Stability assessment of the slopes and side-hills with account of the excess pressure in the pore liquid

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Abstract

**Purpose.** The strength criteria substantiation of water-saturated soils and mine rocks, which make it possible to obtain the analytical solutions necessary for determining the stability of water-flooded soil slopes and side-hills.

**Methods.** The methods are applied of analysis and generalization of the theoretical and numerical experimental studies results. The rocks and soils characteristics are taken into account: specific cohesion $c$, internal friction angle $\phi$, compressive strength $R_c$ and tensile strength $R_p$ of the rock, as well as the bulk density. The load $q$ was imposed to the water-saturated seam roof from the overlying mine rock or soil seams, the weight of equipment or structures located on the surface. It was accepted that the seam is saturated with water (gas) with the excess pressure $P$. A point on the mine working surface (or vertical slope surface), located at a depth $z$ is considered. It is determined at which ratio of $q$, $P$ and $z$ parameters the soil or rock seam will be destroyed. The problem solution is based on the Mohr-Coulomb strength criterion.

**Findings.** The strengths of water-saturated rock and water-free rock are compared. The ratios have been obtained that make possible to determine the critical load on the daylight surface of water-saturated and water-free vertical slopes, side-hills, trenches and foundation pits, as well as various mine workings in soil bases and mine rocks. The analytical solution has been obtained, which makes it possible to determine a value of the critical pressure on the water-flooded vertical surfaces and soil slopes. The generalization has been made of a certain one-dimensional Mohr-Coulomb strength condition for a water-saturated base characterized by the strength characteristics $c$ and $\phi$ for the dimensional case.

**Originality.** It has been theoretically proved that for any pore pressure value in the water-saturated mine rock (or soil) their strength will be less than in their water-free state. New solutions have been formulated for determining the critical height of a water-saturated vertical soil slope or the wall in the vertical mine working.

**Practical implications.** The obtained results make it possible to solve the practical engineering problems on determining the stability of water-saturated slopes and side-hills with a load-free daylight surface, therewith, taking into account the weight of the equipment, stored material and the stability of vertical walls of water-saturated seams of open-cut mine workings.

**Keywords:** strength criterion, pore liquid, pore gas, pressure, slope stability, water-saturated soil, friction angle

1. Introduction

Coal is our most abundant fossil fuel. There is still enough coal underground in Ukraine [1, 2] and foreign countries [3]-[6]. During mining occurs rock pressure [7]-[10]. According to modern scientific data, the presence in the soil and mine rock of excess pressure in the pore liquid leads to such effects [11]-[17]:

- There is a significant change in the stress-strain state of the rock (or soil) massif.
- Even with a constant in time external load, deformations and stresses within the soil (rock) massif and at its boundaries are changed in time. This phenomenon is conditioned by the property of the pore liquid viscosity and, as a consequence, the process of its migration stretched over time in the pores of the soil (rock) stratum under the influence of excess pressure. This phenomenon determines the difference between water-saturated and water-free soils and mine rocks.
- There is also a difference in the behaviour of water-saturated and water-free soils and mine rocks under the influence of an external dynamic load. In this case, in comparison with water-free soils and mine rocks, there is a distortion of eigen-frequencies, vibration amplitudes and stresses acting in soil (rock) massifs.
4. In our opinion, the most important is the fact that the presence in the soil and mine rock of excess pressure in the pore liquid reduces the strength of the soil slopes, side-hills and mine workings.

Therefore, it is of considerable interest the determination of the critical values of such parameters as the pressure in a pore liquid or gas, the depth at which the rock massif destruction is possible (Fig. 1) in the conditions of water and gas saturation of mine rocks.

![Figure 1. The scheme for the determination of the critical pressure \( P \), critical load \( q \) onto the roof of the water-saturated or gas-saturated soil (rock) seam](image)

Up to date, many scientists and research teams are involved in such issues [18]-[21], however, the issue has not been sufficiently studied from this point of view [22]-[26]. Thus, Institute of Geotechnical Mechanics named by N. Poljakov of National Academy of Sciences of Ukraine (Dnipro, Ukraine) employees of Ukraine has performed the studies related to gas-dynamic assessment of coal and rock massif [27]-[28], and particular attention is paid to methane release into the face of underground mine workings, to the gas pressure in the stratum, and other factors [29]-[31]. The physical and chemical processes occurring in the rock massif are quite well presented in the works of V.V. Soboliev and of the teams working under his leadership [32]-[34] at the Dnipro University of Technology (Ukraine).

