel wire by the magnetic field of the Earth. He came to an erroneous assumption that the electrical and magnetic phenomena have nothing in common.

3. Otto Von Guerike (1602-1886) is a German physicist, the mayor of Magdeburg. He built a ball from molten sulfur with a steel axle with winch attached to it. Then, by rotation and friction, he got electrostatic charge. It was the first electrostatic machine, built in 1663.

4. Charles Augustin Coulomb (1736-1806) is a French mathematician, physicist. He discovered the so-called Coulomb's law (1785) about the power of attraction or repulsion of electric charges. He laid the foundation of the modern electrostatics.

5. Luigi Galvani (1737-1798) is an Italian physicist and naturalist. He explored the idea of electric current influence on living organisms (frogs). He put forward the idea that living organisms have an electric current, which is now confirmed.

6. Alessandro Volta (1745-1827) is an Italian physicist, inventor of the first DC power source - "Volta's pile". He discovered contact potential difference of two different metal compounds. Based on this phenomenon Volta built the so-called pile of Volta (1800), which consisted of 20 pairs of silver and zinc disks separated by spacers made of cloth soaked with

salt water. This was the first source of electric DC energy. He built electroscope, invented the flat capacitor, in the mine he discovered the flammable gas - methane, described the project of the future telegraph.

7. Jean-Baptiste Joseph Fourier (1768 –1830) was a French mathematician and physicist born in Auxerre and best known for initiating the investigation of Fourier series and their applications to problems of heat transfer and vibrations. The Fourier transform and Fourier's law are also named in his honour.

8. Hans Christian Oersted (1777-1851) is a Danish physicist, based on the law of electromagnetic induction he built the first machine - a generator of electricity, which made it possible to obtain a large amount of cheap electricity. In 1820, on the lecture in physics he established the effect of electric current on a magnetic needle – he found connection between the magnetic field and electric current.

9. Andre Marie Ampere (1775-1836) is a French physicist, he found the interaction of two current-carrying conductors; wound a coil through which current was passed, and studied its magnetic field; he offered a steel core as the spool to enhance the magnetic field; divided electrotechnics into electrostatics and electrodynamics. He discovered the action of a magnetic field on the current-carrying conductor - Ampere's law. For the first time, he pointed out the connection between the electric and magnetic processes, and proved the idea of magnetism originating from current.

10. George Simon Ohm (1787-1854) is a German physicist. In 1827 he discovered the relationship between current, voltage and resistance which was called Ohm's law. He introduced the concept of resistance as an important characteristic of the conducting medium. At the beginning, this relationship was neglected in physics, and only 16 years later the British physicist Weston confirmed values of the current, voltage and resistance in the electrical circuit following the Ohm's law.

11. Michael Faraday (1791-1867) is an English physicist-experimenter. His research was initially focused on chemistry rather then electricity. In 1831 he discovered the law of electromagnetic induction. This made a revolution in technology, and formed the basis

for the entire electrical engineering, and set the beginning for the second technological revolution (the first was in 1784, connected with the construction of improved steam engine by James Watt, an English inventor). Faraday discovered the law of electrolysis phenomenon, para- and diamagnetism, introduced the concept of the dielectric permeability. Many of his discoveries are related to chemistry and electrolysis.

12. Emil Heinrich Lenz (1804-1865). Born in Tartu (Estonia), worked as a geographer on a ship which sailed around the Earth. In 1828 he was asked to work in the St. Petersburg Academy of Sciences. He studied electric phenomena, which gave him the opportunity to make a number of discoveries: the well-known "rule of Lenz", conversion of electric energy into heat, the dependence of the resistance on the conductor temperature etc.

13. Gustav Robert Kirchhoff (1824-1887) is a German physicist. In 1845, being a university student he offered the mathematical ratio between the currents in the nodes of the circuit as well as voltages and EMFs in the circuit loops that are now called Kirchhoff's laws. He studied telegraph lines and showed that in the lines, other than the direct wave, there is a reverse wave and that they propagate with velocities close to the speed of light.

14. Joseph Henry (1797-1878) is an American physicist. His research is related to electromagnetism. He was the first to construct an electromagnet of considerable force (1828) with lifting capacity of 1,000 kg. He independently discovered the law of electromagnetic induction (Faraday was the first to publish this discovery). He discovered the phenomenon of self-inductance (1832), built a telegraph, determined oscillatory nature of capacitor discharge on resistance and inductance.

15. Nikola Tesla (1856-1943) is an American scientist, electric engineer of Croatian origin, developed a number of multi-phase generators, motors and transformers. In 1888, he discovered the phenomenon of spinning magnetic field. He invented the electric meter, frequency meter and other devices.

16. Michael Dolivo-Dobrovolsky (1862-1919) is a Russian electric engineer. He invented the three-phase

electric motors with slip ring and squirrel cage, ammeter and voltmeter AC, phase meters.

17. James Clerk Maxwell (1831-1879) is a great Scottish physicist. He created the theory of the electromagnetic field, which is presented in the form of equations expressing the basic laws of electromagnetic phenomena. Maxwell came to the conclusion that the alternating electric field causes a magnetic field and an alternating magnetic field is the cause of the electric field. The perturbations thus obtained, are distributing in space in the form of electromagnetic waves with the speed of light. Maxwell deducted electromagnetic waves theoretically, rather than experimentally. In 1864, Maxwell presented his equations describing all electromagnetic phenomena as direct and alternating currents. The existence of electromagnetic waves was confirmed only in 1886 by Hertz’s experiments.

Maxwell's equations are the theoretical basis of electromagnetic field theory and electrodynamics.

18. Heinrich Hertz (1857-1894) is a German physicist. He wrote scientific papers on mechanics and electrodynamics, being one of its founders. He was the first to design and develop the theory of open-vibrator radiating electromagnetic waves. Using a vibrator and a resonator in 1887 he proved experimentally the existence of electromagnetic waves predicted by Maxwell's theory. Hertz observed the emergence, reflection, refraction, interference of electromagnetic waves. He found that the speed of propagation of these waves is equal to the speed of light.

Part I. BASIC THEORY OF THE STEADY-STATE IN ELECTRICAL CIRCUITS

An *electrical circuit* is a collection of electric devices that transmit, distribute and convert electromagnetic energy. Any electrical circuit, as a rule, contains a combination of four elements: energy sources, loads, measuring devices and connecting conductors.

By a *linear electrical circuit* we mean a circuit containing elements whose parameters do not change depending on the magnitude and sign of the flowing current or the applied voltage.

The *steady-state* of the electrical circuit is the state of the circuit, when the effective value of the stored electromagnetic energy in the elements of the circuit does not change with time.

1. BASIC FEATURES AND CALCULATION METHODS OF DC LINEAR ELECTRICAL CIRCUITS

A perfect inductor has zero resistance to DC, and a perfect capacitor has infinitely large resistance to DC in steady-state modes. Therefore, at such operation modes it is necessary and sufficient take into account only perfect ohmic (active) resistance of elements, which greatly simplifies the analysis of the DC circuit without distinguishing by resistance types.

1.1. CURRENT, VOLTAGE, POWER, RESISTANCE AND CONDUCTANCE

Concepts of electric current through the branch, the voltage drop across a circuit branch, generated and consumed power are fundamental in the theory of electrical circuits.

In the theory of electrical circuits, the ordered motion of electric charges under the action of an electric field is referred to as an electric current in a conducting medium.

For example: the conduction current within metals, electrolytes, gases, transfer current through vacuum devices, the bias current through dielectrics, etc.

Quantitatively, the electric current at any time is characterized by a scalar algebraic quantity $i=i(t)$, that is value with a sign.

Quantitatively, the instantaneous current value is characterized by the rate of the charge change over time:

$$i=\lim_{\Delta t\to 0}\frac{\Delta q}{\Delta t}=\frac{dq}{dt}, \tag{1.1}$$

where Δq is the change of electric charge over time Δt through conductor cross-section.

The value of the current at any given instant t is called instantaneous and is denoted by the lowercase letter of the Latin alphabet $i=i(t)$.

In SI system, the charge is measured in coulombs (C), time – in seconds (s), current – in amperes (A).

The electric currents and voltages will be further referred to as currents and voltages.

In accordance with the above definition, the concept of "current" may be used in two meanings: current as a physical process and current as a quantitative characteristic instead of "current strength".

As a function of time, the current $i(t)$ can be positive or negative.

It is commonly supposed that the current $i(t)$ is positive if the movement of positively charged particles coincides with the pre-selected direction, and negative otherwise. Selecting the direction of the current is performed arbitrarily, and positive direction of current is shown with the arrow in the circuit (Fig. 1.1,*a*).

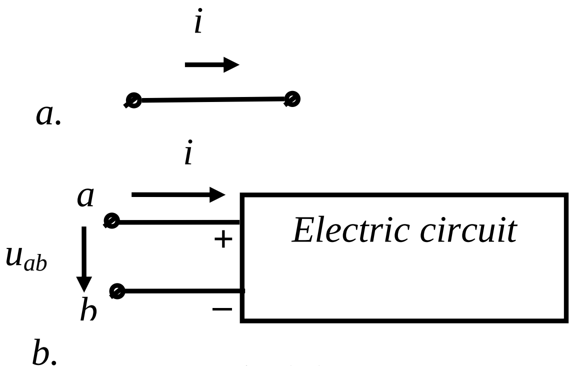


Fig.1.1

In electrical engineering, by the true direction of current is considered to be the direction of positive charges motion, the current flows from the anode (positive terminal indicated by the "+" sign) to the cathode (the negative terminal indicated by the "–" sign).

