Національний технічний університет
ДНІПРОВСЬКА ПОЛІТЕХНІКА
1899

Пістунов І.М.
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The manual discusses the theoretical and practical aspects of analysis, synthesis, modeling and finding optimal solutions for economic systems by means of economic cybernetics.
Examples of the application of theoretical developments are given, methods of calculations with the use of computer applications, such as spreadsheets and mathematical processors are given. Test questions, individual tasks and assignments for course work allow you to better master the material presented and ensure the confident use of cybernetic methods in the real economy.
Designed for students of higher education specialties "Economics".

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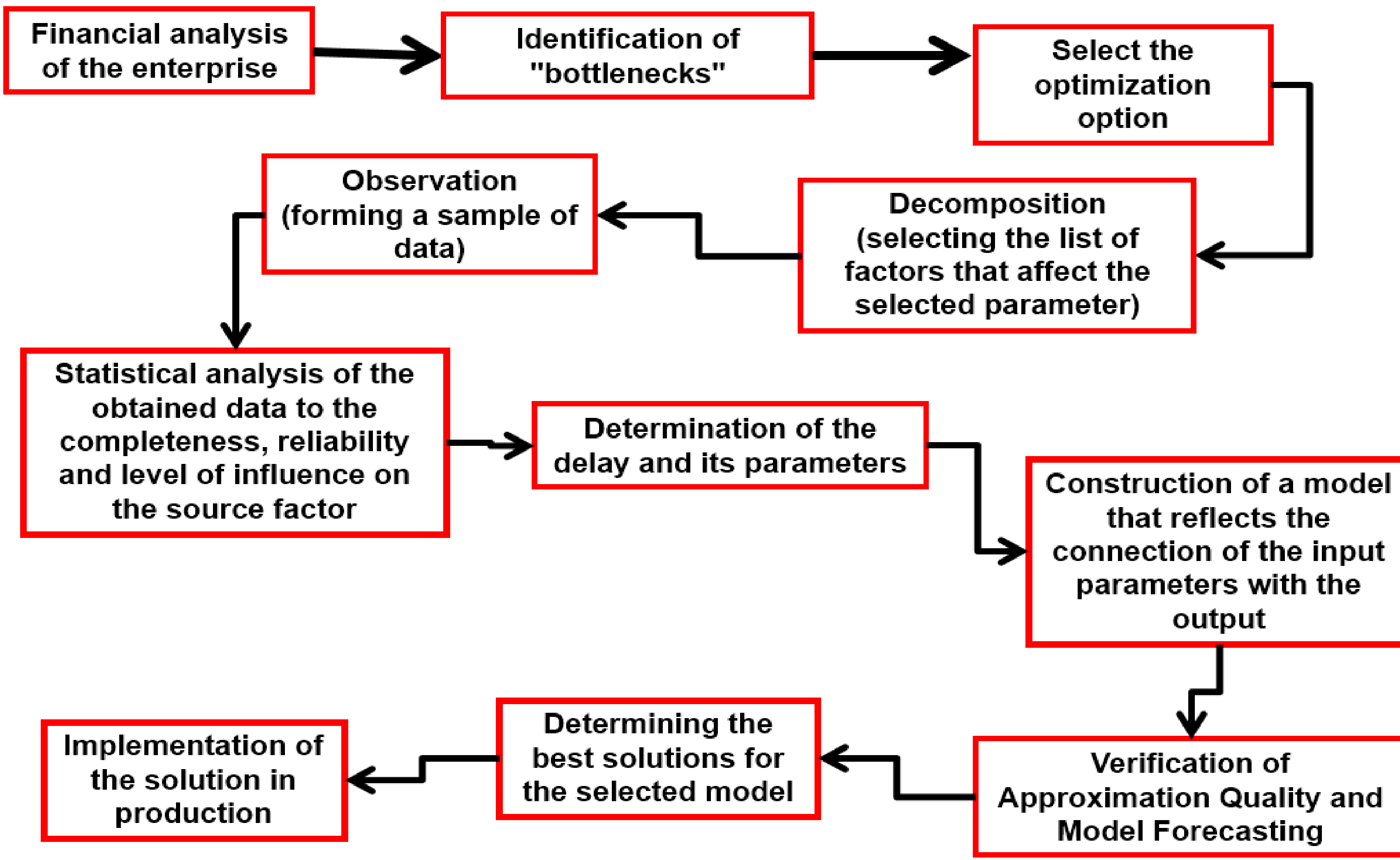
CONTENT

PREFACE

Economic cybernetics, as one of the scientific directions of cybernetics, deals with the application of ideas and methods of cybernetics to economic systems. In a broad sense, economic cybernetics refers to the field of science that emerged at the junction of mathematics and cybernetics with economics, including mathematical programming, operations research, economic-mathematical models, econometrics, and mathematical economics.

Economic cybernetics considers the economy, as well as its structural and functional parts as complex systems in which the processes of regulation and management, which are realized by the movement and transformation of information. Methods of economic cybernetics make it possible to standardize and unify this information, to rationalize the receipt, transmission and processing of economic information, to justify the structure and composition of technical means of its processing.

ALGORITHM OF ECONOMIC CIBERNETICS



The subject of economic cybernetics is the laws, principles and information processes of management of socio-economic systems.

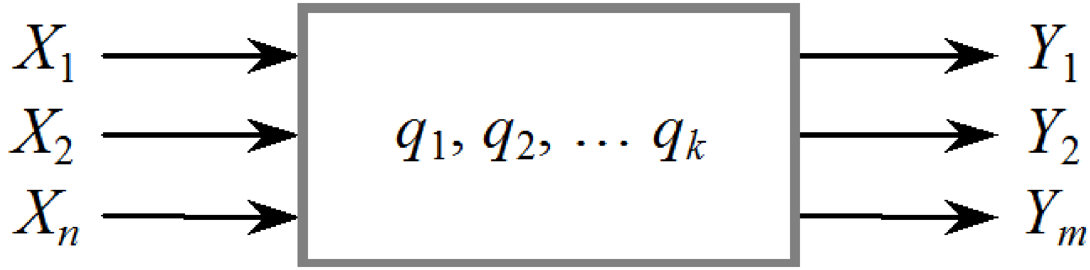
Methods of economic cybernetics:

Mathematical programming

Operations Research

Econometrics

Object

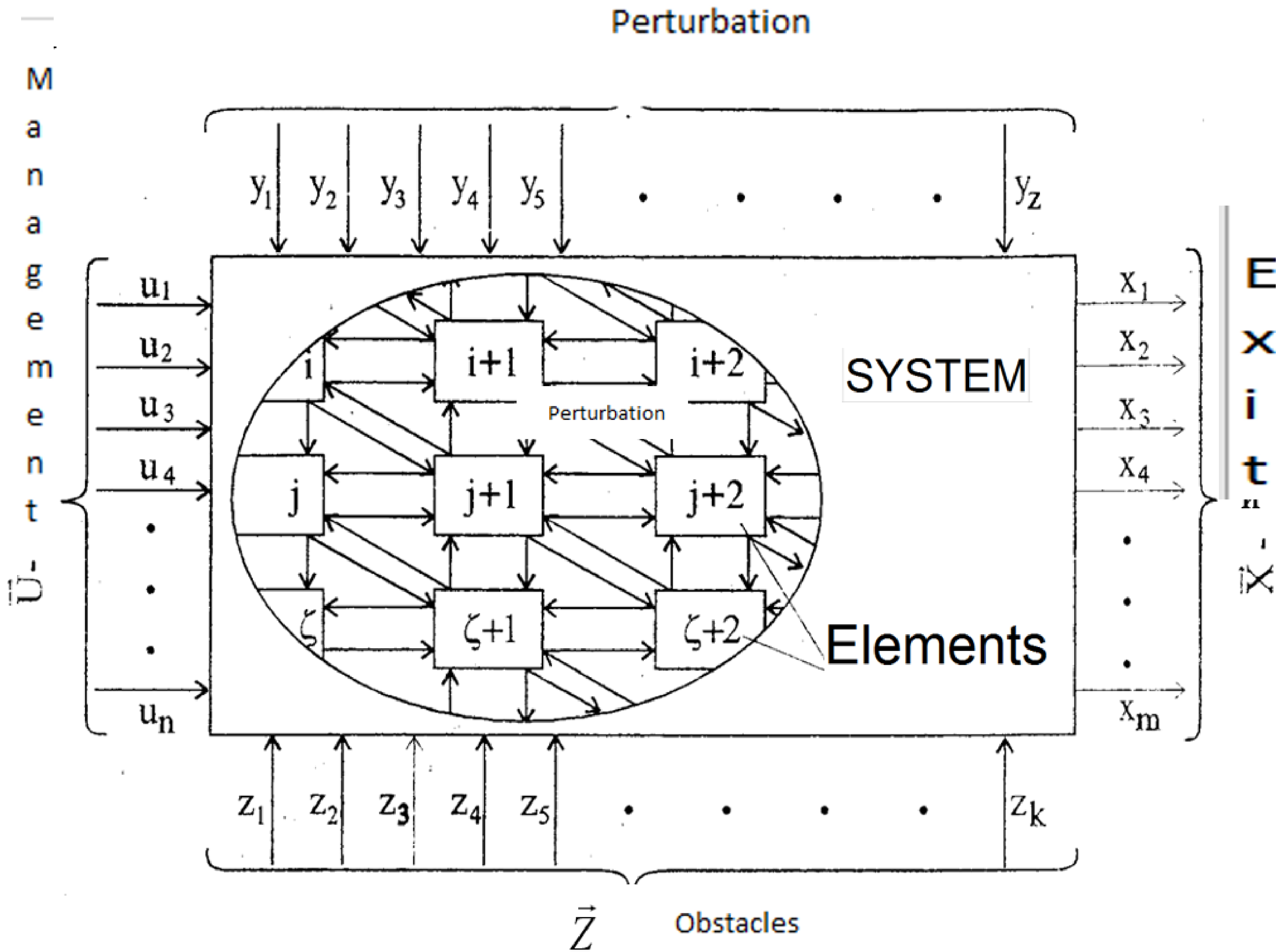


$$y = f\big(z(t), t\big), \qquad z = g\big(z(\tau), x(\tau)\big),\ \tau \le t$$

$$\frac{dy}{dt} = f(t, x, z), \qquad \frac{dz}{dt} = g(t, x, z)$$

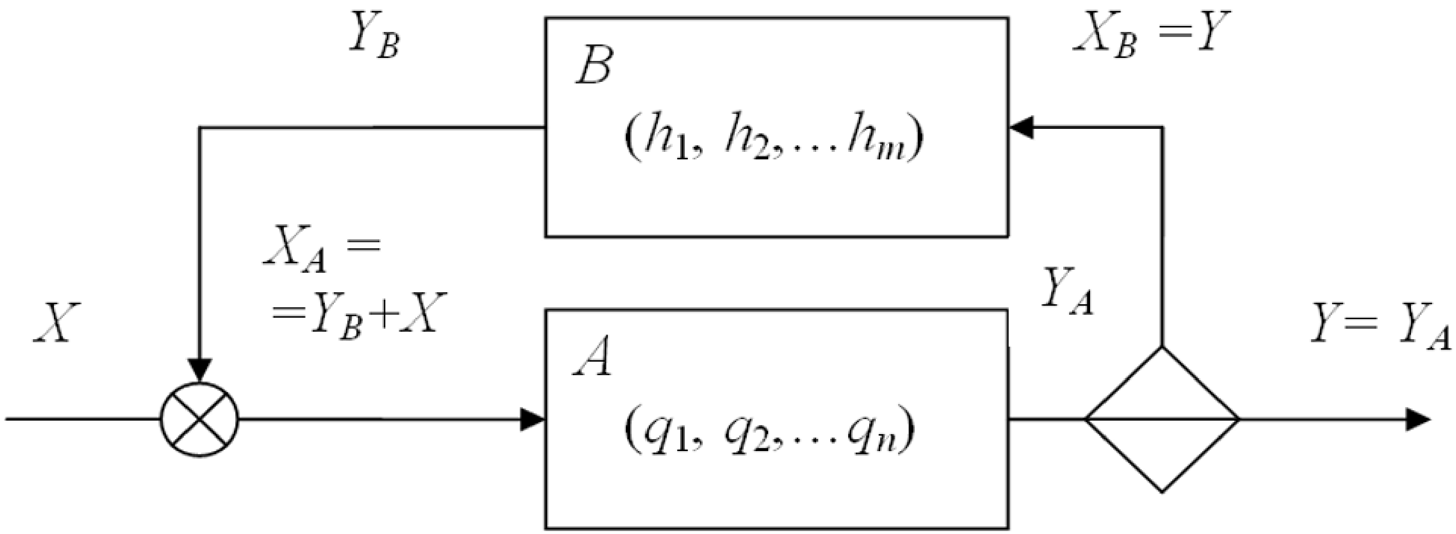
$$z(t_{k+1}) = g\big(t_k, z(t_k), x(t_k)\big)$$

SYSTEM



Perturbation
Management
$\vec{U}$ -
y_1 y_2 y_3 y_4 y_5 . . . . y_z
u_1 u_2 u_3 u_4 . . . u_n
i i+1 i+2
j j+1 j+2
ζ ζ+1 ζ+2
Perturbation
SYSTEM
Elements
x_1 x_2 x_3 x_4 . . . x_m
Exit
$\vec{X}$ -
z_1 z_2 z_3 z_4 z_5 . . . . z_k
$\vec{Z}$
Obstacles

Feedback



YA = *f*(*XA*, *q*1, *q*2,…*qn*), *YB* = *g*(*XB*, *h*1, *h*2,…*hm*)

Y = *f*(*XA* + *g*(*Y*, *h*1, *h*2,…*hm*), *q*1, *q*2,…*qn*),

$$Y_A = 5\frac{dX_A}{dt} + 3X_A - 10, \qquad Y_B = -8X_B + 4.$$

$$Y = 5\frac{d\left[X + (-8Y + 4)\right]}{dt} + 3\left[X + (-8Y + 4)\right] - 10, \quad 40\frac{dY}{dt} + 25Y = 5\frac{dX}{dt} + 3X + 2.$$

USE OF THE COMPUTER (MAXIMA PACKAGE) FOR SOLVING DIFFERENTIAL EQUATIONS

Syntax: $\frac{dy}{dx} \to {}'diff(y,x)$ $\quad x^n \to x\hat{}\,n$ $\quad \sqrt{x} \to sqrt(x)$

Example: Solve the Cauchy problem for differential equation of the first order

$$x^2 \cdot y' + 3 \cdot y \cdot x = \frac{\sin x}{x}$$

```
(%i1) x^2*'diff(y,x) + 3*y*x = sin(x)/x$

(%i2) ode2(%,y,x);
```

(%o2) $y = \frac{\%c - \cos(x)}{x^3}$

Initial Condition: $y(\pi) = 0$

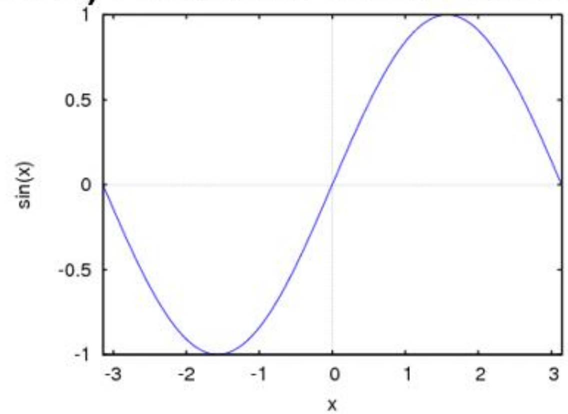
For the initial condition, the operator ***ic***1 is assigned.

```
(%i3) ic1(%o2,x=%pi,y=0);
```

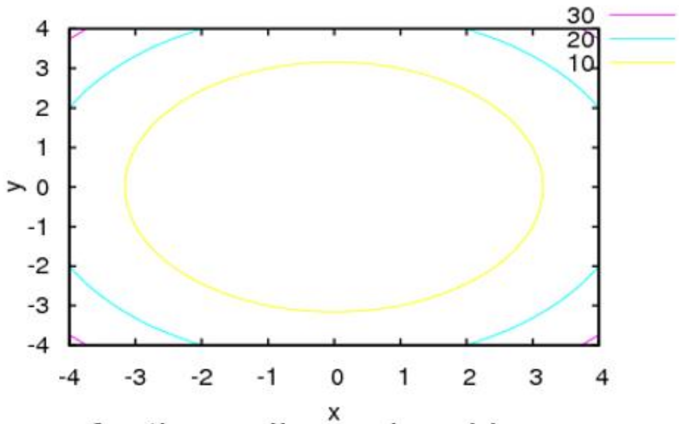
(%o3) $y = -\frac{\cos(x) + 1}{x^3}$

USE OF A COMPUTER (MAXIMA PACKAGE) FOR GRAPHICS BUILDING

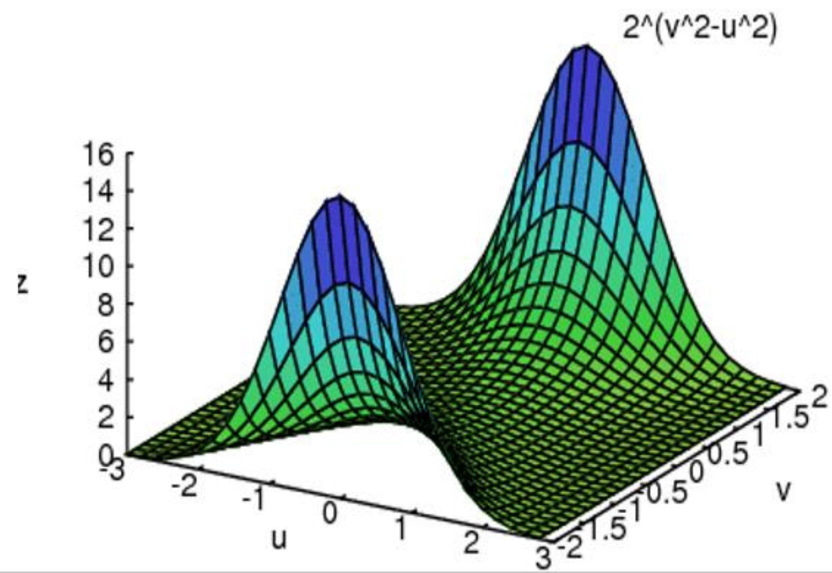
Draw a single variable function:
(%i1) plot2d (sin(x), [x, -%pi, %pi])$
або
(%i1) plot2d (sec(x), [x, -2, 2], [y, -20, 20])$



Draw of the function of two variables
(%i1) contour_plot (x^2 + y^2, [x, -4, 4],
[y, -4, 4])$