Thus, the purpose of the work is to substantiate the strength criteria for the water-saturated soils and mine rocks, which make it possible to obtain analytical solutions necessary for determining the stability of water-flooded soil slopes and vertical mine workings.

2. Materials and methods

The research task has been formulated as follows:

1. The strength characteristics are known of the soil or mine rock (specific cohesion \( c \) and internal friction angle \( \varphi \)) or (compressive strength \( R_c \) and tensile strength \( R_p \) of the rock) [35], [36].

2. The soil bulk density (rock) \( \gamma \) is known.

3. The load \( q \) was imposed to the roof of the water-saturated seam (this can be a load from overlying mine rock or soil seams, the weight of equipment or structures located on the surface of the vertical slope, etc.).

4. The soil (rock) seam is saturated with water (gas), the excess pressure in which is equal to \( P \).

5. A point on the surface of the mine working (or vertical slope), located at a depth \( z \) is considered.

It is necessary to determine, at which ratio of \( q \), \( P \) and \( z \) parameters the rock (or soil) seam will be destroyed.

To solve the problem, we use the Mohr-Coulomb strength criterion [37]:

\[
\tau \leq (\sigma - P) \tan(\varphi) + c,
\]

where:

- \( \tau \) – shear stress;
- \( \sigma \) – the same, but normal stress.

At the same time, we take into account that in the soils and mine rocks mechanics, the compressive normal stresses should be considered with the plus sign, and the tensile normal stresses should be taken with the minus sign.

In addition, we take into account that, according to the Mohr-Coulomb strength criterion, other factors being equal, the destruction of soil, mine rock and coal occurs when the maximum and minimum normal stresses \( \sigma_1 \) and \( \sigma_3 \) reach a certain critical combination. Herewith, the principal normal stress \( \sigma_2 \) does not almost influence the strength.

In order to reduce the one-dimensional Mohr-Coulomb condition (1) to the dimensional case with the principal normal stresses \( \sigma_1 \) and \( \sigma_3 \) in accordance with the information and data presented in the works of V.A Florin [37], [38], we set into (1):

\[
\begin{align*}
\tau &= \tau_m \cos(\varphi); \\
\sigma &= \sigma_m - \tau_m \sin(\varphi); \\
\tau_m &= \frac{\sigma_1 - \sigma_3}{2}; \\
\sigma_m &= \frac{\sigma_1 + \sigma_3}{2}.
\end{align*}
\]

We obtain:

\[
\begin{align*}
\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} - 2P + 2c\tan(\varphi) &\leq \sin(\varphi); \\
\sigma_1 + \sigma_3 - 2P + 2c\tan(\varphi) &\geq 0; \\
\sigma_1 &\geq \sigma_2 \geq \sigma_3.
\end{align*}
\]

Analysis of the expression (3) has allowed us to draw the following conclusions:

1. This expression is different from the well-known in the literature [1], [37], [38] the Mohr-Coulomb strength condition and the presented below ratios (5), by the presence of the “\( P \)” summand in the denominator, i.e. pore pressure, taken with a “minus” sign.

2. With an increase in the pore pressure and the positive value of denominator (3), the left part of the upper equation increases. Thus, in this case there is a decrease in the strength of the soil (rock).

3. At significant values of the pore pressure \( P \) and the fulfilment of the condition:

\[
\sigma_1 + \sigma_3 - 2P + 2c\tan(\varphi) < 0,
\]

the strength condition (3) changes to the super-critical area and has not the physical meaning.

4. If to set in (3) the pore pressure equal to zero (i.e. \( P = 0 \)), we will obtain the generally accepted record of the Mohr-Coulomb strength condition:
\[
\begin{align*}
\sigma_1 - \sigma_3 & \leq \sin(\varphi); \\
\sigma_1 + \sigma_3 + 2cctg(\varphi) & = \sigma_1 + \sigma_2 \geq \sigma_3.
\end{align*}
\]  

(5)

In such a way, the generally accepted record of the Mohr-Coulomb strength condition (5) is a special case of the obtained solution (3).

5. If in (3) to change the sign before the pore pressure \( P \) (i.e., not to inject the liquid or gas into the base, but to pump them out), then the base will be strengthened.

Then, we will compare the strength conditions (3) and (5).