The electric voltage between two points of an electrical circuit is determined by the amount of energy required to move a unit charge from one point to another:

$$u = \lim_{\Delta q \to 0} \frac{\Delta W}{\Delta q} = \frac{dW}{dq}, \qquad (1.2)$$

where W is the energy of electric field.

The voltage measurement unit in SI system is volt (V).

In potential electric field the voltage between two points coincides with the value of potentials difference between them. For example, the voltage between terminals *a* and *b* of a circuit is shown in Fig.1.1,*b*

$$u_{ab}=V_a-V_b, \qquad (1.3)$$

where V_a and V_b are the terminal potentials of *a* and *b* of the circuit.

Electric potential is a scalar energy characteristic, quantitatively defining the potential energy, which has a unit positive test charge located at a given point of the field. The unit of the potential in the International System of Units (SI) is the volt.

The voltage value at any given instant t is called instantaneous and is denoted by the lowercase letter of the Latin alphabet $u=u(t)$.

Being a scalar value, $u(t)$ can be both positive and negative. To uniquely determine the voltage sign direction, the positive reference direction is chosen which is shown by the arrow in the circuit, Fig.1.1,*b* directed from one terminal of the circuit to the other. For definiteness, we assume that the positive direction of reference coincides with the direction of the arrow from the higher potential, i.e. "+" to the lower one, i.e. "–", Fig.1.1,*b*. With this choice the positive counting directions of the voltage and current coincide, as a positive direction of the voltage counting u_{ab} corresponds to the movement direction of positively charged particles from the higher potential V_a (+) to lower one V_b (–). Obviously, $u_{ab}=-u_{ba}$. The voltage across the branch of a circuit through which the current flows is frequently called a "voltage drop".

Electric energy consumed by the movement of a unit positive charge between two points of the circuit branch with voltage u (potential difference), over the time t will be determined according to relations (1.1) and (1.2) with the equation

$$W=\int_{0}^{q} udq=\int_{-\infty}^{t} uidt\,, \tag{1.4}$$

where it is assumed that $W=0$ when $t\rightarrow-\infty$.

Energy in SI system is measured in joules (*J*).

Derivative from energy with respect to time determines the instantaneous *power consumed* by the circuit branch:

$$p=dW/dt=ui\,. \tag{1.5}$$

Power is measured in watts.

The sign of power p is determined by the signs of voltage and current. If $p > 0$, power is consumed by the circuit branch and when $p < 0$, power is delivered by the circuit branch.

The energy consumers are characterized by electric resistance, i.e. counteraction of a branch to the electric current.

The resistance R to DC is called the perfect (or ohmic, active) resistance, which is measured in Ohms.

The reciprocal value of the DC ohmic resistance R is called conductance G and is measured in Siemens (*S*) or in mho (ohm spelled back-ward or reciprocal ohm with symbol of inverted omega ℧)

$$G=1/R. \tag{1.6}$$

By the behavior over time, there are distinguished constant, harmonic, periodic sinusoidal, non-periodic currents and voltages. In many cases, for example in circuits with distributed parameters, currents and voltages can be not only functions of time, but also functions of the spatial coordinates.

DC and voltage that is unchanged over the time are denoted by capital letters of the Latin alphabet: *I*, *U*, respectively.

1.2. EQUIVALENT CIRCUITS FOR ELECTRICAL ENERGY SOURCES

To obtain the mathematical models of real (practical, actual) energy sources, the concepts of ideal source of electromotive force (EMF) and current are introduced.

An ideal source of EMF is a source of electric energy which delivers unchanged value of the potential difference at the output terminals, its internal resistance strictly equals to zero, so that the output voltage is independent of the current through it.

An ideal current source is such electric energy source that delivers output current with constant value, its internal resistance is strictly equal to infinity and therefore output voltage is directly dependent on the value of the load resistance.

In Fig. 1.2 are schematically shown the symbolic notation of the ideal sources of EMF (Fig. 1.2,*a*) and of ideal current source (Fig. 1.2,*b*).

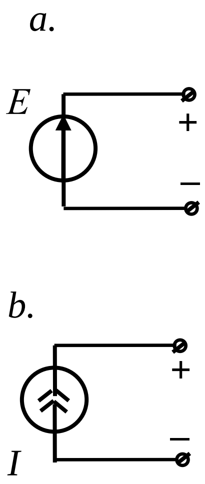


Fig.1.2

We will obtain the mathematical model for the real electric energy source.

The initial source is represented by the active two-terminal network *A*, Fig. 1.3. Here, under the active two-terminal network or active one-port network *A* we mean the part of the circuit which is considered with respect to two terminals, and which has at least one source of energy. If a two-terminal network does not contain sources of energy, it is a passive one-port network.

The operating modes of active one-port network will be set by means of two switches S_1 and S_2. Four combinations of switches position are possible.

1. The switches S_1 and S_2 are opened. The active one-port network operates in the open-circuit mode.

The open-circuit mode of the power source is the mode when the load resistance equals infinity, the output current is equal to zero and the output voltage of the power source is equal to the EMF.

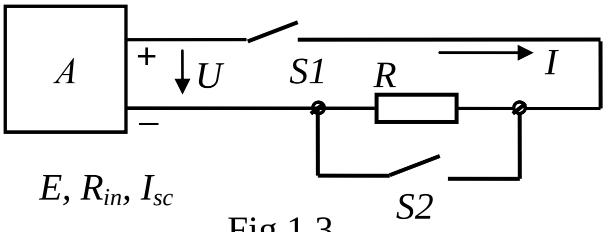


Fig.1.3

2. The switches S_1 and S_2 are closed. The active one-port network operates in the short-circuit mode.

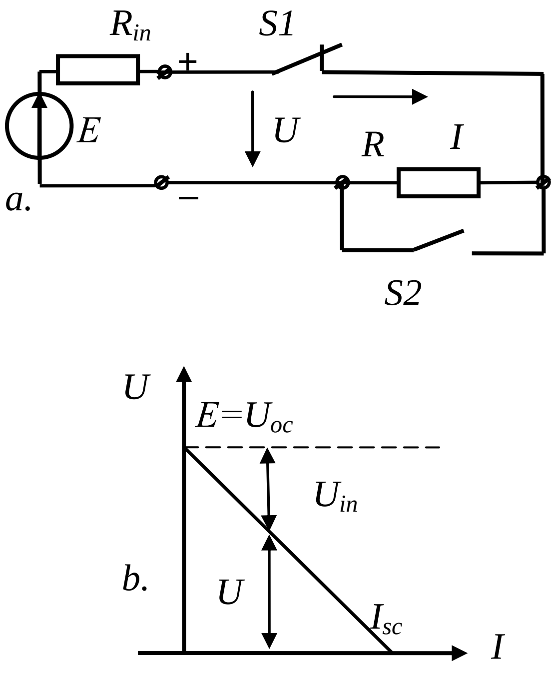


Fig.1.4

The short-circuit mode of the power source is the mode when the load resistance is zero, the output current is limited by power source internal resistance only, and the output voltage of the power source is equal to zero.

3. The switch S_1 is opened, and S_2 is closed. The active one-port network operates in the open circuit mode, as in case 1.

4. The switch S_1 is closed, and S_2 is opened. The active one-port network operates under load. The less the load resistance, the greater the load current through active one-port network.

The mode of the power source under the load is the mode when the load resistance is less than infinity but greater than zero, the output current is limited by the sum of power source internal resistance and of load resistance, and the output voltage of the power source becomes less than the EMF of one-port network by value of the drop voltage across the internal resistance.

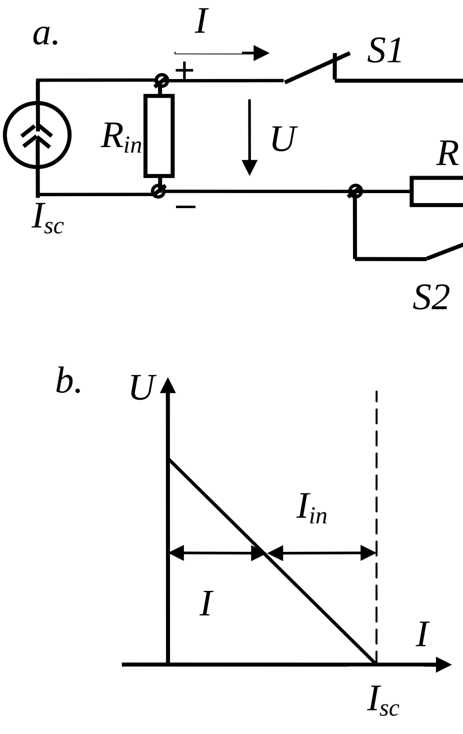


Fig.1.5

We will consider case 4 when the real power source operates under the load. At the increase of the load current through the active one-port network, its output voltage is reduced, accordingly. A mathematical model is represented for this case, which is described by linear algebraic equation

$$U=E-IR_{in} \tag{1.7}$$

where $U=IR$ is the voltage drop across load resistance; $\Delta U_{in}=IR_{in}$ is the voltage drop across internal resistance R_{in} of active one-port network; E is EMF.

After transformation we obtain

$$I=E/(R+R_{in}).$$

According to Ohm's law the above equation is assigned to equivalent circuit comprising the deal EMF source, internal resistance and load resistance connected in series, Fig.1.4,*a*.