Draw of a three-dimensional image
(%i1) plot3d (2^(-u^2 + v^2), [u, -3, 3],
[v, -2, 2])$



Information

$$H(X) = -\sum_{i=1}^{n} p_i \log p_i.$$

MAIN LAWS AND PRINCIPLES OF CIBERNETICS

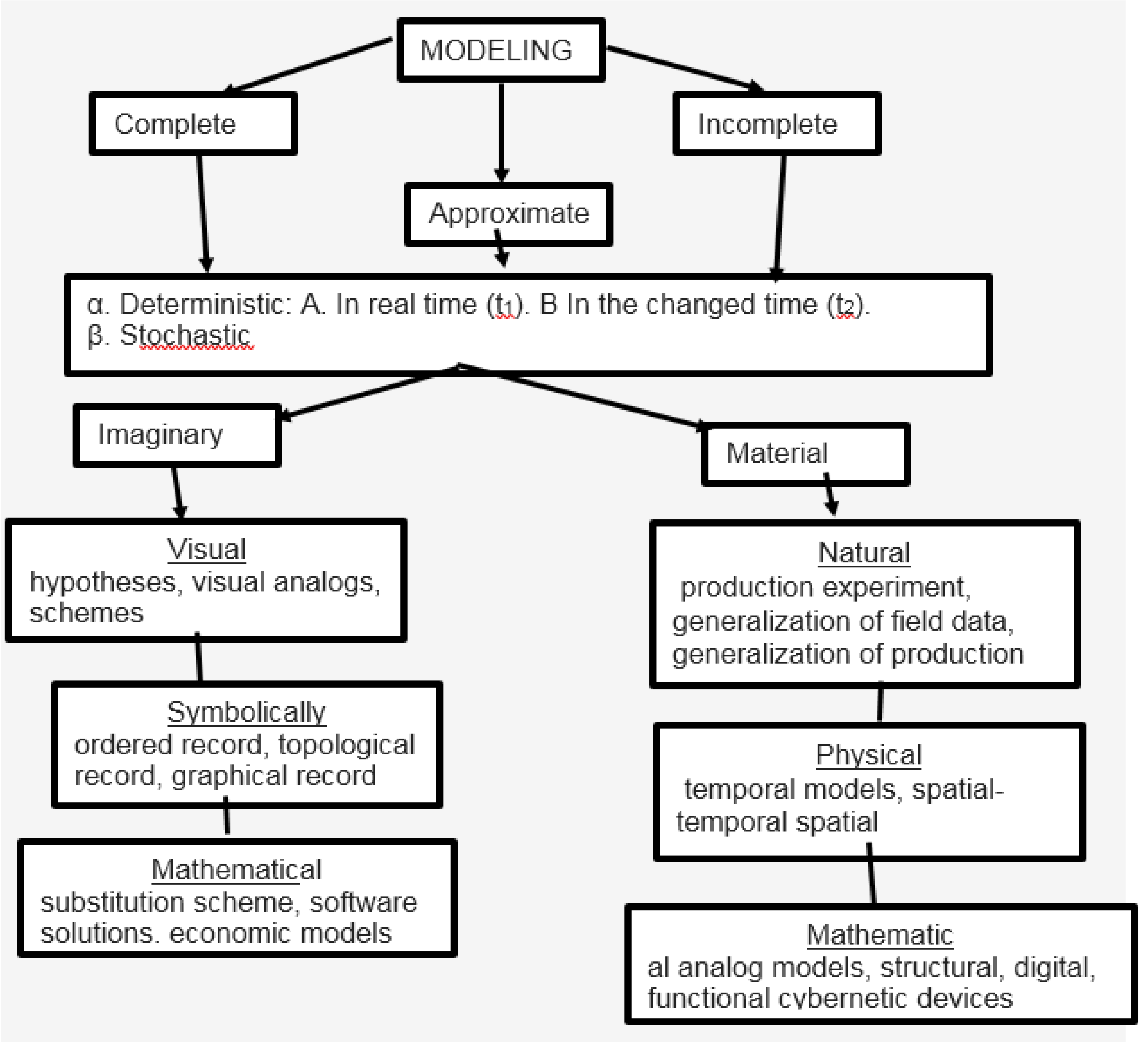
The law of the necessary diversity.

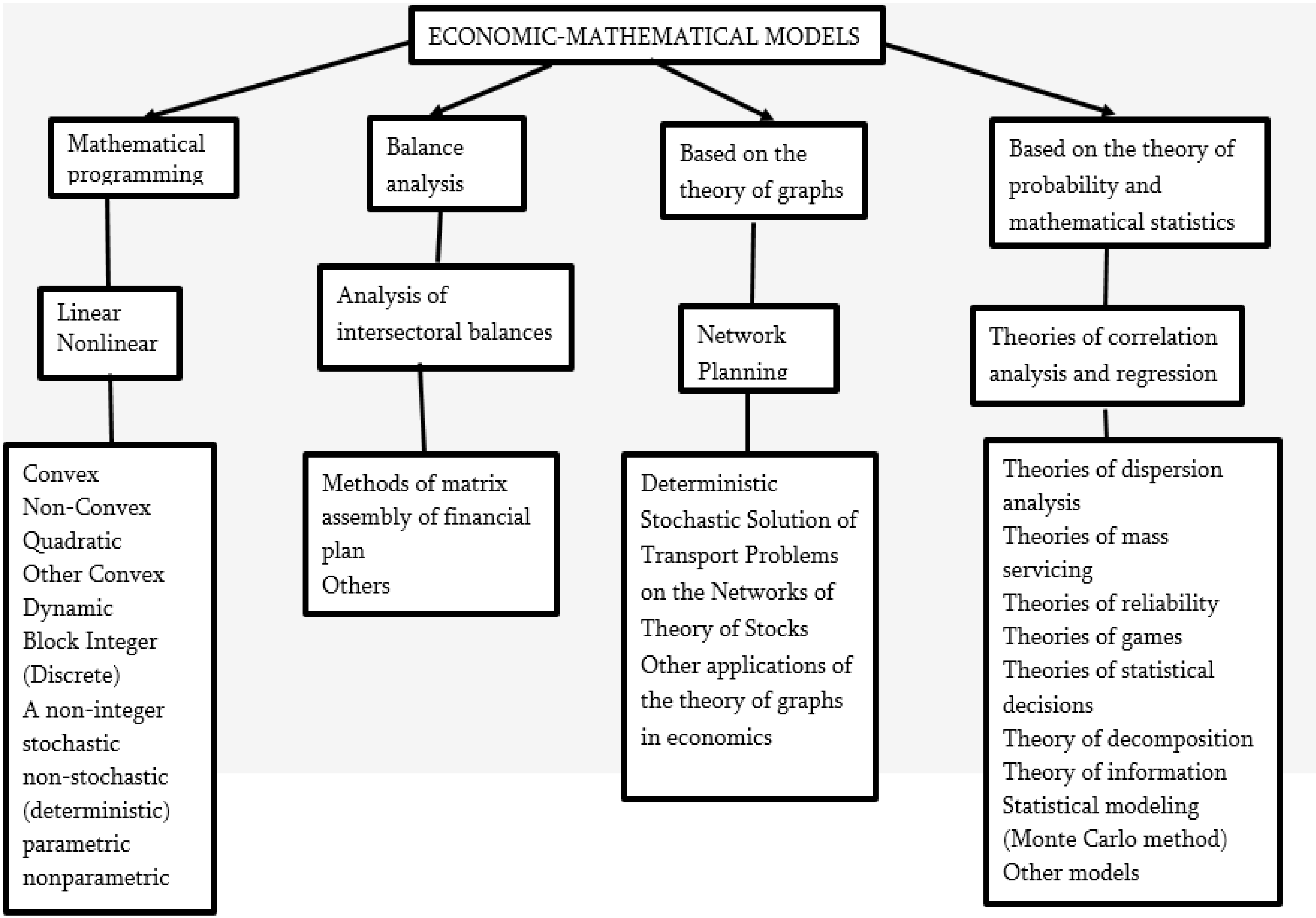
Principle of decision making based on selection and transforming information

The principle of mandatory feedback

Principle of external addition

2. MODELING OF SOCIO-ECONOMIC SYSTEMS AS THE MAIN METHOD OF ECONOMIC CYBERNETICS




ECONOMIC-MATHEMATICAL MODELS
Mathematical programming
Balance analysis
Based on the theory of graphs
Based on the theory of probability and mathematical statistics
Linear
Nonlinear
Analysis of intersectoral balances
Network Planning
Theories of correlation analysis and regression
Convex
Non-Convex
Quadratic
Other Convex
Dynamic
Block Integer
(Discrete)
A non-integer
stochastic
non-stochastic
(deterministic)
parametric
nonparametric
Methods of matrix assembly of financial plan
Others
Deterministic
Stochastic Solution of Transport Problems on the Networks of
Theory of Stocks
Other applications of the theory of graphs in economics
Theories of dispersion analysis
Theories of mass servicing
Theories of reliability
Theories of games
Theories of statistical decisions
Theory of decomposition
Theory of information
Statistical modeling
(Monte Carlo method)
Other models

Examples of cybernetic models of socio-economic systems

Interest calculation model

$$r_e = n\left(e^{\frac{r}{n}} - 1\right) \qquad r = n\ln\left(1 + \frac{r_e}{n}\right)$$