We will divide term-by-term the upper equation (5) by the upper equation (3). Given the fact that the left and right parts of (3) and (5) are greater than zero, we have:

\[
\psi = \frac{\sigma_1 + \sigma_3 - 2P + 2cctg(\varphi)}{\sigma_1 + \sigma_3 + 2cctg(\varphi)} \leq 1.
\]  

(6)

Then we will set in (6):

\[
\chi = \frac{2P}{\sigma_1 + \sigma_2 + 2cctg(\varphi)}.
\]  

(7)

With account of inequation (7), the second condition (3) and inequation (6) will take the form:

\[
\psi = 1 - \chi \leq 1; \\
0 \leq \chi < 1.
\]  

(8)

In such a way, the range of the dimensionless group \( \chi \) definition is all the positive numbers on the interval \( \chi \in (0,1) \), the range of function \( \psi \) values is on the interval \( \psi \in (0,1) \) (Fig. 2).

3. Results and discussion

The analysis of dependences presented in Figure 2 made it possible to conclude that at any pore pressure value in the water-saturated mine rock (or soil) their strength will be less than in their water-free state. It should be noted that equation (3) can also be obtained in a simpler way, based upon quite general considerations – Figure 3.

Consider some elementary volume of soil (or rock) to which the principal stresses are applied externally \( \sigma_1, \sigma_2 \) and \( \sigma_3 \).

Inside the sample, pressure acting in pore liquid or gas is numerically equal to \( P \) (in soil mechanics it is called neutral, Fig. 2).

In accordance with the Pascal’s law, this pressure acts in all directions, and its values in all directions are equal to each other.

![Figure 3. Scheme for the determination of effective stresses in the soil (rock) matrix](image)

### Figure 3. Scheme for the determination of effective stresses in the soil (rock) matrix

Furthermore, in accordance with the principle of K. Terrzaghi’s moisture capacity, the matrix of the soil (rock) is under the influence of an effective pressure, which is numerically equal to the difference between the principal stresses and the pore pressure \( P \):

\[
\begin{bmatrix}
\sigma_{1,eff} & 0 & 0 \\
0 & \sigma_{2,eff} & 0 \\
0 & 0 & \sigma_{3,eff}
\end{bmatrix} =
\begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3
\end{bmatrix} -
\begin{bmatrix}
P & 0 & 0 \\
0 & P & 0 \\
0 & 0 & P
\end{bmatrix},
\]

(9)

Here \( \sigma_{i,eff} \) – the effective values of principal stresses.

For the final solution of the problem, we replace in (5) the principal stresses \( \sigma_i \) by their effective values \( \sigma_{i,eff} \). Thus, we arrive at the ratios obtained earlier (3).

It should be noted that the same condition of strength (3) for water-saturated soil (mine rock) has been obtained in various ways. This indicates the validity of the result obtained by us.

The strength characteristics \( c \) (specific cohesion) and \( \varphi \) (internal friction angle) are commonly used in prediction of soil destruction [1], [37]. Thus, in mechanics of mine rocks, their strength at uniaxial compression \( R_c \) and strength at uniaxial tension \( R_t \) are used as strength characteristics [38].

According to the data presented in the works of V.A. Florin and work [38], between the compressive strengths \( R_c \) and tensile strengths \( R_t \) as well as between the trigonometric functions of the internal friction angle of the mine rock and its specific cohesion the following ratios occur:

\[
\begin{align*}
c &= \frac{1}{2} \sqrt{R_c - R_p}; \\
\sin(\varphi) &= \frac{R_c - R_p}{R_c + R_p}; \\
\cos(\varphi) &= \frac{2 \cdot R_c - R_p}{R_c + R_p}; \\
tg(\varphi) &= \frac{\sin(\varphi)}{\cos(\varphi)} = \frac{\frac{R_c - R_p}{2 \cdot \sqrt{R_c - R_p}}}{\frac{2 \cdot R_c - R_p}{R_c + R_p}}; \\
ectg(\varphi) &= \frac{\cos(\varphi)}{\sin(\varphi)} = \frac{2 \cdot \sqrt{R_c - R_p}}{R_c - R_p}.
\end{align*}
\]  

(10)
When substituting the ratios (10) into equation (3), we have:

\[
\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2P + 2R_c \cdot \frac{\varphi}{1 - \varphi}} \leq \frac{R_c - R_p}{R_c + R_p};
\]

\[
\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2P + 2R_c \cdot \frac{\varphi}{1 - \varphi}} \geq 0;
\]

\[
\sigma_1 \geq \sigma_2 \geq \sigma_3.
\]

wherefrom, with account of notation \( \xi = \frac{R_p}{R_c} \), we will find:

\[
\xi = \frac{R_p}{R_c};
\]

\[
\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2P + 2R_c \cdot \frac{\varphi}{1 - \varphi}} \leq \frac{1 - \xi}{1 + \xi};
\]

\[
\sigma_1 + \sigma_3 - 2P + 2R_c \cdot \frac{\varphi}{1 - \varphi} \geq 0;
\]

\[
\sigma_1 \geq \sigma_2 \geq \sigma_3.
\]

The conditions (11) obtained by us are another form of the record of obtained above Mohr-Coulomb strength condition for water-saturated soil (3), where instead of strength material constants \( c \) and \( \varphi \), other material constants were used, such as the compressive strengths \( R_c \) and tensile strengths \( R_p \) of the mine rock.