Let us find a mathematical model of the real source of energy using an ideal current source. To do this, the original equation (1.7) can be represented through the voltage drop

$$IR=E-IR_{in},\ IR/R_{in}=E/R_{in}–I,\ I_{in}=I_{sc}–I \text{ or } I_{sc} = I_{in} +I \quad (1.8)$$

where $I_{in}=IR/R_{in}=U/R_{in}$ is internal leakage current in current source; $I_{sc}=E/R_{in}$ is short-circuit current of real energy source.

According to the first Kirchhoff's law, the equivalent circuit, which contain ideal current source, Fig.1.5, is assigned to the equation (1.8).

When the load resistance R increases, the open circuit voltage U_{oc} increases. Therefore, in actual circuits because of the hazard of isolation breakdown it is not recommended to disconnect the output terminals of current sources.

Thus, in the circuit, which is calculated, any actual source of electromagnetic energy can be presented by an equivalent mathematical model having ideal source of EMF and internal resistance connected in series or having ideal current source and internal resistance connected in parallel.

The greater the load resistance exceeds internal resistance of energy source, the closer characteristics of the real source approaches to the ideal source of EMF. In case of the inverse ratio of resistances, the characteristics of the real source are closer to the perfect current source.

Using the parameters E_{OC}, I_{SC}, R_{INT}, real power source can be represented via two equivalent circuits for calculus, whose parameters are connected by relation $E_{\text{OC}}=I_{\text{SC}}R_{I\text{NT}}$.

Therefore in a circuit analysis it is possible to substitute an ideal EMF source having internal resistor connected in series with an ideal current source with internal resistor connected in parallel or vice versa.

1.3. VOLTAGE DROP ACROSS THE CIRCUIT SECTION. OHM'S LAW

A voltage drop across the section or element of the circuit is the electric potential difference occurring under action of flowing current between given points of the circuit section *R* element.

If current *I* flows through resistor *R* from left to right, then the resistor potential of the left lead V_a is higher than that of the right lead V_b ($V_a > V_b$) by the value of voltage drop across the resistor *R* (*IR*), Fig.1.6,*a*

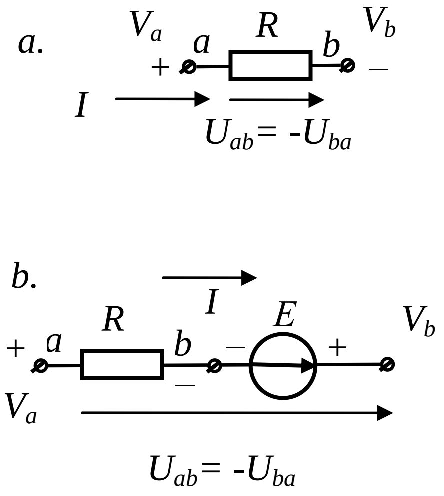


Fig.1.6

$$V_a = V_b + IR \text{ or } V_a - V_b = IR = U_{ab} = -U_{ba}.$$

Hence we obtain Ohm's law for the circuit section without power source

$$I = U / R.$$

In the presence of an ideal source of EMF (Fig. 1.6, *b*) Ohm's law is transformed to the form

$$V_a = V_b + IR - E \text{ or } V_a - V_b = IR - E = U_{ab} = -U_{ba}.$$

Hence we obtain Ohm's law for the circuit section having a power source

$$I = (U \pm E)/R.$$

The above equation is written down in general form. The plus sign is selected when the direction of current flow coincides with the direction of the EMF. If they do not coincide, the minus sign is chosen.

Current *I* through the section of a complicated circuit comprising power sources may be directed from the point *a* of the branch to point *b* and vice versa. If actual current direction is not known in advance, then to draw up a design equation it is necessary to choose the current direction arbitrarily. If *I* is negative as a result of current calculation, it means that actual current direction does not coincide with previously chosen positive direction.

1.4. POTENTIAL DISTRIBUTION ALONG THE ELECTRICAL CIRCUIT. POTENTIAL CIRCLE

Potential circle is a graphical representation of the change of potential along any part of the circuit as a function of the resistance of this part.

Suppose we have an isolated electrical circuit loop with two ideal power sources connected in series and oppositely, as well as three resistors, Fig. 1.7,*a*.

Given that any single point of the circuit can be grounded, we connect point *a* to ground. If the potential of the point *a* previously was nonzero, then after its grounding the point *a* potential has zero value, i.e. the potential of point *a* is changed by value ±V_a. All the rest of the circuit points changed the potentials proportionally by the value ±V_a. Potential difference between all points of the circuit remains constant. The current through the loop is not changed

$$I=(E_1-E_2)/(R_1+R_2+R_3).$$

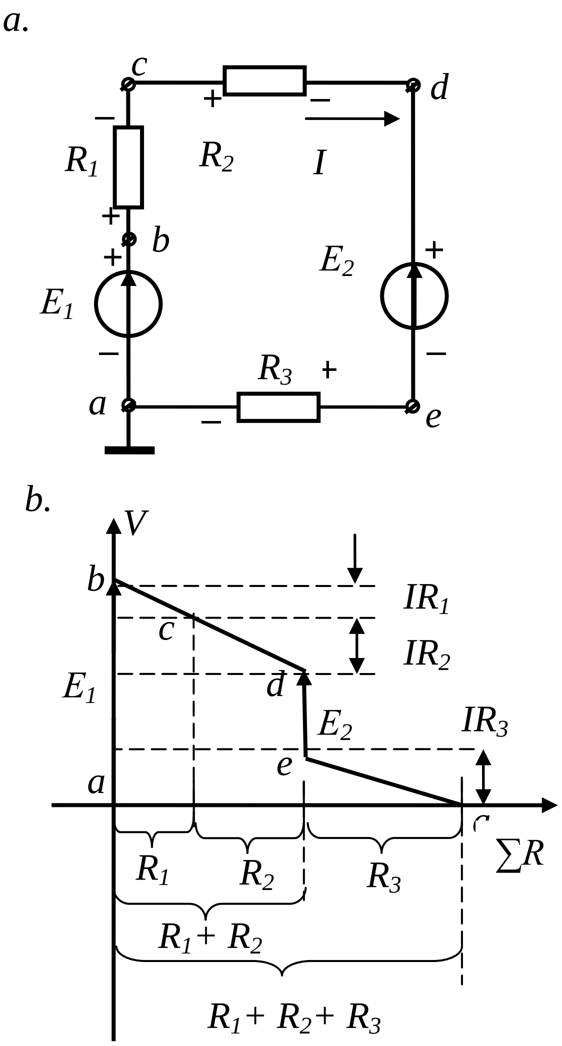


Fig.1.7

Potentials calculation begins from the starting point *a* where $V_a=0$. The potential value of this point *a* is placed at origin of the potential circle, Fig. 1.7,*b*.

We move clockwise in the circuit in Fig.1.7,*a*. When moving from point *a* to point *b* potential in circuit increases and in point *b* takes on the value

$$V_b= V_a +E_1.$$

The internal resistance of ideal source of EMF E_1 is zero; therefore in potential circle the value of the point *b* potential is plotted along the ordinate axis.

We are moving from point *b* to point *c* through the resistor R_1 in the direction of current flow. In this case, the potential of point *a* is less than the potential at point *b* by the voltage drop

$$V_c = V_b - IR_1.$$

The values of resistance R_1 and the potential V_c in potential circle are plotted in Fig.1.7,*b*.

From point *c* we are moving to point *d*. Potential in this part of circuit decreases by the voltage drop value

$$V_d = V_c - IR_2.$$

Position of point *d* is plotted according to the calculated coordinates in the potential circle, Fig.1.7,*b*.

From point *d* we are moving to point *e*. Resistance in this loop part is equal to zero, and potential decreases by value E_2

$$V_e = V_d - E_2.$$

We plot the coordinates of the point *e* in the potential circle.

Moving from points *e* to *a* reduces the potential to zero value, Fig. 1.7,*b*

$$V_a = V_d - IR_3.$$

1.5. ENERGY BALANCE IN ELECTRICAL CIRCUIT. TELLEGEN'S THEOREM

Tellegen's theorem: the algebraic sum of voltages and currents products on the circuit elements satisfying to Kirchhgoff's laws is equal to zero.

Consequence of Tellegen's theorem is the following: in any electrical circuit, the powers balance is fulfilled, i.e. quantities of delivered and consumed powers are always equal.

We prove this condition on the example of DC generator *GDC* and accumulator battery *ACB,* which are oppositely connected, Fig.1.8.

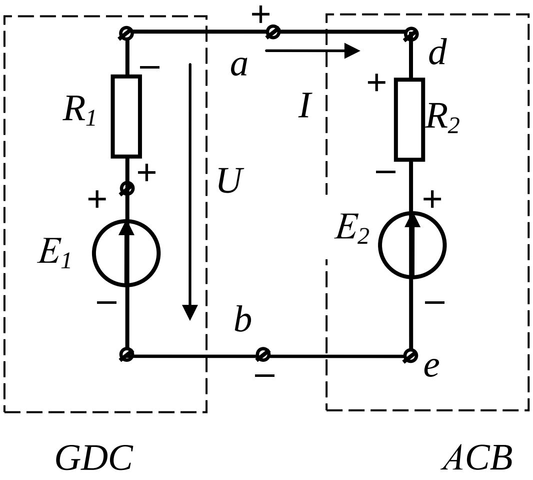


Fig.1.8

Equivalent circuit of a DC generator *GDC* is represented by an ideal source of EMF E_1 with internal resistance R_1. Battery is represented by an ideal source of EMF E_2 with internal resistance R_2. EMFs of machine and battery are directed towards each other. Depending on the ratio of the EMFs E_1 and E_2 the following operation modes are possible: electrical balance when E_1 and E_2 are equal, $E_1 < E_2$ – then energy is transferred from the battery to electric machine, $E_1 > E_2$ – the energy is transmitted from the DC generator to accumulator battery.