Models of financial calculations

$$NPV = \sum_{t=0}^{T} \frac{In_t - Ot_t}{(1+r_t)^t}$$

$$NPTV = \sum_{t=0}^{T} [In_t - Ot_t](1+r_t)^t$$

Cobb-Douglas Model

$$z = b_0 x^{b1} y^{b2}$$

Dynamic models

$$\frac{dx}{dt} = kx(N - x) \qquad x = \frac{N}{1 + (\alpha - 1)e^{-Nkt}}$$

Pistunova-Chukhlev's propensity to bankruptcy of trade and transport enterprises of Dnipropetrovsk region

$$Рейтинг = 0,9 \cdot \frac{ЧДР^{0,1} \cdot ВА^{0,63} \cdot СРП^{0,07}}{КЗ^{0,73} \cdot З^{0,07}} \left(\frac{(ЧП(З))}{ВА} + 3\right)^{0,29} \left(\frac{(ЧП(З))}{ЧДР} + 4\right)^{0,08},$$

Theory of sets

Operations over sets

Comparison $A \subset B :\Leftrightarrow x \in A \Rightarrow x \in B$

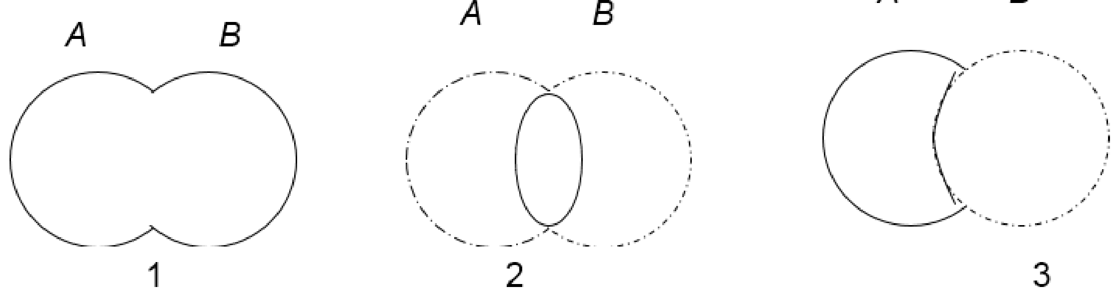
Association $A \cup B := \{x | x \in A \vee x \in B\}$

Crossing $A \cap B := \{x | x \in A \wedge x \in B\}$

Difference $A \setminus B := \{x | x \in A \wedge x \notin B\}$

Symmetric difference

$$A \triangle B \equiv A \dot{-} B := (A \cup B) \setminus (A \cap B) = \{x | (x \in A \wedge x \notin B) \veebar (x \notin A \wedge x \in B)\}$$



Example of concepts (indicated by solid lines): the union (1), the intersection (2) and the difference (3) of the sets A and B.

Tables of special mathematical symbols

Symbol	Name / Pronunciation / The section of mathematics	Value	Example
$\wedge$	Conjunction «and» Mathematical logic	$A \wedge B$ true if and only if A and B are both true.	$(n > 2) \wedge (n < 4) \Leftrightarrow (n = 3)$ if n is a natural number.
$\vee$	Disjunction «or» Mathematical logic	$A \vee B$ true, if at least one of the conditions A and B is true.	$(n \leqslant 2) \vee (n \geqslant 4) \Leftrightarrow n \neq 3$ if n is a natural number.
$\neg$	Denial "not" Mathematical logic	$\neg A$ true if and only if A is false.	$\neg(A \wedge B) \Leftrightarrow (\neg A) \vee (\neg B)$
$\forall$	Quantum of universality "For Any", "For All" Mathematical logic	$\forall x, P(x)$ denotes "*P (x)* true for all *x*".	$\forall n \in \mathbb{N},\ n^2 \geqslant n$
$\in$	Affiliation / not belonging to the plural "Belongs", "c" does not belong Theory of sets	$a \in S$ means "a is an element of the set S" $a \notin S$ means "a is an not element of the set S"	$2 \in \mathbb{N}$

$\subseteq$	Subset "Is a subset", "included in" Theory of sets	$A \subseteq B$ means "every element of A is also an element of B". $A \subset B$ usually means the same as that $A \subseteq B$. However, some authors use to indicate rigorous inclusion (i.e. $\subsetneq$).	$(A \cap B) \subseteq A$
$\cup$	Association "Association. and. ",".., united with. " Theory of sets	$A \cup B$ means a plurality of elements belonging to A or B (or both at once).	$A \subseteq B \Leftrightarrow A \cup B = B$
$\cap$	Crossing "Crossing. and ",".., Crossed with. " Theory of sets	$A \cap B$ means a plurality of elements belonging to both A and B.	$\{x \in \mathbb{R} \mid x^2 = 1\} \cap \mathbb{N} = \{1\}$
$\mathbb{N}$	Natural numbers «En» Numbers	$\mathbb{N}$ means plural $\{1,\ 2,\ 3,\ \ldots\}$ or $\{0,\ 1,\ 2,\ 3,\ \ldots\}$ (depending on the situation).	$\{\lvert a\rvert \mid a \in \mathbb{Z}\} = \mathbb{N}$
$\mathbb{Z}$	Whole numbers «Zed» Numbers	$\mathbb{Z}$ means plural $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$	$\{a,\ -a \mid a \in \mathbb{N}\} = \mathbb{Z}$
$\mathbb{Q}$	Rational numbers «Qu» Numbers	$\mathbb{Q}$ means plural $\left\{ \frac{p}{q} \middle\vert p \in \mathbb{Z} \wedge q \in \mathbb{Z} \wedge q \neq 0 \right\}$	$3{,}14 \in \mathbb{Q}$

An example of the format of the observation table by factors socio-economic system

Time or date of observation	Incoming factors				Output factors			
	X_1	X_2	…	X_n	Y_1	Y_2	…	Y_m

Statistical

$$M_X = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$$D_X = \frac{1}{N-1}\sum_{i=1}^{N} X_i^2 - M_X^2$$

$$\sigma_X = \sqrt{D_X}$$

$$\varepsilon_m = \ddot{\sigma}_x \Phi^{-1}(\beta)$$

$$\varepsilon_D = \ddot{D}_x \Phi^{-1}(\beta)\sqrt{\frac{0,8N+1,2}{N(N-1)}}$$

$$\ddot{\sigma}_m = \sqrt{\frac{D_X}{N}}$$

$$\mathrm{var}(X) = \frac{\ddot{D}(x)}{\ddot{M}(X)}$$

$$K\,\mathrm{var}(X) = \frac{\ddot{\sigma}(x)}{\ddot{M}(X)}$$

Dispersion analysis

$$r_{xy} = \frac{\sum (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \cdot \sum (y_i - \bar{y})^2}}$$

$$z = \frac{1}{2} \ln \frac{1+r}{1-r},$$

$$z' - t_\gamma \sqrt{\frac{1}{n-1-3}} \le z \le z' + t_\gamma \sqrt{\frac{1}{n-1-3}}$$