Analysis of the expression (11) has allowed us to draw the following conclusions:

1. The obtained strength condition of water-saturated mine rock (11) is different from the well-known in the literature Mohr-Coulomb strength condition [1], [37], [38] by the presence of the “\( P \)” summand in the denominator, i.e. pore pressure, taken with a ‘minus’ sign and by the strength material constants.

2. In other words, instead of the strength material constants \( c \) and \( \varphi \) accepted in (3), other material constants are used in (11), such as the compressive strengths \( R_c \) and tensile strengths \( R_p \) of the mine rock.

3. At significant values of the pore pressure \( P \) and the fulfilment of the condition:

\[
\sigma_1 + \sigma_3 - 2P + 2R_c \cdot \frac{\varphi}{1 - \varphi} \geq 0;
\]

the strength condition (11) changes to the super-critical area and has not the physical meaning.

4. If to set in (11) the pore pressure equal to zero (i.e. \( P = 0 \)), we will obtain the record of the Mohr-Coulomb strength condition for water-free mine rock in this form:

\[
\frac{\sigma_1}{\sigma_1 - 2P + 2\cctg \phi} \leq \sin \phi;
\]

\[
\frac{\sigma_1}{\sigma_1 - 2\cctg \phi} \leq P.
\]

In such a way, the generally accepted record of the Mohr- Coulomb strength condition (13) is a special case of the obtained solution (3).

5. If in (11) to change the sign before the pore pressure \( P \) (i.e., not to inject the liquid or gas into the mine rock, but to pump them out), then there will be the strengthening of the rock massif.

Then, we will compare the strengths of water-saturated rock and water-free rock.

We will divide term-by-term the second equation from above (11) by the second equation from above (13). Given the fact that the left and right parts of (11) and (11) are greater than zero, we have:

\[
\psi_1 = \frac{1 + \sigma_3 - 2P + 2R_c \cdot \frac{\varphi}{1 - \varphi}}{1 + \sigma_3 + 2R_c \cdot \frac{\varphi}{1 - \varphi}} \leq 1.
\]

Then we will set in (14):

\[
\chi_1 = \frac{2P}{\sigma_1 + \sigma_3 + 2R_c \cdot \frac{\varphi}{1 - \varphi}}.
\]

With account of (15), the inequation (14) and the third condition from above (11) will take the form:

\[
\psi_1 = 1 - \chi_1 \leq 1,
\]

\[
0 \leq \chi_1 \leq 1.
\]

In such a way, the range of the dimensionless group \( \chi_1 \) definition is all the positive numbers on the interval \( \chi_1 \in (0,1) \), and the range of function \( \psi_1 \) values is on the interval \( \psi_1 \in (0,1) \). Further on, we use the obtained result (3) to solve a number of specific engineering problems of soil mechanics and geotechnical mechanics.

First, we will find the principal normal critical stress \( \sigma_{1,cr} \), at which the water-saturated soil seam will be destroyed (Fig. 1).

Assume that the rate of mine working drivage and the permeability coefficient are such that the pore pressure dispersion can be neglected near the wall of mine working. This property is appropriate for over-consolidated clays, coal strata with closed porosity, and also for all soils and mine rocks with a low value of the permeability coefficient [39].

In this case, the horizontal principal stress is equal to zero (\( \sigma_3 = 0 \)), so that the strength condition (3) will take the form:

\[
\frac{\sigma_1}{\sigma_1 - 2P + 2\cctg \phi} \leq \sin \phi.
\]

\[
\frac{\sigma_1}{\sigma_1 - 2\cctg \phi} \leq P.
\]
Having solved (17), according to the principal stress \( \sigma_1 \), we will finally find:

\[
\begin{align*}
\sigma_{1,cr} & \geq 2P - 2\text{cctg}(\phi), \\
\sigma_{1,cr} & \geq \frac{2c \cos(\phi)}{1 - \sin(\phi)} - 2P \sin(\phi).
\end{align*}
\] (18)

To analyse the solution (18) obtained by us, first examine the behaviour of the internal friction angle functions \( \phi \)

\[
\begin{align*}
f_1 & = \frac{\cos(\phi)}{1 - \sin(\phi)} \quad \text{(line 1)} \quad \text{and} \\
f_2 & = \frac{\sin(\phi)}{1 - \sin(\phi)} \quad \text{(line 2)} \quad \text{on the internal friction angle} \quad \phi.
\end{align*}
\]

The range of the internal friction angle variation is accepted from those considerations that for absolutely plastic soils and mine rocks (for example, for silts) the internal friction angle \( \phi \approx 0 \), and for very hard rocks (granites, diorites) \( \phi \approx 43-45 \) degrees [40]-[42].