To be specific, we assume that $E_1 > E_2$. Under this condition, the actual current direction *I* coincides with the direction of EMF E_1. Voltage *U* across the external terminals of both sources is less than EMF E_1 by internal voltage drop IR_1 in the generator, and greater than EMF E_2 by the internal voltage drop IR_2 across the battery.

In the chosen ratio of EMFs, the value of potential in point *a* is greater than potential value in point *b*

$$V_a = V_b + E_1 – IR_1 \text{ and } V_a = V_b + E_2 + IR_2.$$

The voltage $U = V_a – V_b$, therefore

$$U = E_1 – IR_1 \text{ and } U = E_2 + IR_2.$$

For the transition from voltages balance to the powers balance, it is necessary to multiply the last equations by circuit current value *I*

$$UI = IE_1 – I_2R_1 \text{ and } UI = IE_2 + I_2R_2,$$

where $UI=P$ is power transmitted through terminals *a* and *b* from the generator to battery; $IE_1=P_{GDC}$ is electromagnetic power delivered by GDC generator; $IE_2=P_{ACB}$ is electromagnetic power consumed by the battery charging; $I^2R_1=P_1$, $I^2R_2=P_2$ are the heat loss powers at internal resistances R_1 and R_2 , respectively.

Thus, the algebraic sum of powers $P_{GDC} – P_{ACB}$ delivered by power sources is equal to the sum of heat losses in electrical circuit I^2R_1, I^2R_2

$$P_{GDC} - P_{ACB} = I^2R_1 + I^2R_2,$$

i.e. delivered power is always equal to consumed one, quod erat demonstrandum.

Power loss in ohmic resistance is always positive, i.e. always consumed. Powers of energy sources can be not only delivered but also consumed. To draw up a powers balance it is necessary to take into account the energy source operating modes.

To determine the operation mode of any part of the circuit it is necessary and sufficient to compare the actual directions of the current and voltage, if they match than the part of the circuit consumes power, otherwise the power is delivered.

The consumed power is taken into account in calculations with plus sign and delivered power – with minus sign. This is known as the power sign convention.

Based on the power balance one can assess the correctness of calculations of current and voltage circuit.

In the circuit at transition from EMF sources to current sources and vice versa, powers of initial and transformed sources may not equal. In addition, transformed source may change the work mode with respect to initial source.

1.6. CALCULATION METHODS OF COMPLICATED OHMIC CIRCUITS

1.6.1. CIRCUITS CALCULATION BY TRANSFORMATION METHOD

Electrical circuit transformation is based on the equivalence condition under which the voltages and currents in the circuit branches not affected by the transformation remain unchanged before and after the transformation.

In electrical circuits there can be distinguished simplest parts with the following connection types: series, parallel, wye and delta.

1. Series connection of resistors.

The *condition of series connected* elements in the circuit branch is the equality of current value through each element.

Suppose that the initial part of the circuit contains three resistors R_1, R_2, R_3 through which current I is flowing, Fig.1.9.*a*. Applied voltage U is the sum of the voltage drops across each separate resistor

$$U=U_1+U_2+U_3$$

or through voltage drops across the elements of the segment

$$IR_e=IR_1+IR_2+IR_3,$$

where the applied voltage U is presented through the input current I and certain equivalent resistor R_e, Fig.1.9.*b*.

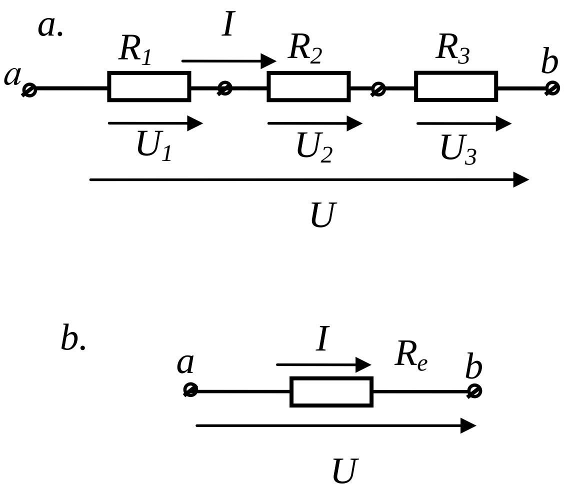


Fig.1.9

From this, one can receive

$$R_e=R_1+R_2+R_3.$$

Generalizing the result for equivalent resistor with n resistors connected in series, we obtain

$$R_e = \sum_{k=1}^{n} R_k$$

where k is the present number of a series resistor.

When the elements are connected in series the equivalent resistance is equal to resistances sum.

The equivalent resistance is always larger than any separate resistance.

2. Parallel connection of resistors.

The *condition of parallel connected* elements in the circuit part is the equality of voltage values across each element.

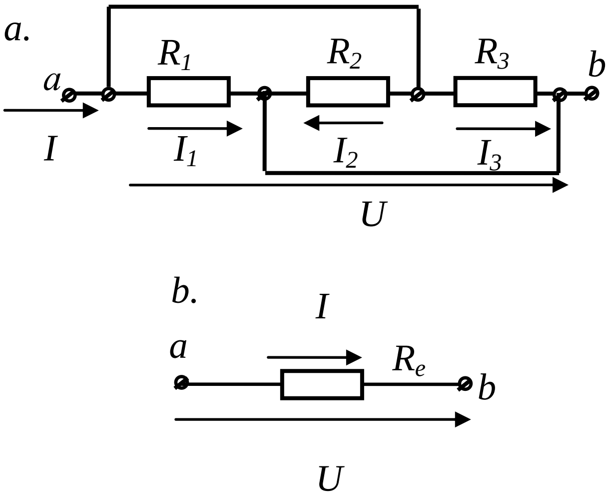


Fig.1.10

Let us assume that at the initial circuit part of network in Fig.1.10.*a* the resistors R_1, R_2, R_3 are connected in parallel, because the same voltage U is applied to every branch. Input current I is equal to the sum of currents through branches I_1, I_2, I_3. The currents through applied voltage and the branch resistors can be expressed as

$$I = I_1 + I_2 + I_3 = \frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3} = \frac{U}{R_e},$$

where the input current I is expressed through the applied voltage U and certain equivalent resistor R_e, Fig.1.10.*b*.

From the last equation one can receive

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_e}.$$

Generalizing result for equivalent resistor with n resistors in parallel, we obtain

$$\frac{1}{R_e} = \sum_{k=1}^{n} \frac{1}{R_k},$$

where k is the present number of a parallel resistor.

The above equation can be written in terms of branch conductivity

$$G_e = \sum_{k=1}^{n} G_k .$$

If the two resistors are in parallel, the equivalent resistance is determined by the relation

$$R_e = \frac{1}{G_e} = \frac{R_1 R_2}{R_1 + R_2}.$$

In case of equality of the resistances R_1 and R_2, equivalent resistance is equal to half the resistance value of R_1 or R_2.

The equivalent resistance is always smaller than the smallest of all the resistances connected in parallel.

3. Delta-wye conversion. Transformations of the initial circuit with resistors connected in delta into equivalent wye circuit.

Three or more elements connected in one node form a threebeam or multibeam circuit collected in wye-connection.

Three or more elements that are connected into a circuit with three or more nodes form a circuit collected in a delta- or polygon-connection.

Further on, we consider only delta and threebeam wye-connection circuits.

For equivalence of the initial resistors connected in delta and equivalent resistors connected in wye, it is necessary and sufficient that the resulting resistances between each pair of points with the third disconnected point are equal in both circuits.

We apply this rule for points *1* and *2* when points *3* is disconnected, Fig.1.11

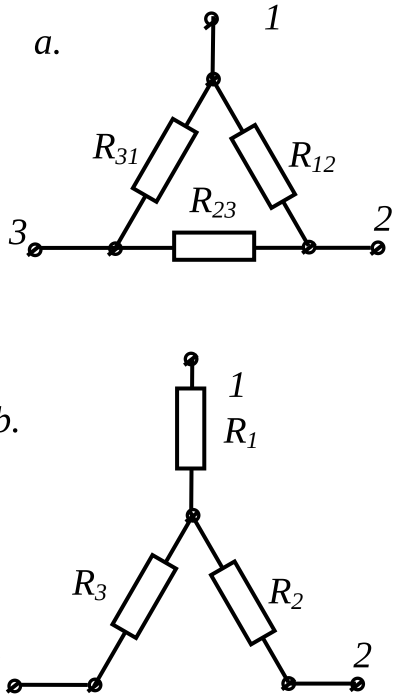


Fig.1.11

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}. \qquad (1.9)$$

Similar equations for the points *2, 3* and *3, 1* can be written, if we use a circular permutation of the indices *1, 2, 3*

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}}, \qquad (1.10)$$

$$R_{31} + R_1 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}}. \qquad (1.11)$$