Analysis of expert conclusions

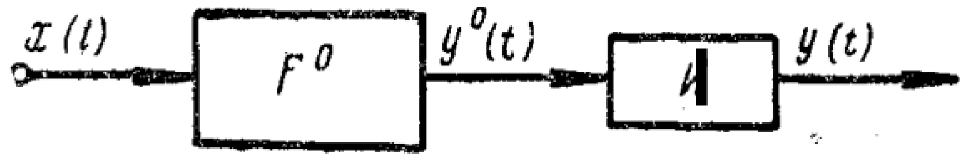
According to Spearman

$$\rho = 1 - \frac{6 \sum d_i^2}{n^3 - n}$$

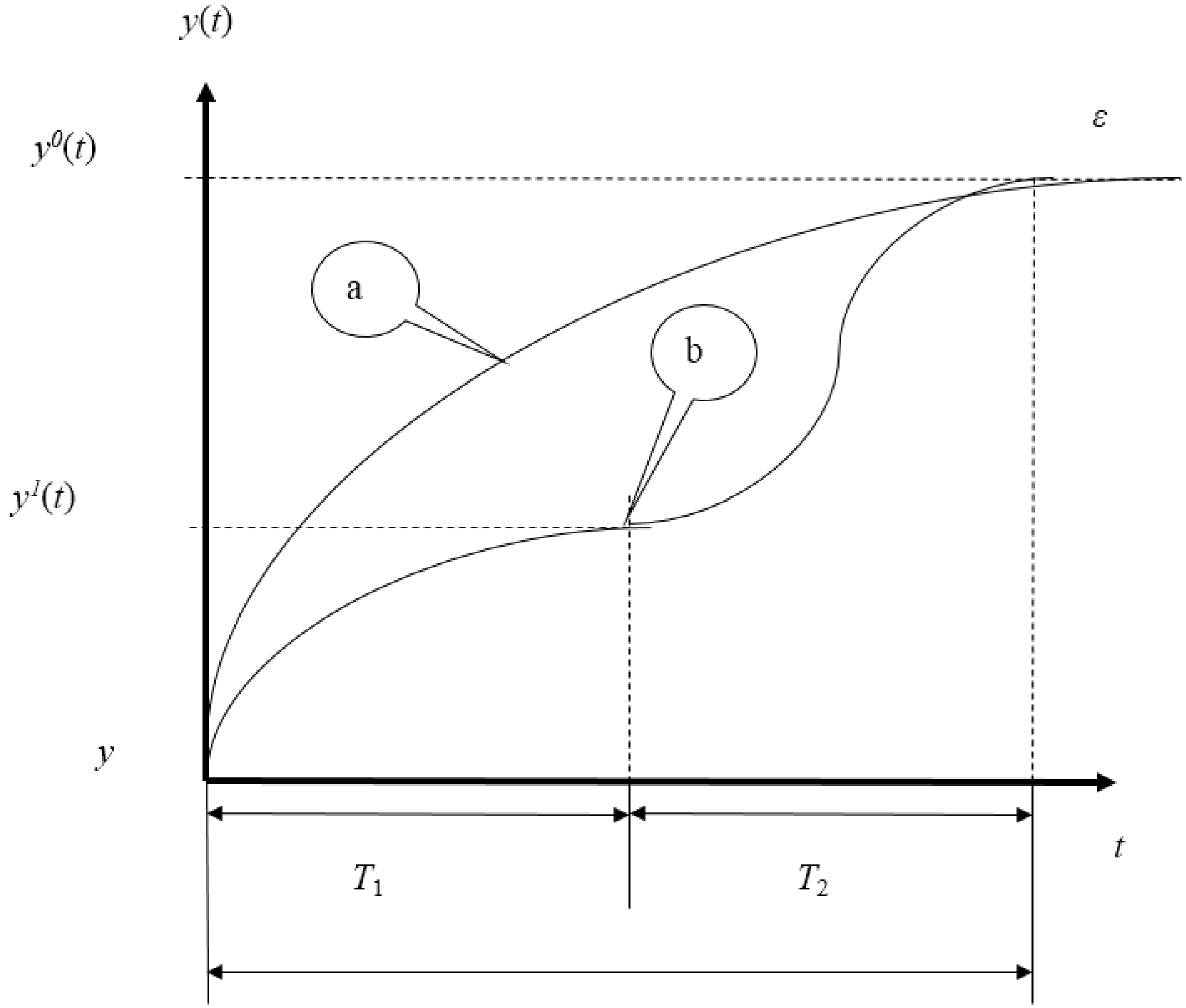
Behind Kendal

$$W = \frac{12 \sum_{i=1}^{m} \left(\sum_{j=1}^{d} \rho_{ij} - \frac{d(m+1)}{2} \right)^2}{d^2 (m^3 - m)}$$

Late analysis



Types of inertial processes in socio-economic systems



$$T = \frac{1}{\lambda} \ln \frac{y^0}{\varepsilon}$$

$$\lambda = \frac{1}{\tilde{T}},$$

$$\varepsilon = \frac{y^0}{e} \simeq 0{,}379 y^0$$

Spectral analysis

The Fourier series - in mathematics - is a way to represent an arbitrary complex function with a simpler sum. In the general case, the number of such functions can be infinite, with the more such functions are taken into account in the calculation, the higher the final accuracy of the representation of this function becomes. In most cases trigonometric functions of sinus and cosine are used as the simplest. In this case, the Fourier series is called trigonometric, and the calculation of such a series is often called a harmonic expansion.

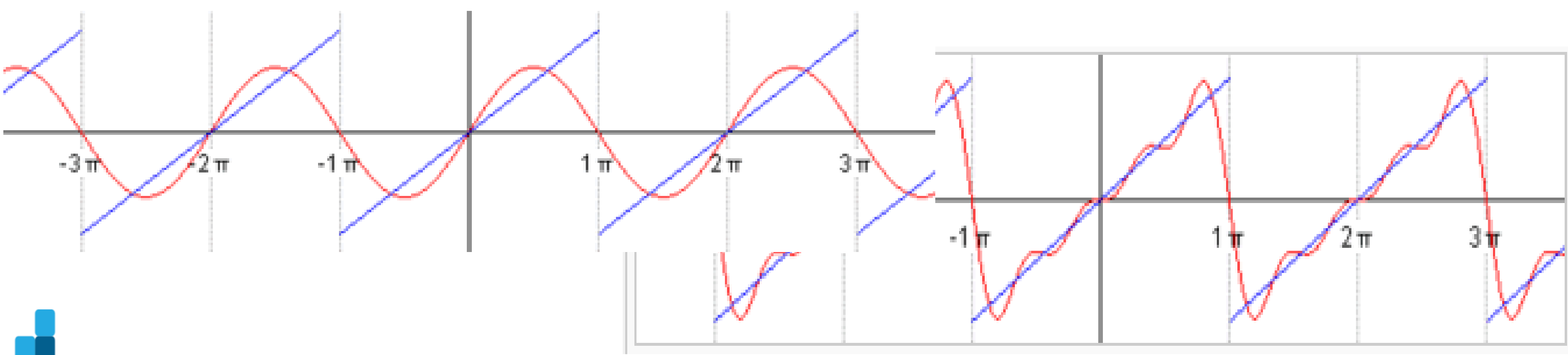
A Fourier trigonometric series is called a series of form

$$y = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(nx) + b_n \sin(nx)\right]$$

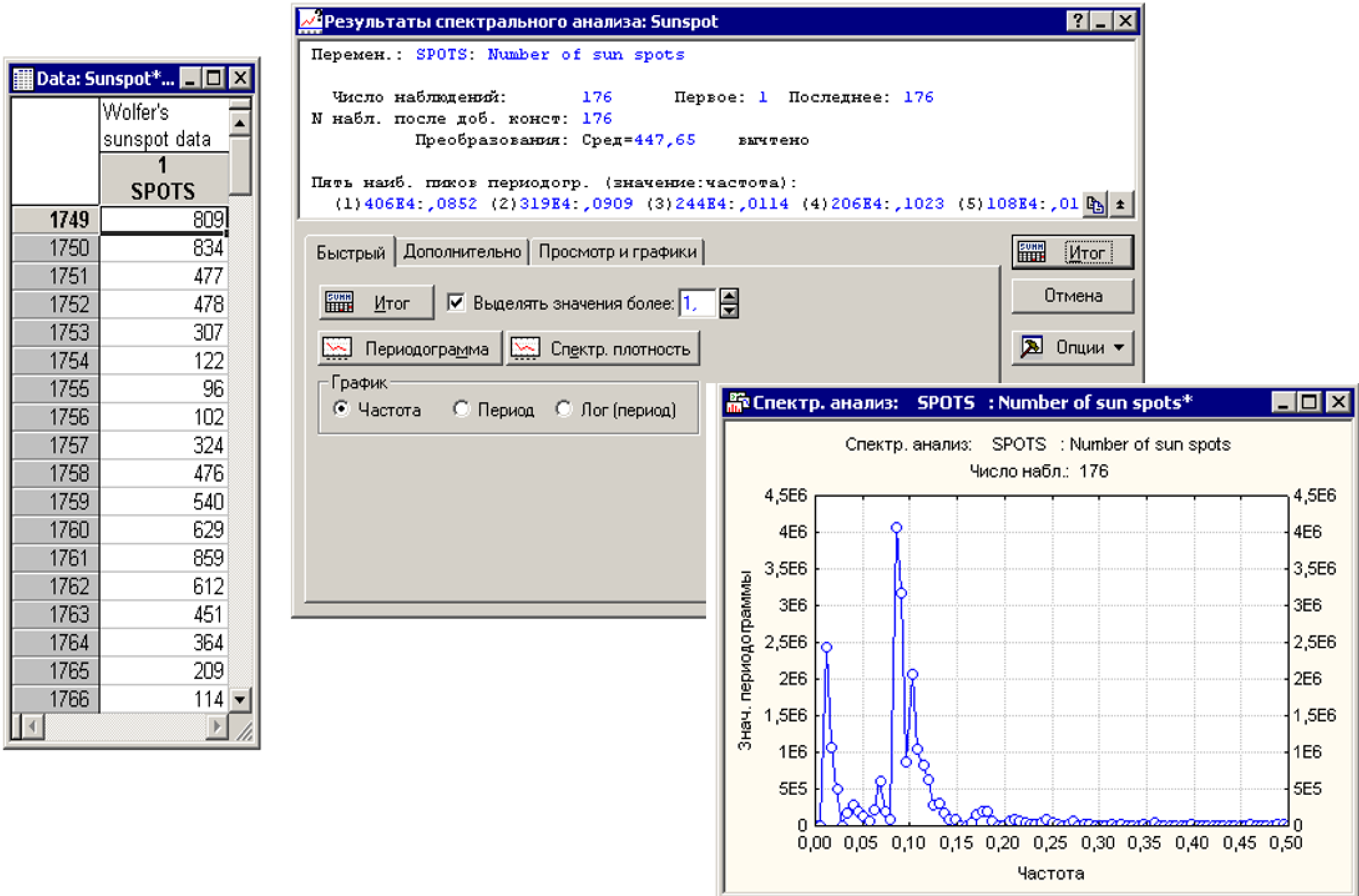
Stain numbers $a_0, a_n, b_n \quad (n \in \mathbb{N})$ called coefficients trigonometric series and are like

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx, \qquad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx.$$