It follows from the Figure 4, that on the interval of the internal friction angle variation \( \phi \in (0...45) \) functions \( f_1 \) and \( f_2 \) increase monotonically, and \( f_1(\phi) \geq f_2(\phi) \).

The analysis of inequations (18), with account of the data presented in Figure 4, made it possible to conclude that the critical value of the principal stress \( \sigma_{1,cr} \) is greater, if:

- the greater is the specific cohesion \( c \) and internal friction angle \( \phi \);
- the pressure value in the pore liquid \( P \) is less.

It should also be noted that the inequations (18) do not make sense when the internal friction angle \( \phi \) decreases to zero (i.e., when \( \phi \to 0 \)), since in this case the pressure in the pore liquid (gas) does not affect the strength of soil (mine rock). This effect is explained by various mechanisms of destruction of ideally-plastic soils (for example, silt), and soils with specific cohesion and internal friction [1]. A similar result we get, when setting into (1) the internal friction angle equal to zero.

Then, we will find the principal normal critical stress \( \sigma_{1,cr} \), at which the vertical slope wall in the water-saturated or gas-saturated soil seam will be destroyed (Fig. 1).

Having set into (11) the principal stress \( \sigma_1 = 0 \) (Fig. 3) and having solved the inequations obtained in this way with account of the principal stress \( \sigma_1 \), we will find:

\[
\begin{align*}
\xi & = \frac{R_P}{R_c}; \\
\sigma_1 & \leq R_c - \frac{1 - \xi}{\xi} P; \\
\sigma_1 & \geq 2P - 2R_c \cdot \frac{\xi}{1 - \xi}; \\
\sigma_1 & \geq \sigma_2 \geq \sigma_3.
\end{align*}
\] (19)

The solution (18) allows us to determine the critical height \( q_{cr} \) of a water-saturated vertical soil slope or of the wall in the vertical mine working (Fig. 5).

**Figure 5. To the determination of the critical height of a water-saturated vertical slope**

It follows from the Figure 5, that if the bulk density of the soil is equal to \( \gamma \), then at depth of \( h_{cr} \), the principal stress \( \sigma_{1,cr} \) is equal to:

\[
\sigma_{1,cr} = q + \gamma h_{cr}.
\] (20)

When substituting (20) into (18), we will find:

\[
\begin{align*}
h_{cr} & \geq \frac{2P - 2\text{cctg}(\phi) - q}{\gamma}, \\
h_{cr} & \geq \gamma \left( 1 - \sin(\phi) \right) - \frac{q}{\gamma}. \\
\end{align*}
\] (21)

Having set into (21) the pore pressure \( P = 0 \), we will arrive at the well-known formula for determining the critical height of water-free soil slope [37]:

\[
\begin{align*}
h_{cr} & \leq \frac{2c \cos(\phi)}{\gamma \left( 1 - \sin(\phi) \right)} - \frac{q}{\gamma}.
\end{align*}
\] (22)

The obtained ratios (21) make it possible to solve the following engineering problems:

1. The determination of the critical height of water-saturated vertical slopes, side-hills, trenches and foundation pits and, in general, of various mine workings in soil bases and mine rocks.
2. The determination of the stability of water-saturated vertical slopes, side-hills, trenches and foundation pits and, in general, of various mine workings in soil bases with a surface free from the load (for this we should set into (21) \( q = 0 \)).
3. The determination of the critical height of water-free vertical slopes, side-hills, trenches and foundation pits and, in general, of various mine workings in soil bases and mine rocks.