Adding (1.9), (1.11) and subtracting (1.10) results we obtain the desired resistance connected in wye R_1

$$R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}.$$

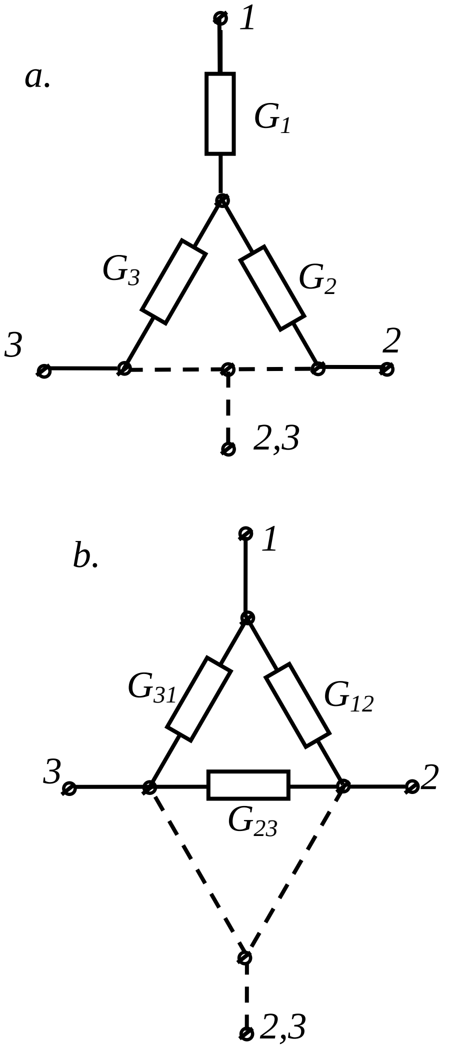


Fig.1.12

Further, using a circular permutation of the indices, we find the rest of resistors of equivalent wye-connected circuit

$$R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}},$$
$$R_3 = \frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}.$$

We do not need to memorize these equations. To transform a delta circuit to wye, we create an extra node *n* and follow this *delta-wye conversion rule*: each resistor in the wye circuit is the product of the resistors in the two adjacent delta circuit branches, divided by the sum of the three wye resistors.

4. Wye-delta conversion. Transformations of the initial circuit with resistors connected in wye into equivalent delta circuit.

For equivalence of the initial wye connected resistors and the equivalent delta connected resistors it is necessary and sufficient to connect mentally two pairs of similar points and equate conductivities between this common point and the third point.

This rule is applied to points 2 and 3, Fig.1.12

$$G_{12} + G_{31} = \frac{G_1(G_2+G_3)}{G_1+G_2+G_3}. \quad (1.12)$$

Similar equations for the points *1*, *2* and *3,1* can be written, if we use a circular permutation of the indices *1,2, 3*

$$G_{23} + G_{12} = \frac{G_2(G_3+G_1)}{G_1+G_2+G_3}, \quad (1.13)$$
$$G_{31} + G_{23} = \frac{G_3(G_1+G_2)}{G_1+G_2+G_3}. \quad (1.14)$$

Adding (1.12), (1.14) and subtracting (1.13), we obtain the desired conductance in delta-connected circuit G_{12}

$$G_{12} = \frac{G_1 G_2}{G_1 + G_2 + G_3}.$$

Further, using a circular permutation of the indices we find the rest of conductance of equivalent delta-connection

$$G_{23} = \frac{G_2 G_3}{G_1 + G_2 + G_3},$$
$$G_{31} = \frac{G_3 G_1}{G_1 + G_2 + G_3}.$$

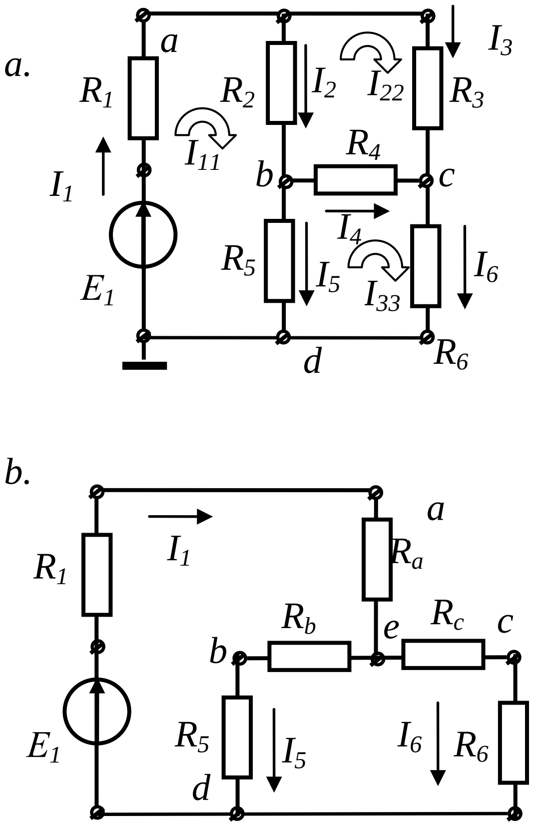


Fig.1.13

We do not need to memorize these equations. To transform a wye circuit to delta follow to *wye-delta conversion rule*: each conductor in the delta circuit is the product of the conductors in the two adjacent wye network branches, divided by the sum of the three wye conductors.

When you transform a wye circuit into an equivalent delta, the common node in wye circuit disappears.

The effectiveness of transformation method will be shown by example calculation of the input resistance in bridge circuit, Fig.1.13.

In the initial circuit Fig.1.13.*a* there are no resistors connected in series or parallel. To find the input resistances of the circuit, the resistors R_2, R_3, R_4 connected in delta circuit of the initial circuit are transformed into the wye connection resistors R_a, R_b, R_c of equivalent circuit.

In the transformed circuit Fig.1.13.*b* series and parallel connected elements appear in the circuit branches. The equivalent input resistance of the circuit relative to power supply is

$$R_e = R_1 + R_a + \frac{(R_c + R_6)(R_b + R_5)}{R_c + R_6 + R_b + R_5}.$$

It should be noted that the circuit in Fig.1.13.*a* is a four-arm DC Wheatstone bridge with two diagonals and four arms. The resistors R_2, R_3, R_5, R_6 form the bridge arms – a quadrangular electric loop, with one of diagonals having power supply E_1, and the other having a load resistor R_4. Bridge circuit has the following property: under equal potentials of nodes *b* and *c*, the current through the diagonal of the bridge with a resistor R_4 equals zero and the bridge will be in a balanced state. The equilibrium condition of the bridge is defined by equality of resistances ratio in the bridge arms

$$R_2/R_5 = R_3/R_6.$$

When the balance of the bridge is disturbed, then the potentials of nodes *b* and *c* are not equal and current flows through a diagonal with a resistor R_4.

1.6.2. CIRCUITS CALCULATION USING KIRCHHOFF’S LAWS

In any junction of electrical circuit the quantity of entering charges is equal to the quantity of leaving charges (continuous charge law). The consequence of this regularity is the Kirchhoff's first law which is formulated relatively of circuit node.

A node is a point of connection of three or more branches.

The branch is a series connection of elements between two nodes carrying the same current flowing through them.

First Kirchhoff's law (Kirchhoff's current law, KCL): the algebraic sum of currents through branches that converge into one electrical circuit node is equal to zero

$$\sum_{k=1}^{n} I_k = 0,$$

where k – is the present number, n – is the number of branches connected to the node and I_K – is the k-th current entering (or leaving) the node.

The *sign rule of KCL*: the currents directed to the node (entering) are taken into account in the equations with the like signs, and directed away from the node (leaving) – are taken into account with the opposite signs.

The number of simultaneous equations under KCL: under the first Kirchhoff's law the number of linear independent equations that can be drawn up is one less than the number of nodes in the circuit.

Along any closed electrical circuit segment the algebraic sum of voltage drops is equal to zero. Consequence of this regularity is the second Kirchhoff's law with respect to the circuit loop.

A loop is any closed path along electrical circuit that does not pass through any node more than once.

A mesh it is an elementary loop which does not contain any other loops within it.

Second Kirchhoff's law (Kirchhoff's Voltage Law KVL): the algebraic sum of the EMFs in the loop is equal to the algebraic sum of the voltage drops across the elements of this loop

$$\sum_{k=1}^{n} E_k = \sum_{k=1}^{n} I_k R_k \,,$$

where k is the branch present number in the loop; n – the quantity of branches in the loop.

The *sign rule of KVL*: signs of EMF and voltage drops in algebraic sums are selected in accordance with the chosen positive direction of the loop traversal, that is, if EMFs and voltage drops coincide with the chosen direction of traversal they are taken into account in the equations with identical signs, otherwise they are taken into calculation with opposite signs.

Under the second Kirchhoff's law there can be drawn up as many linear simultaneous equations as there are independent loops in the circuit. The independent loop is selected using one of the following rules:

– the form of mesh is chosen;

– the chosen loop must comprise at least one previously undescribed branch.

Order of calculation by using KCL and KVL: the circuit calculation under Kirchhoff's laws is carried out in the following order:

– the number of nodes *q* and branches *p* in the circuit are calculated;

– current directions through branches are chosen arbitrarily;

– independent loops are determined, in which positive directions of traversal around the loops are chosen arbitrarily;

– under the first Kirchhoff's law, we draw up *q-1* equations, and using the second Kirchhoff's law, we draw up *p - (q-1)* simultaneous equations;

– *p* unknown currents are defined by solving the system of linear simultaneous equations.