Convergence of the Fourier series

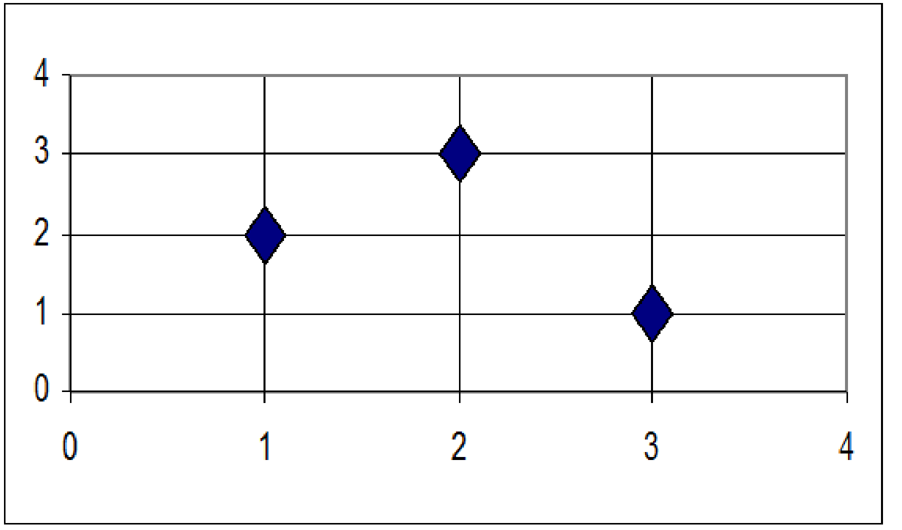


CONSTRUCTION OF THE AMPLITUDE-FREQUENCY CHARACTERISTICS OF PROCESS BY STATISTICA PACKET



Cluster analysis

Distances between two objects



Representation of three objects, as points on a plane

$$d_S(x_i;y_i)=\left(\sum_{i=1}^{Nf}|x_i-y_i|^p\right)^{\frac{1}{r}}$$

$$d_M(x_i;y_i)=\sqrt{\sum_{i=1}^{Nf}(\sqrt{x_i}-\sqrt{y_i})^2}$$

$$d_E(x_i;y_i)=\sqrt{\sum_{i=1}^{Nf}(x_i-y_i)^2}$$

$$d_{XEM}(x_i;y_i)=\sum_{i=1}^{Nf}(x_i-y_i)$$

$$d_{SUP}(x_i;y_i)=SUP|x_i-y_i|$$

$$d_L(x_i;y_i)=\sum_{i=1}^{Nf}|x_i-y_i|$$

Clustering with complete overview of objects

$$Z = \sum_{i=1}^{N_0} \sum_{j=1}^{N_0} q_{ij} d_{ij} \to \max$$

$$\sum_{i=1}^{N_0} q_{ij} \le N_0$$

$$\sum_{j=1}^{N_0} q_{ij} = 1$$

$$\sum_{i=1}^{N_0} \sum_{j=1}^{N_0} q_{ij} = N_0$$

	1	2	3	4	5
1	0	0,89	0,41	0,74	0,46
2	0,89	0	0,86	0,87	0,61
3	0,41	0,86	0	0,96	0,66
4	0,74	0,87	0,96	0	0,62
5	0,46	0,61	0,66	0,62	0

		Objects					Amount by clusters
		1	2	3	4	5	
Clusters	1	0	1	0	0	0	1
	2	1	0	0	0	0	1
	3	0	0	0	1	1	2
	4	0	0	1	0	0	1
	5	0	0	0	0	0	0
Amount by columns		1	1	1	1	1	5

Dispersion analysis (continued)

- Student's criterion is used to determine how likely it is that two samples from the general population have the same mean.
- If the probability is small (<0.55), we can assume that the samples have significantly different meanings, and therefore they can not be combined in one to construct a mathematical model.
- To implement this distribution law, there is the TTEST function
- TTEST (Data 1; Data 2; Mode; Type)

- The F-test (Fisher) determines the one-way probability that the variance of the arguments array1 and array2 differ insignificantly. This function is used to determine if there are two different variances in the sample. If the probability is small (<0.55), this means that the variance of the sample varies significantly, and therefore, simulation can not use data from the samples at the same time.
- To implement this test, there is the FTEST function
- FTEST (array1; array2)

Creation of models of social and economic systems

This direction of scientific activity can be divided into three types:

The first type is the development of models when the structure of the socio-economic system is known. For example, develop a profit model for an enterprise that produces TVs, video recorders and music centers, if the price of these products is 1200, 800 and 950 UAH respectively. Each type of product includes four components, which have a price of 0.8, respectively; 1.2; 3.3; and 4.4 UAH. In each type of product, these components come in the following quantities: for TV - 1, 5, 8, 4 pcs .; for VCR - 0, 8, 6, 2 pcs .; for the music center - 4, 8, 3, 1 pc. Availability of spare parts in stock, respectively - 12000, 15000, 14000, 45000 pcs. The price of working time per unit of production is 12, 15 and 18 UAH respectively. Continuous costs are not counted.

The second type of scientific activity in the development of models refers to the case when the structure of the socio-economic system is unknown, but the researcher has a hypothesis as to what this structure is. For example, a well-known model of an advertising campaign based on such basic hypotheses. Let N (t) be the number of consumers who know about the product and intend to buy it (t - the time since the beginning of the advertising campaign), the value of dN / dt is the rate of change in the number of already-informed clients. It is assumed that dN / dt is proportional to the number of buyers who are not yet aware of this product (service), that is, the value $a_1(t)$ (Np - N (t)), where Np is the total number of potential paying customers, $a_1(t) > 0$ characterizes the intensity of the advertising campaign (which is determined by the cost of advertising at a given time).? It is also assumed that those who find out about the product, one way or another spread the information received among the ignorant, acting as additional advertising agents of the firm. Their contribution is determined by the value $a_2(t)$(N (t) (Np - N (t))) and will be greater than the larger number of agents. The value $a_2(t) > 0$ characterizes the degree of customer communication among themselves (it can be established by the survey) .The result is the following equation

$$\frac{dN}{dt} = \left[\alpha_1(t) + \alpha_2(t)N(t)\right](N_p - N(t)).$$

The third type of scientific activity in the development of models refers to the case where the structure of the socio-economic system is completely unknown. Then the researchers resort to regression analysis, which allows obtaining a formula for communication between input and output factors. The procedure described below is statistically not precise, but it is quite sufficient for practical application in solving microeconomic problems.

Synthesis of statistical linear and quasilinear models

Possible transformations

$$y = a_0 + \sum_{i=1}^{K} a_i x_i \qquad y = a_0 \ell^{a_i x_i} \qquad y = a_0 \log_n x$$

$$y = a_0 + \sum_{i=-n}^{-1} a_i x^i$$

$x_1\, x_2$, x_1/x_2, $x_1 - x_2$, $\log x_1\, x_2$,

An example of normalization-denormalization

$y = a_0 + a_1 x + a_2 x^2$

$$y = \left[M_y + \sigma_y \left(a_0 - \frac{a_1 M_x}{\sigma_x} - \frac{a_2 M_{x2}}{\sigma_{x2}} \right) \right] + \frac{a_1 \sigma_y}{\sigma_x} x + \frac{a_2 \sigma_y}{\sigma_{x2}} x^2$$

$\mathrm{Ln}y = a_0 + a_1 {*} \mathrm{Ln}x_1 + a_2 {*} \mathrm{Ln}x_2$

$$\mathrm{Ln}y = \left[M_y + \sigma_y \left(a_0 - \frac{a_1 M_x}{\sigma_x} - \frac{a_2 M_{x2}}{\sigma_{x2}} \right) \right] + \frac{a_1 \sigma_y}{\sigma_x} \mathrm{Ln}x_1 + \frac{a_2 \sigma_y}{\sigma_{x2}} \mathrm{Ln}x_2$$