In conclusion, we note that if the water-saturated soil seam in which the mine working is arranged is at a certain depth \( H \) below the water-free soil seams, which have the thicknesses \( h_l \) and bulk densities \( \gamma_l \), then the parameter \( q \) entering the formula (18) should be determined as follows:
\[ q = \sum_{i=1}^{n} \gamma_i h_i. \]  

Here \( \gamma_i \) – bulk density of the soil seam; \( h_i \) – its thickness; \( n \) – number of soil seams. Besides, there is an equation \( H = \sum_{i=1}^{n} n_i. \)

The obtained ratios (18) also make possible to determine the critical load \( q_{cr} \) on the daylight surface of a water-saturated vertical soil slope or vertical mine working in the base (Fig. 5). Having substituted (20) into (18) and solving the inequalities obtained in this way with respect to the load \( q \), we will find:

\[
q_{cr} \geq 2P - 2c\text{ctg} (\phi) - \gamma h; \\
q_{cr} \leq \frac{2c \cos(\phi) - 2P \sin(\phi) - \gamma h}{1 - \sin(\phi)}. \tag{24}
\]

The ratios (24) make it possible to solve the following engineering problems:

1. The determination of the critical load on the daylight surface of water-saturated vertical slopes, side-hills, trenches and foundation pits and, in general, of various mine workings in soil bases and mine rocks.

2. The determination of the critical load on the daylight surface of water-free vertical slopes, side-hills, trenches and foundation pits and, in general, of various mine workings in soil bases and mine rocks (for this we should set \( q = 0 \)).

3. The determination of the critical depth at which an arrangement of the mine working with vertical walls is possible in a water-saturated soil (or rock) seam, above which there are soil (or rock) seams with bulk densities \( \gamma_i \). In this case, the inequalities (24) should be represented in the form:

\[
H_{kr} \geq \frac{2P - 2c\text{ctg} (\phi) - \gamma h}{\sum_{i=1}^{n} \gamma_i}; \\
H_{kr} \leq \frac{2c \cos(\phi) - 2P \sin(\phi) - \gamma h}{\sum_{i=1}^{n} \frac{\gamma_i}{1 - \sin(\phi)}}. \tag{25}
\]

When solving the practical problems for construction of foundation and mines, the problem often arises of the critical pressure determination in the pore liquid or gas in the base, in which it is necessary to arrange the mine working with vertical walls. Therein, when strengthening the bases using the high-pressure injection method [37], it is also important to know the critical pressure value near the underground mine workings, utility systems and other structures.

To determine the critical value in a pore liquid (gas), we will solve (17) with respect to the pore pressure \( P \). We have:

\[
P = \frac{2c \cos(\phi) - 1 - \sin(\phi)}{2 \sin(\phi)} \sigma_1; \\
P \leq \frac{1}{2} \left[ \sigma_1 + 2c\text{ctg} (\phi) \right]. \tag{26}
\]

The least of the pore \( P \) values corresponds to the critical value in the pore liquid (gas) in (25). Since for any value of the internal friction angle \( \varphi \), there is an inequality \( \frac{1 - \sin(\varphi)}{2 \sin(\varphi)} \leq \frac{1}{2} \), then the critical pressure in the pore liquid (gas) \( P_{cr} \) is equal:

\[
P_{cr} = \frac{2c \cos(\phi) - \left[ 1 - \sin(\varphi) \right] \sigma_1}{2 \sin(\phi)}. \tag{27}
\]

Further on, we adapt the results obtained for the strength material constants \( c \) and \( \varphi \) (i.e., the specific cohesion and the internal friction angle) to the material constants \( R_c \) and \( R_p \) which have been accepted in the mine rock mechanics (i.e., the rock strengths for uniaxial compression and uniaxial tension).

At first, with the use of (19) and (20), we will find the critical height \( q_{cr} \) of the water-saturated vertical soil slope or the wall of vertical mine working (Fig. 5). We obtain:

\[
q_{cr} = \frac{R_p}{R_c} \left\{ \frac{R_p}{R_c} - \frac{q}{\gamma} \right\}. \tag{28}
\]

Having set the pore pressure equal to zero in (28), we find the critical height of the water-free mine rock. We obtain:

\[
q_{cr} = \frac{R_p}{R_c} \left\{ \frac{R_p}{R_c} - \frac{q}{\gamma} \right\} \tag{29}
\]

It can also be noted that formulas (28) and (29) are completely analogous to formulas (21) and (22), respectively. The only difference is in the recording of material constants. The obtained ratios (19) make it possible to determine the critical load \( q_{cr} \) on the daylight surface of a water-saturated vertical soil slope or vertical mine working in the base (Fig. 5).