We draw up calculated system of equations for the bridge circuit Fig.1.13.*a*. The quantity of nodes and branches in circuit are *p=4*, *q=6*. We choose the clockwise direction of traversal around the loops. The circuit has three independent loops chosen in the form of the meshes: $E_1 – R_1 – R_2 – R_5$, $R_2 – R_3 – R_4$, $R_4 – R_5 – R_6$.

Under the first Kirchhoff's law we draw up *p–1=3* simultaneous equations for the circuit nodes *a*, *b*, *c*

$$\begin{aligned} I_1 - I_2 - I_3 &= 0; \\ I_2 - I_4 - I_5 &= 0; \\ I_3 + I_4 - I_6 &= 0. \end{aligned}$$

Under the second Kirchhoff's law we draw up *p-(q-1) =3* linear simultaneous equations for the chosen meshes

$$\begin{aligned} R_1 I_1 + R_2 I_2 + R_5 I_5 &= E_1; \\ R_3 I_3 - R_2 I_2 - R_4 I_4 &= 0; \\ R_4 I_4 - R_5 I_5 + R_6 I_6 &= 0. \end{aligned}$$

The resulting equations under the Kirchhoff's laws are solved jointly with respect to branch currents.

1.6.3. MESH METHOD OF CIRCUIT CALCULATION (MAXWELL'S METHOD)

Mesh currents analysis allows reducing the number of equations to be solved from p to the number of independent loops p - (q-1) (p, q – the number of branches and nodes in the circuit).

The mesh currents method is based on introducing some conditional mesh current in every independent loop, the direction is chosen coinciding with the direction of traversal around the loop.

Algebraic sum of all the EMFs in the loop along the traversal direction is called loop EMF, the sum of all loop resistances is called eigen loop resistance, and resistance between loops is adjacent loop resistance.

For each loop, second Kirchhoff's law is applied, which leads to drawing up p-(q-1) equations with respect to mesh currents

$$\left.\begin{array}{c} R_{11}I_{11} - R_{12}I_{22} - \cdots = E_{11} \\ -R_{21}I_{11} + R_{22}I_{22} - \cdots = E_{22} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{array}\right\},$$

where I_{11}, I_{22}, …, I_{kk} – are mesh current; R_{11}, R_{22}, …, R_{kk} – are eigen mesh resistance; R_{12}, R_{21}, …, R_{km} – are adjacent mesh resistance; E_{11}, E_{22}, …, E_{kk} – are loop EMFs.

The mesh currents flowing through the external branches are actually existing ones flowing through these branches and mesh currents of internal branches are fictitious quantities introduced for convenience of calculation. The actual currents through the internal branches are found as the algebraic sum of the mesh currents that are closed through these branches.

Thus, the calculation of branch currents is done using the following rule (*the rule of transition from the mesh current to the branch current*): through nonadjacent branches the current is equal to mesh current, and through adjacent branches the current is equal to the algebraic sum of the mesh currents that are closed through this branch.

We apply the principles of calculation using mesh current method to the initial circuit Fig.1.13,*a*. The mesh currents I_{11}, I_{22}, I_{33} are introduced into every circuit loop – and we assume they are directed along loop traversal (as indicated by thickened arrows). We apply Kirchhoff's voltage law for every mesh current

$$\left.\begin{aligned} R_{11}I_{11} - R_{12}I_{22} - R_{13}I_{33} = E_{11} \\ -R_{21}I_{11} + R_{22}I_{22} - R_{23}I_{33} = E_{22} \\ -R_{31}I_{11} - R_{32}I_{22} + R_{33}I_{33} = E_{33} \end{aligned}\right\},$$

where eigen R_{kk}, adjacent R_{km} mesh resistances and loop EMFs E_{kk} are equal:

$$R_{11}=R_1+R_2+R_5;\ R_{22}=R_2+R_3+R_4;\ R_{33}=R_4+R_5+R_6;$$
$$R_{21}=R_{12}=R_2;\ R_{31}=R_{13}=R_5;\ R_{23}=R_{32}=R_4;\ E_{11}=E_1;$$
$$E_{22}=0;\ E_{33}=0.$$

The solution of the drawn equations set gives three mesh currents I_{11}, I_{22}, I_{33}. From the mesh currents, one can pass to the branch currents

$$I_1=I_{11};\ I_2= I_{11}–I_{22};\ I_3= I_{22};$$
$$I_4=I_{33}–I_{22};\ I_5= I_{11}–I_{33};\ I_6= I_{33}.$$

If along with EMF sources the circuit contains branches with current sources, then it is necessary to choose independent loops so that current source is included only in one loop. The number of loop equations in that case decreases by the number of current sources.

1.6.4. CIRCUITS CALCULATION USING NODAL POTENTIALS METHOD

Nodal potentials method allows bringing down the number p of equations to be solved to one less than the number of nodes, i.e. $q-1$ (p, q – the number of branches and nodes in the circuit).

The method is based on the calculation of the potential at the nodes of the circuit with respect to the reference node. Then, the currents or voltages in corresponding branches are calculated under Ohm's law.

Let’s consider the principles of calculation using method of nodal potentials by the example of finding the currents in the circuit, Figure 1.13,*a*. Node *d* is chosen as the reference or datum node. We ground node *d*. A potential equation for each branch is drawn up, from which we find the currents using the branches parameters

$$\begin{aligned}
&V_d = 0; V_d = V_a + R_1 I_1 - E_1;\\
&V_a = V_b + R_2 I_2; V_a = V_c + R_3 I_3;\\
&V_b = V_c + R_4 I_4; V_b = V_d + R_5 I_5;\\
&V_c = V_d + R_6 I_6.
\end{aligned}$$

From the obtained potential equations we define branch currents

$$\begin{aligned}
&I_1 = \left(V_d - V_a + E_1\right)/R_1 = (-V_a + E_1)G_1;\\
&I_2 = (V_a - V_b)/R_2 = (V_a - V_b)G_2;\\
&I_3 = (V_a - V_c)/R_3 = (V_a - V_c)G_3;\\
&I_4 = (V_b - V_c)/R_4 = (V_b - V_c)G_4;\\
&I_5 = (V_b - V_d)R_5 = (V_b - V_d)G_5;\\
&I_6 = (V_c - V_d)/R_6 = V_c G_6.
\end{aligned}$$

For nodes *a*, *b*, *c* under the first Kirchhoff's law we have

$$\begin{aligned}
&I_1 - I_2 - I_3 = 0;\\
&I_2 - I_4 - I_5 = 0;\\
&I_3 + I_4 - I_6 = 0.
\end{aligned}$$

We express the currents in terms of the found potential values at the nodes

$$\begin{aligned}
&(-V_a + E_1)G_1 - (V_a - V_b)G_2 - (V_a - V_c)G_3 = 0;\\
&(V_a - V_b)G_2 - (V_b - V_c)G_4 - (V_b - V_d)G_5 = 0;\\
&(V_a - V_c)G_3 + (V_b - V_c)G_4 - V_c G_6 = 0.
\end{aligned}$$

Grouping the terms in the above equations with respect to circuit node potentials results in

$$\begin{aligned}
&V_a(G_1+G_2+G_3)-V_bG_2-V_cG_3=E_1G_1;\\
&-V_aG_2+V_b(G_2+G_4+G_5)-V_cG_4=0;\\
&-V_aG_3-V_bG_4+V_c(G_3+G_4+G_6)=0.
\end{aligned}$$

We introduce concepts of eigen conductance as the sum of the conductance of the branches connected to the node

$$\begin{aligned}
&G_{11}=G_1+G_2+G_3; G_{22}=G_2+G_4+G_5;\\
&G_{33}=G_3+G_4+G_6,
\end{aligned}$$

mutual conductance as the sum of conductances between nodes

$$G_{12}=G_{21}=G_2; G_{13}=G_{31}=G_3; G_{23}=G_{32}=G_3=0,$$

and nodal currents as the algebraic sum of the currents converging in nodes

$$\sum_a I=E_1G_1; \sum_b I=0; \sum_c I=0.$$

After the introduction of eigen conductance, mutual conductance and nodal currents, we can write equations of nodal potentials in an orderly manner

$$\begin{aligned}
&V_aG_{aa}-V_bG_{ab}-V_cG_{ac}=\sum_a I;\\
&-V_aG_{ba}+V_bG_{bb}-V_cG_{cc}=\sum_b I;\\
&-V_aG_{ca}-V_bG_{cb}+V_cG_{cc}=\sum_c I.
\end{aligned}$$

The set of equations written in orderly manner allows making the following generalization:

1. On the left side of the equations is situated the algebraic sum of the positive value of the given nodal potential multiplied by the sum of branch conductances connected to this node, and of negative values of all the rest nodal potentials multiplied by conductances between these nodes and the given node;

2. On the right side of the equations is the algebraic sum of EMFs multiplied by the conductance of the corresponding branch, the sign of the last multiplier is chosen positive if EMF (or current source) is directed to the given node, otherwise the sign is chosen negative.

In the presence of current sources in the circuit the number of equations to be solved decreases by the number of current sources, and current sources are taken into calculation with its own sign on the right side of the equations.

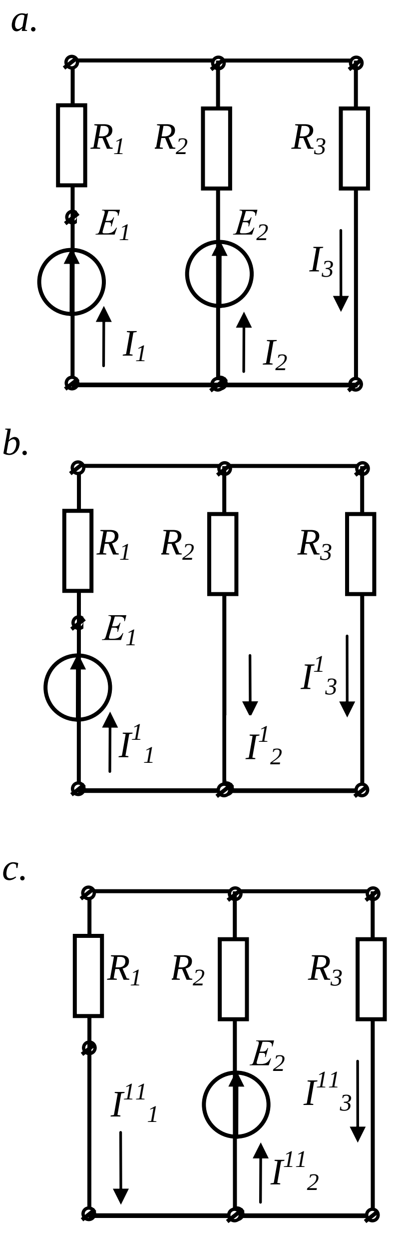
After finding the nodal potentials of the circuit, branch currents are calculated under Ohm's law for circuit branches. For the selected circuit we have branch currents

$$I_1 = \left(V_d - V_a + E_1\right) / R_1 = (-V_a + E_1) G_1;$$
$$I_2 = (V_a - V_b) / R_2 = (V_a - V_b) G_2;$$
$$I_3 = (V_a - V_c) / R_3 = (V_a - V_c) G_3;$$
$$I_4 = (V_b - V_c) / R_4 = (V_b - V_c) G_4;$$
$$I_5 = (V_b - V_d) R_5 = (V_b - V_d) G_5;$$
$$I_6 = (V_c - V_d) / R_6 = V_c G_6.$$

1.6.5. CIRCUIT CALCULATION BY THE SUPERPOSITION METHOD. RULE OF CURRENT DIVIDER

If a circuit has two or more independent sources, one way to determine the value of specific quantity (current or voltage) is to use nodal and loop analysis. Another way is to define partial quantities from each independent source in total quantity, and then to sum these partial values. The latter approach is known as the superposition calculation method.

The superposition method uses the principle of superposition and is essential in the theory of linear circuits. The superposition method is based on the fact that through any circuit branch the current is equal to algebraic sum of currents through this branch from each power source taken separately.



a.=b.+c.

Fig.1.14

The superposition method can be used at defining the currents and voltages in the linear circuit with several sources, as well as with a single source with output voltage or current that has complex arbitrary (non-harmonic) form.

Consider the case when in linear circuit several energy sources are acting. Under the principle of superposition for defining the current I and voltage U, in any branch we alternately apply each source, and we find the corresponding partial currents and voltages. Then the actual currents and voltages are defined as the algebraic sum of the partial currents and voltages.

We illustrate the principle of superposition by the example of a circuit shown in Fig.1.14.*a*. Let us find the branch currents I_1, I_2, I_3.

At the first stage of calculation, we suppose that only the source of EMF E_1 acts in the circuit. Source of EMF E_2 is turned off from the circuit, but its internal resistance in the branch is retained, Fig.1.14.*b*.

The partial currents I^1_1, I^1_2, I^1_3 are flowing in the direction defined by terminal polarity of the EMF source E_1. The values of partial currents I^1_1, I^1_2, I^1_3 are equal

$$I_1' = \frac{E_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}; I_2' = \frac{I_1' R_3}{R_2 + R_3}; I_3' = I_1' - I_2'.$$

The partial current I^1_1 is calculated by the transformation method, the current I^1_2 – according to rule of current divider, and the current I^1_3 – under the first Kirchhoff's law.

Rule of currents divider: current through either of two parallel branches is directly proportional to the product of the input current by the resistance of the opposite branch, and inversely proportional to the sum of resistances connected in parallel.

At the second stage of calculation, we suppose that only the source of EMF E_2 acts in the circuit. Source of EMF E_1 is turned off from the circuit, but its internal resistance in branch is retained, Fig.1.14.*c*.

The partial currents $I^{11}{}_{1}$, $I^{11}{}_{2}$, $I^{11}{}_{3}$ are flowing in direction defined by terminal polarity of the source of EMF E_2. The values of partial currents $I^{11}{}_{1}$, $I^{11}{}_{2}$, $I^{11}{}_{3}$ are equal

$$I_2'' = \frac{E_2}{R_2 + \frac{R_1 R_3}{R_1 + R_3}}; I_1'' = I_1' \frac{R_3}{R_2 + R_3}; I_3'' = I_2'' - I_1''.$$

At the third stage of the calculation the actual current branches are defined as the algebraic sum of the partial currents. The *signs rule in determining the actual currents*: determining the resultant (actual) currents, partial currents coinciding with the chosen positive direction of the actual current are taken into calculation with "+" sign, and with "–" sign in the opposite case

$I_1 = I_1' - I_1''; I_2 = -I_2' - I_2''; I_3 = I_3' + I_3''.$

As follows from the considered example, in drawing up equations for the partial electrical circuits, the energy sources are turned off, but their internal resistances in the circuit are retained.

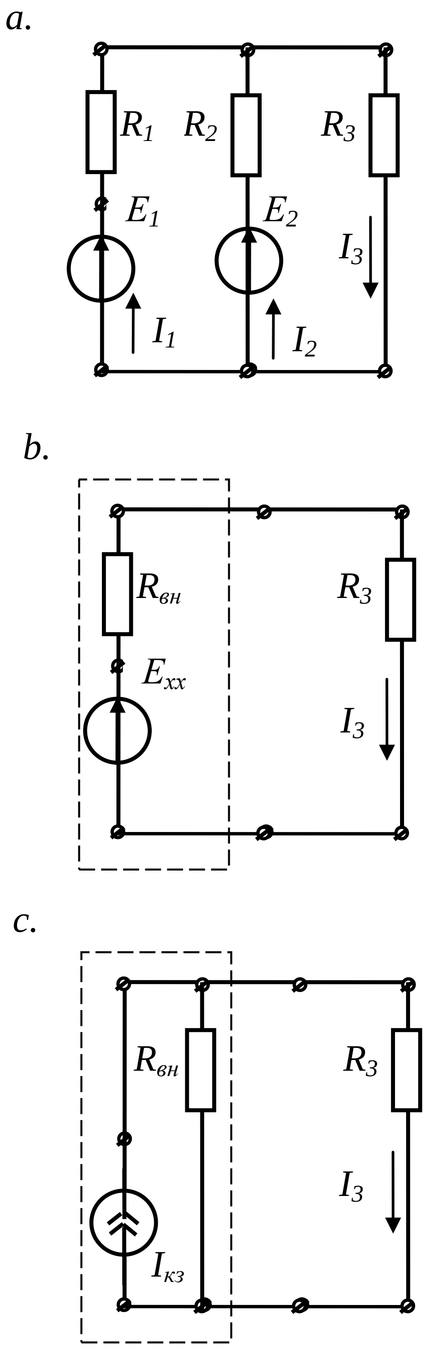
1.6.6. CIRCUIT CALCULATION BY EQUIVALENT GENERATOR METHOD

The equivalent generator method is effective in case when it is necessary to find the current, voltage or power in a single branch. At that, the rest part of the circuit this branch is connected to will be considered in the form of an active one-port network.

Two modifications of the equivalent generator are distinguished: an equivalent EMF source method and the method of equivalent current source.

The equivalent EMF source method. This method is based on *Thevenin's theorem*, according to which the current in any branch of the linear circuit will not change if the active one-port network this branch is connected to is replaced by an equivalent source of EMF with an output voltage equal to the open circuit voltage, and with internal resistance equal to the equivalent input resistance of passive one-port network.

The equivalent current source method. This method is based on Norton's theorem according to which the current in any branch of linear electrical circuit will not change, if active one-port network this branch is connected to is replaced by an equivalent current source with an output current equal to short circuit current of this branch, and with internal conductance equal to equivalent input conductance from the side of open branch.



a.=b.=c.

Fig.1.15

We apply the equivalent generator method to find the current through the third branch of the circuit in Fig.1.15.*a*.

At the beginning this problem will be solved using an EMF equivalent source method by Thevenin's theorem.

We open the branch with R_3 and determine EMF of the open-circuit, Fig.1.15.*b*

$$E_{oc} = \frac{E_1 - E_2}{R_1 + R_2} R_2 - E_2.$$

Equivalent internal resistance R_{in} of passive one-port network is

$$R_{in} = R_1 R_2 / (R_1 + R_2).$$

From circuit Fig.1.15.b, the current through the third branch is found

$$I_3 = E_{os} / (R_{in} + R_3).$$

Let us now find the current through the third circuit branch Fig.1.15.a by equivalent current source method using Norton's theorem. The resistor R_3 is shunted, and short-circuit current I_{sc} of active one-port network is found

$$I_{sc} = E_1 / R_1 + E_2 / R_2.$$

Equivalent internal resistance R_{in} of passive one-port network, Fig.1.15.*c* is

$$R_{in} = R_1 R_2 / (R_1 + R_2).$$

From circuit Fig.1.15.*c* , current of the third branch is calculated by the principle of current division

$$I_3 = I_{oc} R_{in} (R_{in} + R_3).$$

1.7. POWER TRANSFER FROM ACTIVE TO PASSIVE ONE-PORT

In the low-power electric engineering devices there arises the problem of transmission of the highest possible power from the energy source to the load, while the transmission efficiency factor has minor importance. In this case we resort to the matching of the source and load characteristics.