Having substituted (20) into (19) and solving the inequalities obtained in this way with respect to the load \( q \), we will find:

\[
q_{cr} = \frac{R_p}{R_c} \left\{ \frac{R_p}{R_c} - \frac{q}{\gamma} \right\}. \tag{30}
\]

The ratios (30) are completely analogous to (24). The only difference between them is in the material constants. If the thicknesses of the soil seams above the considered point \( h_i \) and their bulk densities \( \gamma_i \) are known, the equation (30) makes it possible to determine the critical depth at which the mine working with vertical walls without support can be arranged in the water-saturated rock. By analogy with (25), we have:
we will find the dependence of the critical value of the principal stress on the pore pressure $P$. We have:

$$
\xi = \frac{R_p}{R_c};
$$

$$
\sigma_1 \leq R_c - 1 - \frac{\xi}{\xi}, P;
$$

$$
\sigma_1 \geq 2P - 2R_c - \frac{\xi}{1 - \xi};
$$

$$
\sigma_1 \geq \sigma_2 \geq \sigma_3.
$$

It should be noted, that the ratios (32) are completely analogous to (25). The only difference between them is in the material constants.

Further on, we will find the dependence of the critical (destructive) principal stress $\sigma_1 = q + \gamma$ on the pore pressure $P$ in the coal seam (see the computational scheme in Figure 1).

So, for example, if the compressive strengths are $R_c = 10.8$ MPa and the tensile strength is $R_p = 0.84$ MPa, then from (19), with account of these values, we will get:

$$
\xi = 0.078;
$$

$$
\sigma_1 \leq 10.8 - 1.186P;
$$

$$
\sigma_1 \geq 2P - 2R_c - 1.82;
$$

$$
\sigma_1 \geq \sigma_2 \geq \sigma_3.
$$

The graphical interpretation (33) is represented in Figure 6. It follows from the figure, that at zero pore pressure the critical value of the principal stress $\sigma_1$ is equal to 11 MPa. With an average bulk density of 20 kN/m$^3$, this pressure corresponds to a depth of 550 meters.

Explanatory notes:

1. The area is marked with a gray color in which the values are located of the principal stress $\sigma_1$ and the pore pressure $P$ in the coal seam, corresponding to the stable state of the mine working.

2. Black solid lines indicate the boundaries of the stable zone of the mine working.

At a pore liquid (or gas) pressure equal to $P = 0.6$ MPa or (6 atmospheres), the critical value of the principal stress $\sigma_1$ is 3.7 MPa. With an average value of the bulk density of 20 kN/m$^3$, this pressure corresponds to a depth of 185 meters.

Thus, the presence of excess pressure in the coal seam resulted in a decrease in the depth of the mine working drivage without support from 550 to 185 meters.

**Figure 6. The dependence of the principal stress $\sigma_1$ on the pore pressure $P$ in the coal seam**

Herewith, with account of the coefficient $k_d$, the problem of structural weakening, the value of which, for example, is 0.3, the value of the ultimate depths will be 165 and 55.5 m, respectively.

**4. Conclusions**

In general, the research materials presented in this article made it possible to draw such conclusions:

1. A generalization has been performed of the known one-dimensional Mohr-Coulomb strength condition for a water-saturated base characterized by the strength characteristics $c$ and $\varphi$ to the dimensional case. With this, the same result has been obtained in two completely different ways.

2. These results have been generalized to the case of water-saturated and water-free mine rock, characterized by strength characteristics $R_c$ and $R_p$.

3. The difference of these strength criteria from analogous ones for water-free soils and mine rocks is their ambiguity:

3.1. For a water-free soil (or mine rock), we have only one critical point, separating the area in which the strength of the soil (or mine rock) is protected from the area in which destruction occurs.

3.2. For water-saturated soils (or mine rocks), we have two critical points instead of one. The first point separates the area, in which the strength of the soil (or mine rock) is protected from the area in which the obtained strength criteria (3) and (11) are not applicable. With this, the second point separates the area in which the strength of the soil (or mine rock) is protected from the area in which the destruction occurs.

4. An analytical solution has been obtained for the problem of determining the critical vertical pressure at which the vertical walls of mine workings and vertical water-saturated slopes are destroyed.

5. The obtained results make it possible to solve such practical problems:

5.1. The stability determination of water-saturated slopes and side-hills with a load-free daylight surface.

5.2. The stability determination of water-saturated slopes and side-hills on the daylight surface of which there is a load (including the weight of the equipment, stored material and the ground, mined during extraction, etc.).

5.3. The assessment of the vertical walls stability of water-saturated seams of mine workings (including the walls of mine shafts and pit walls).

5.4. The assessment of the vertical walls stability of water-saturated coal seams during mine workings drivage.
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Оцінка стійкості укосів і схилів з урахуванням наділлючого тиску в порожній рідині

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Мета. Обґрунтування критеріїв міцності водонасичених ґрунтів і гірських порід, що дозволяють отримувати аналітичні рішення, необхідні для визначення стійкості обводнених ґрунтових укосів та схилів.