To study the characteristic features of the power transfer from the power supply to the load, we present the power source as an active one-port network (E_{oc}, R_{in}, Fig.1.16), and the load as a passive one-port network (R, Fig.1.16) with respect to terminals *a*, *b*.

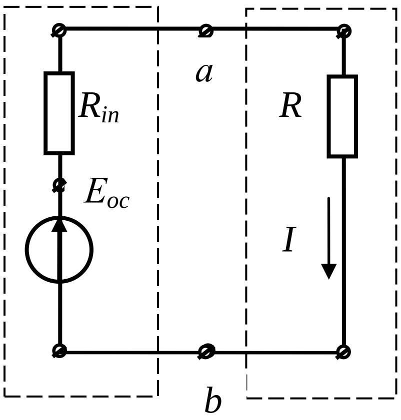


Fig.1.16

The EMF E_{oc} of the power supply is greater than the voltage at load by the voltage drop value across internal resistance R_{in}

$$E_{oc} = U + IR_{in}.$$

Multiplication by the current value transforms the balance voltages equations to the equation of the power balance

$$IE_{oc} = IU + I^2 R_{in} \text{ or } P_g = P_l + P_{loss},$$

where P_g – is power delivered by the generator; P_l – is power consumed by the load; P_{loss} – is power of loss for heating the internal resistance of the generator.

All electromagnetic power $P_g = IE_{oc}$ delivered by energy source is expended onto the power transmitted to an external circuit $P_l = IU$ and power loss inside of power source $P_{loss} = I^2 R_{in}$.

We study the conditions of maximum power transfer from the power source to the external circuit and the dependence of the efficiency factor of the source on the value of external circuit resistance.

At the open-circuit mode ($R=\infty$) load current is zero ($I=0$) and load power is zero ($P_l = IU$). As the load resistance decreases the current increases, and power dissipated in the load ($P_l = IU$) increases as well as the losses in the power supply ($P_{loss} = I^2 R_{in}$). The current through the circuit reaches its maximum value at short-circuit at the load ($R=0$), at that load power in the short-circuit mode is zero ($P_l = IU$) because load voltage is zero.

If under limiting operating modes of the energy source the power is equal to zero, while in all other operating modes the power is nonzero ($P_l=IU$), then there must exist the inflection point in dependence $P_l = f(I)$. For any continuous function, extremum occurs at points where the derivative is zero. This point will be found

$$P_l = UI = RI^2 = RE_{oc}^2 / \left(R + R_{in}\right)^2,$$

$$\frac{d}{dR} P_l = \frac{d}{dR}\left(R \frac{E_{oc}^2}{\left(R + R_{in}\right)^2} \right) = E_{oc}^2 \frac{\left(R + R_{in}\right) - 2R}{\left(R + R_{in}\right)^3} = 0.$$

The fraction is equal to zero when numerator is zero

$$\left(R + R_{in}\right) - 2R = 0,\ R = R_{in}.$$

Thus, if the load resistance is equal to the internal resistance of the power supply, then maximum power is delivered in load. This operating mode of electrical circuit is called matched mode. In this mode

$$P_{l\,\max} = RE_{oc}^2 / R_{in}, I_{mr} = I_{sc} / 2,$$

where P_{lmax} – is the maximal load power; I_{mr} – is current in matched mode; I_{sc} – is short-circuit current of the power source.

The efficiency factor of power transmission from active to passive one-port network is estimated according to the ratio of the circuit resistances

$$\eta = 1/(1 + R_{in} / R).$$

At the open-circuit mode the efficiency factor tends to unity, in matched mode it is 0.5, and in short-circuit mode it tends to zero, Fig.1.17.

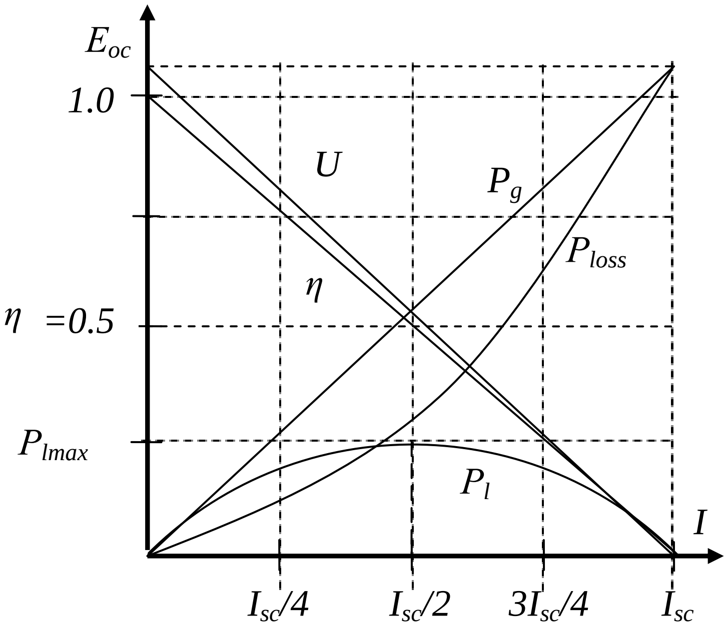


Fig.1.17

1.8. CONCLUSIONS

When analyzing electrical DC circuits with active resistances it is necessary to take into account the following:

1. All variables are the set of real numbers with the sign, which are located on the single number line. The calculation is based on algebraic equations using matrix apparatus for the solutions of system equations.
2. The current of ideal current source is not dependent on the load resistance. The inner resistance of ideal current source is always equal to infinity.
3. The output voltage of ideal source of EMF is not dependent on load current. The internal resistance of ideal source of EMF is always equal to zero.
4. In the steady-state modes of DC electrical circuits, only the ohmic resistances of elements are taken into account in the calculation. The perfect inductive elements do not have ohmic resistance, and perfect capacitive elements disconnect the branches in DC circuits because they have infinitely large internal ohmic (active) resistance.
5. The calculation of linear DC circuits in steady-state operation is performed on the basis of algebraic equations, which are drawn up using Kirchhoff's laws,

and in some cases, when the circuit has the only source of energy – on the basis of Ohm's law.

6. In the cases, when the quantity of independent loops is less than the number of branches, it is rational to perform circuit calculation on the basis of the mesh currents method.

7. If the number of nodes in circuit is less than the number of independent loops, then it is advisable to do currents calculation on the basis of the nodal potentials method, and in particular case when circuit contain only two nodes – on the basis of the method of two nodes.

8. When it is necessary to calculate the parameters of the circuit only in single branch then it is rational to perform calculation on the basis of the equivalent generator method.

9. The verification of the calculation correctness is performed by means of drawing up the power balance.

10. When the power balance is drawn up it is necessary to take into account that ohmic resistance elements are always consuming electric energy, and it is irreversibly converted into heat.

11. Energy sources can both consume and deliver electric energy. If the voltage and current at the terminals of the energy source have the same direction, then source consumes energy, otherwise source delivers it.

12. Transformation of the current source to the source of EMF and inverse transformation may change the operation mode of initial and converted power source.

13. Potential circle is the graph of the voltage distribution along any part of circuit. If the circuit part is not closed, then it is graphic representation of Ohm's law, otherwise it is graphic representation of second Kirchhoff's law.

AFTERWORD

The planned program of the course is completed, which allows to draw some conclusions and makes it possible to evaluate the presented material, as well as to plan further study of this discipline.

The course "Theoretical Fundamentals of Electrical Engineering" contains fundamental concepts that are used in all applied electrical disciplines. The term "fundamental concept" should be interpreted not as a synonym for completeness and self-sufficiency. This term rather means that this course sets out those scientific concepts and methods which make the basis for constant evolving and improving of theoretical arsenal, as well as numerous application areas of electrical engineering.

Modern theoretical electrical engineering uses concepts and techniques from different scientific areas, first of all mathematics, physics, electronics, electrical machines, simulation of circuits and fields, and engineering software. It should be noted that all of these concepts and methods form an interrelated unity and should be considered as a single unit within a systemic approach adopted by modern science. The concept of a mathematical model is used as the basic concept, which allows talking about the systemic nature of Theoretical Electrical Engineering.

In this textbook, a number of deterministic mathematical models of AC and DC circuits at steady-state and transient modes in circuits with lumped and distributed parameters are reviewed. All these models are such that they allow fulfilling the basic operations of circuit analysis with of a greater or lesser degree of certainty, depending on the assumptions that have been made.

Theoretical electrical engineering, as well as many other scientific and technical fields of knowledge, is developing in such a way that methods of analysis often overtake the methods of synthesis. Recently, this situation has begun to change drastically, mainly influenced by the widespread introduction of PC software in the practice of scientific research.

The style and sequence of presentation adopted in this book are dictated largely by considerations of the world-view as well as the traditions developed in electrical engineering in the last decades. This book, written in accordance with the course "Theoretical Fundamentals of Electrical Engineering," certainly does not aim to give an exhaustive description of all the issues that can be attributed to the field of theoretical electrical engineering. For example, out of focus were left such important and rapidly developing fields as electrical network synthesis, methods of analysis of circuits at random and discrete signals, engineering electrodynamics and many others. However, we hope that having mastered the material of this book and using literary sources, you can continue to work on your theoretical and practical problems yourself.

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