Методика. Застосовано методи аналізу та узагальнення результатів теоретичних і чисельних експериментальних досліджень. Враховано характеристики пори і ґрунтів: питоме зчеплення с, кут внутрішнього тертя φ, міцність породи на стиск R_s і розріз R_t, питому вагу g. До покриття водонасиченого шару прикладалася навантаження q від верхніх шарів пород (ґрунту), ваги розташованого на поверхні обладнання або конструкцій. Під час, що відносився до водонасичених вертикальних ґрунтових відкосів або стінки вертикальної гірничої виробки. Розглядається точка на поверхні виробки (або вертикального укосу), розташована на глибині z. Визначається, при якому співвідношенні параметрів q, R_s і z відбувається руйнування ґрунтового або породного шару. Рішення завдання здійснюється на підставі критеріїв міцності Мора-Кулона.

Результати. Проведено зіставлення міцності водо- і неводонасиченої пород відкосів. Отримані співвідношення дозволяють визначити критичне навантаження на денну поверхню водонасичених і неводонасичених вертикальних укосів, схилів, траншей та котлованів, а також інших виробок у ґрунтових основах і гірських породах. Отримано аналітичні розв’язки, що дозволяють визначити критичну стійкість обводнених вертикальних укосів та гірничих виробок. Виконано узагальнення відомої однозвіпмової умови міцності Мора-Кулона для водонасиченої основи, що характеризується с і φ на просторовий випадок.

Наукова новизна. Теоретично доведено, що при будь-якому значенні порового тиску у водонасиченій гірській породі (або ґрунти) її міцність буде менше, ніж у неводонасиченій породі. Сформульовано нові рішення задачи із визначення критичної висоти водонасичених вертикальних ґрунтових укосів або стінки вертикальної гірничої виробки.

Практичне значення. Отримані результати дозволяють вирішувати практичні інженерні завдання з визначення стійкості водонасичених укосів і схилів з відношенням на повноцінальну поверхню і охорони ваги обладнання, складових матеріалів і стійкості вертикальних стін водонасичених шарів відкритих виробок.

Ключові слова: критерій міцності, порода рідина, порожній газ, тиск, стійкість укос, кут тертя

Оцінка устойчивости откосов и склонов с учетом избыточного давления в поровой жидкости

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Цель. Обоснование критериев прочности водонасыщенных грунтов и горных пород, позволяющих получать аналитические решения, необходимые для определения устойчивости обводненных ґрунтовых откосов и склонов.

Методика. Применены методы анализа и обобщения результатов теоретических и численных экспериментальных исследований. Учитывались характеристики пород и грунтов: удельное сцепление с, угол внутреннего трения φ, прочность породы на сжатие R_s и растяжение R_t, удельный вес. К кровле водонасыщенного слоя прикладывалась нагрузка q от вышележащих слоев пород или грунта, вес расположенного на поверхности оборудования или конструкций. Показано, что после снаряжения водой (газом) с избыточным давлением P. Рассматривается точка на поверхности выработки (или вертикального откоса), расположенная на глубине z. Определется, при каком соотношении параметров q, P и z произойдет разрушение ґрунтового или породного слоя. Решение задачи осуществляется на основании критерия прочности Мора-Кулона.

Результаты. Проведено сопоставление прочности водо- и неводонасоченной породы. Получены соотношения позволяющие определить критическую нагрузку на дневную поверхность водонасыщенных и неводонасыщенных вертикальных откосов, склонов, траншей и котлованов, а также прочих выработок в грунтовых основах и горных породах. Получено аналитическое решение, позволяющее определить величину критического давления на обводненные вертикальные поверхности и грунтовые откосы. Выполнено обобщение известного одномерного условия прочности Мора-Кулона для водонасыщенного основания, характеризующегося прочностными характеристиками с и φ на пространственном случае.

Научная новизна. Теоретически доказано, что при любом значении порового давления в водонасыщенной горной породе (или грунте) их прочность будет меньше, чем в неводонасыщенном состоянии. Сформулированы новые решения задач по определению критической высоты водонасыщенного вертикального откоса или стенки вертикальной горной выработки.

Практическое значение. Полученные результаты позволяют решать практические инженерные задачи по определению устойчивости водонасыщенных откосов и склонов со свободной от нагрузки дневной поверхностью и с учетом веса оборудования, складированных материалов и устойчивости вертикальных стенок водонасыщенных слоев открытых выработок.

Ключевые слова: критерий прочности, поровая жидкость, порожній газ, давление, устойчивость откоса; угол трения

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