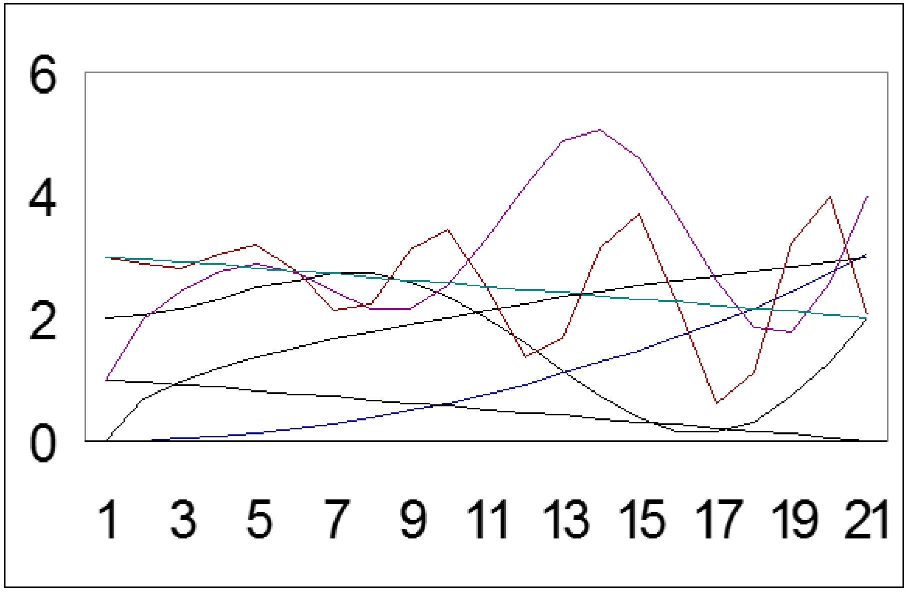
Synthesis of autoregressive models

Profitability	Preliminary value of profitability
5%	-
4%	5%
1%	4%
12%	1%

Synthesis of periodic models

$$y = Ax^B + C(1 - e^{Dx})$$

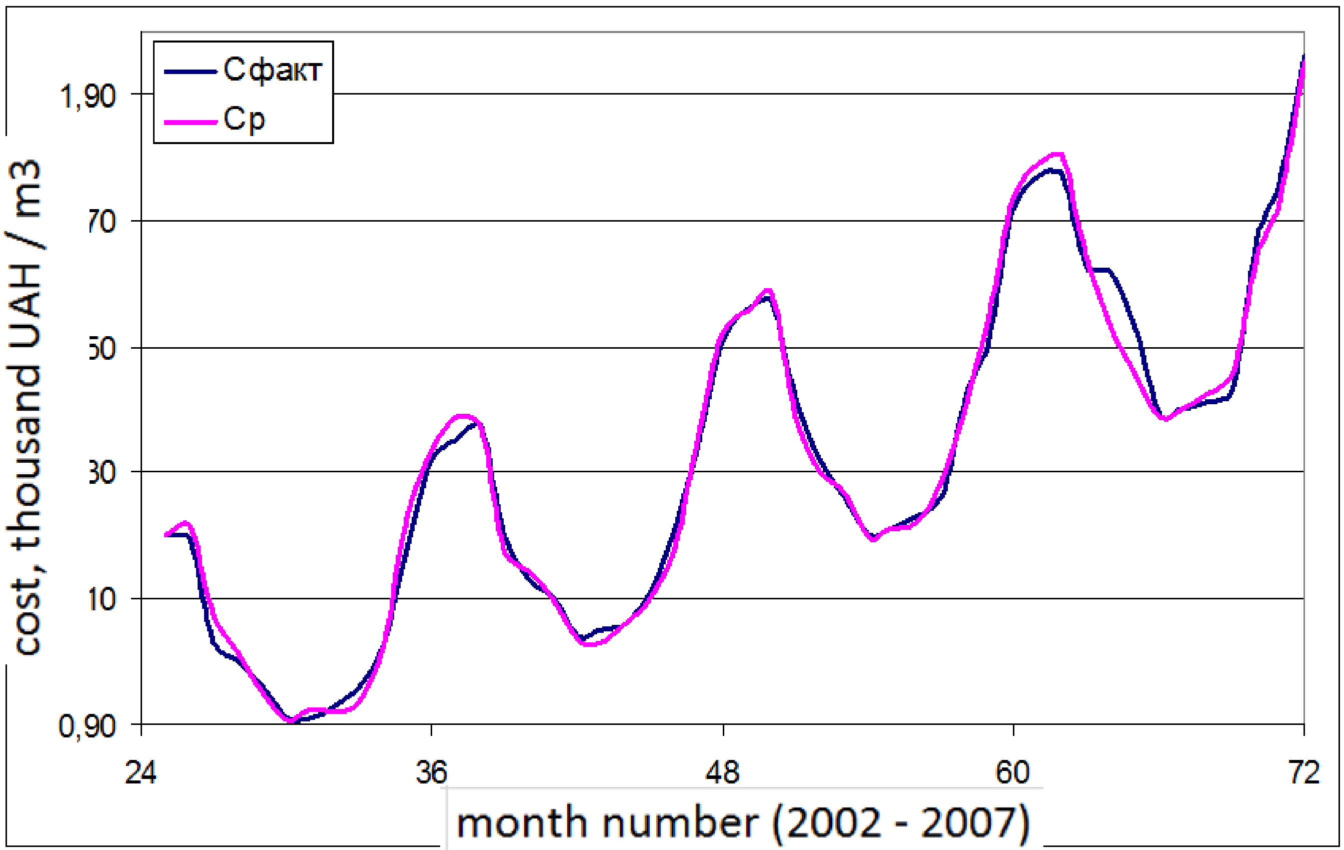
$$Sin(Ex^F + G) + H$$



An example of a periodic model

$$C_{t,1}=1{,}342c_{t-12,1}^{\;0{,}887}+0{,}525(1\text{-}e^{-0{,}976c_{t-12,1}})\sin(0{,}524c_{t-12,1}^{\;1{,}187}+0{,}664)\text{-}$$

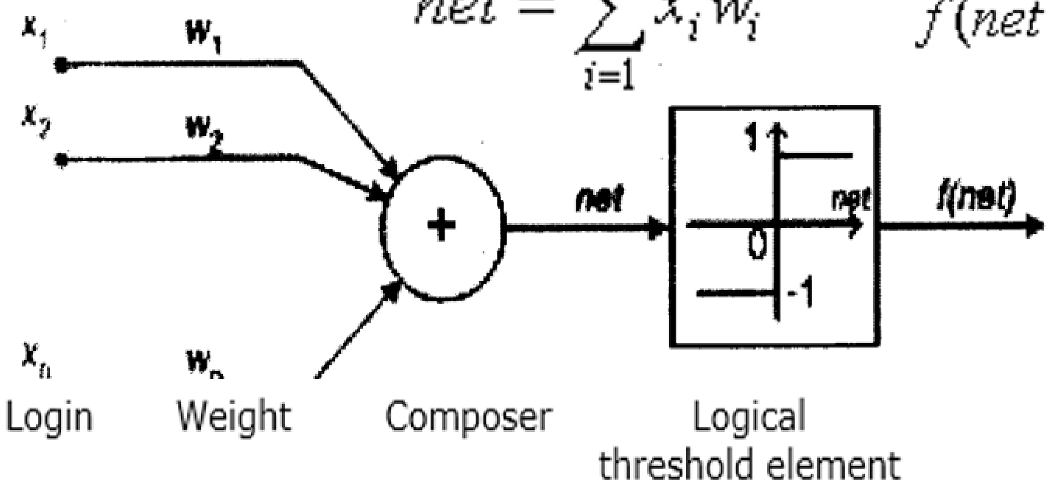
$$-0{,}402c_{t-24,1}^{\;0{,}686}+0{,}288(1\text{-}e^{-0{,}106c_{t-24,1}})\sin(0{,}524c_{t-24,1}^{\;0{,}808}+0{,}195)-0{,}146$$



Neural Networks

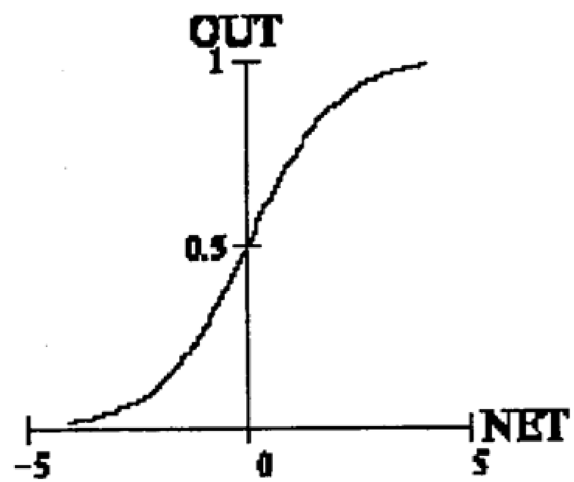
$$net = \sum_{i=1}^{n} x_i w_i$$

$$f(net) = \operatorname{sgn}(net) = \begin{cases} +1, & net > 0 \\ -1, & net < 0 \end{cases}$$



McCulloch-Pits boundary neuron model

$$f(net) = \frac{1}{1 + e^{-\lambda \cdot net}}$$



Type of logistics function

Algorithm of perceptron training

1. Primary weights may be any. The correction is proportional to the value of the derivative of the given coordinate. The derivative is taken from the activation function. Adjustment of j weight for i neuron is carried out according to the formula

$$\Delta w_{ij} = \eta [d_i - f(net_i)] \cdot f'(net_i) \cdot x_j ,$$

where j = 1,2, ..., n η - learning factor, is selected heuristic

2. Error while studying at step k:

$$E_k = \frac{1}{2}[d_i - f(net_i)]^2$$

where di - expected exit

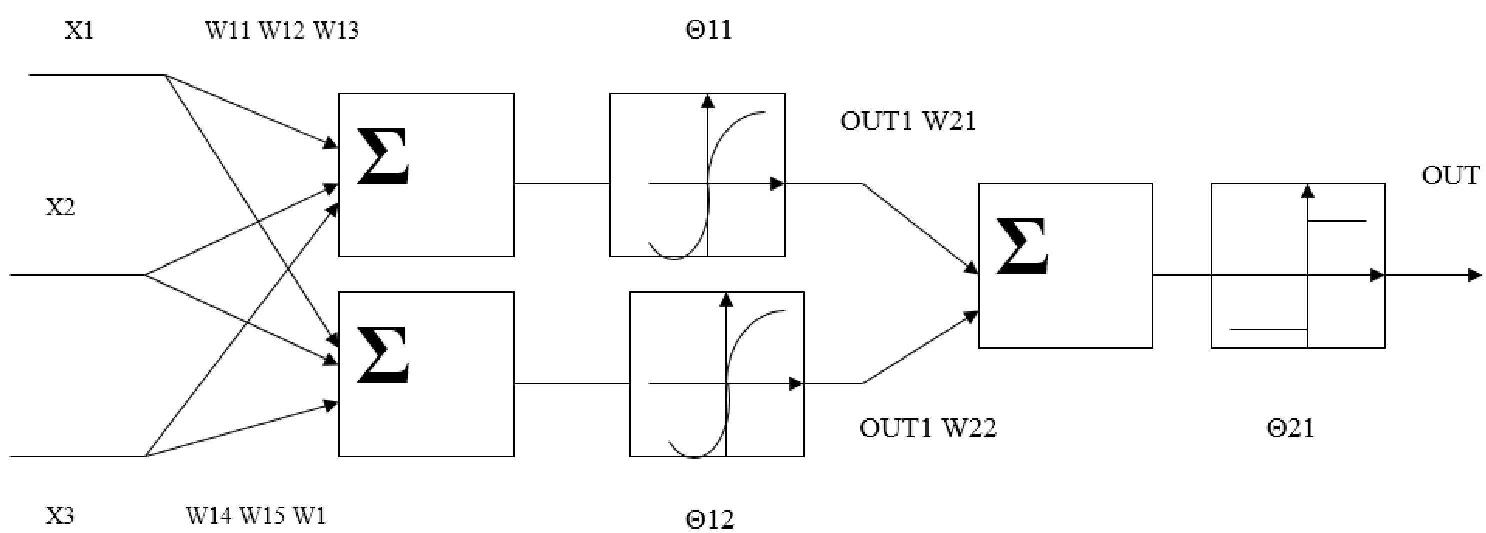
3. General learning error:

$$E = \frac{1}{2 \cdot p}\sum_{k=1}^{p}[d_i - f(net_i)]^2$$

where p is the number of examples in the training sample

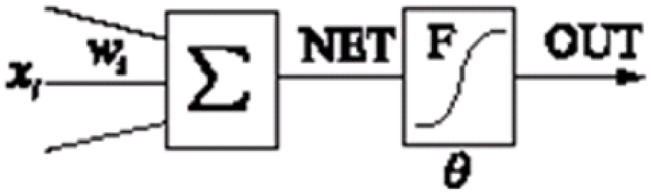
4. Derived from sigmoid where *p* - number of examples in the training sample.

$$f'(net) = \lambda \cdot f(net) \cdot [1 - f(net)]$$

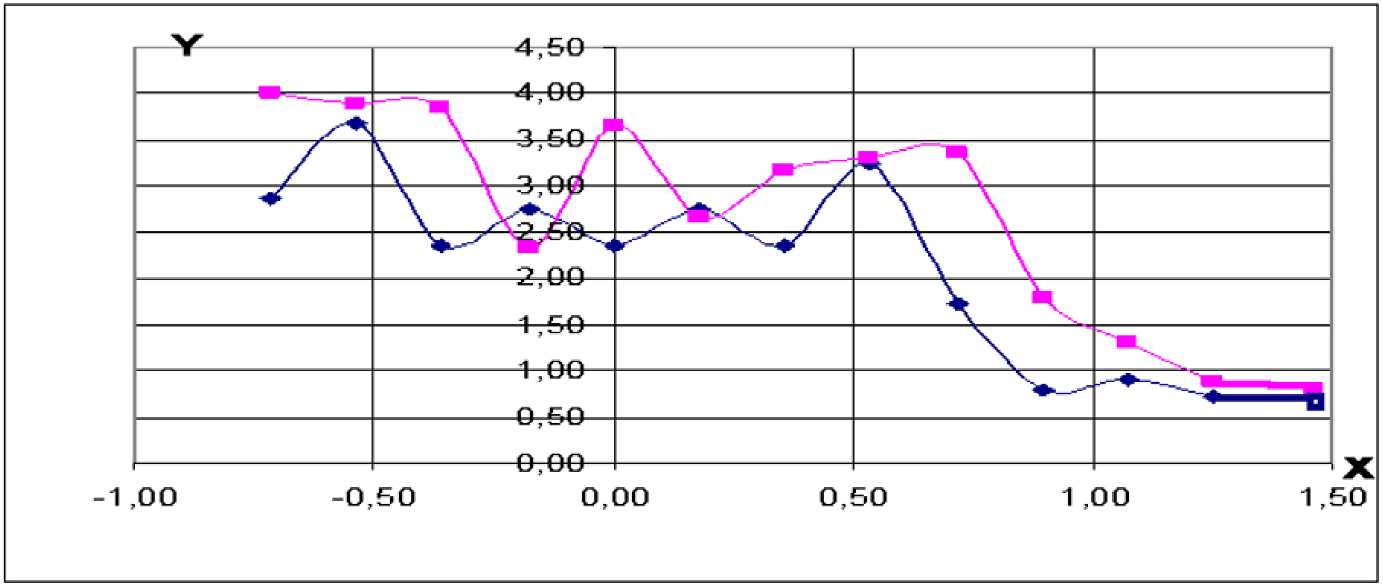


Examples of neural nets

General view of the perceptron scheme with one neuron and adder at the input

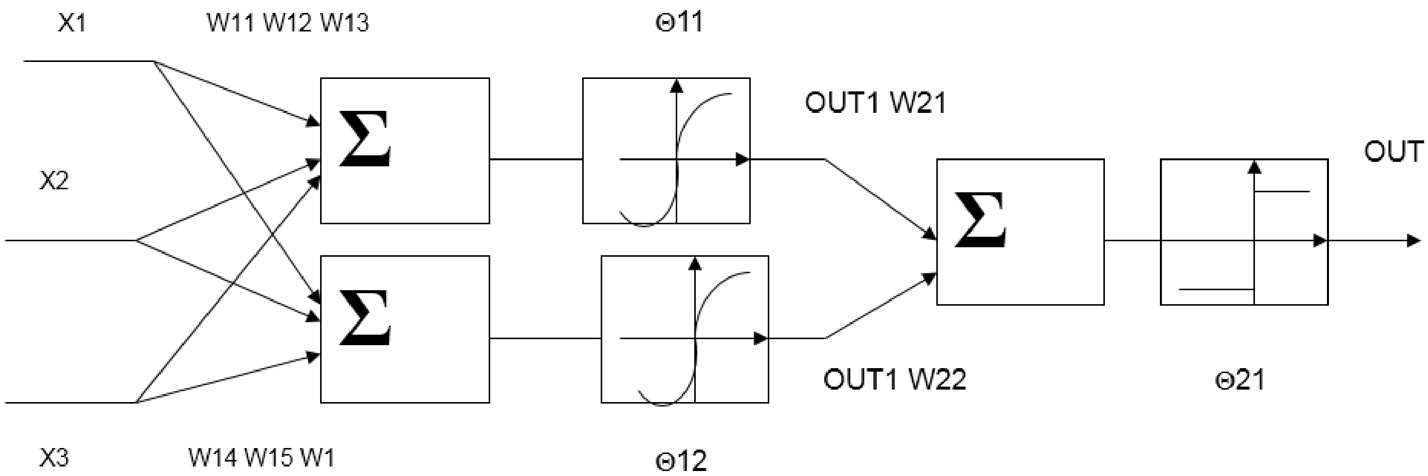


$$OUT = \frac{1}{1+\ell^{-(w_1 x_i + w_2 x_{i+1} + w_3 x_{i+2})}}$$



Graph of the number of calls Y
per hour of working hours X
◆ – experimental curve
■ – calculated curve

Scheme of a two-layer perceptron with three inputs on each neuron



$$OUT = Sign\left[\frac{10,37669298}{\left[1+\ell^{-(-1,443525248X_1+1,561436345X_2+1,582503396X_3+4,683681367)}\right]}+\frac{7,940045065}{\left[1+\ell^{-(1,556474752X_1+4,561436345X_2+4,582503396X_4+-3,683681367)}\right]}+2,883681367\right]$$

ASSESSMENT OF QUALITY OF APPROXIMATION AND FORECASTING

The most convenient test is the Pearson criterion, which is conducted in the following order:

1. To verify the adequacy of the approximation, the calculation of the initial values of the mathematical model is made, substituting in it the real input values for which this model was constructed. To determine the quality of prognosis, the values of input factors that are not used in calculating the model coefficients are subjected to the model.

2. For each pair calculated $y_{\Pi i}$ *and real values* y_{Pi} *are calculated by the criterion "hi-square" by the formula*

$$\chi_P^2 = \sum_{i=1}^{n} \frac{(y_{Pi} - y_{\Pi i})^2}{y_{Pi}}.$$

3. Determine the number of degrees of freedom as $r = n - 2$.

4. The theoretical value of the "chi-squared" is determined by the pre-determined probability of confidence. This confidence probability should be sufficiently high so that the researcher could trust the results (0,8-0,99). If this value is more than calculated, the model is adequate with the determined probability of probability. Otherwise - the model is not adequate, that is, it badly describes the process. The quality of forecasting is also understood by this probability.

5. In the case where it is necessary to determine the quality of forecasting, for the construction of the model, only part of the actual values of input and output factors are used. And for the remaining values, the predicted values of the source factor are calculated. then act on pp. 2-4.

Synthesis of dynamic models

Simple delay $$y(t) = y^0(t - \tilde{T}); \quad t \geqslant \tilde{T},$$

Frequently used model $$\frac{dy(t)}{dt} = -\lambda [y(t) - y^0(t)].$$

$$\frac{dy(t)}{dt} + \lambda y(t) = \lambda y^0, \qquad y(t) = (1 - e^{-\lambda t})\, y^0.$$

Discrete latency models

$$y_\tau = \beta_1 y^0_{\tau-1} + \beta_2 y^0_{\tau-2} + \ldots = \sum_{\theta=1}^{N} \beta_\theta y^0_{\tau-\theta}, \qquad \beta_\theta \leqslant \beta_{\theta+1}; \quad \sum_\theta \beta_\theta = 1.$$

$$\Delta y_\tau = \lambda [y^0_\tau - y_\tau],$$

$$\lambda = 1 - r, \text{ a } y_{\tau+1} = y_\tau + \Delta y_\tau; \; \tau = 1, 2, \ldots$$

Algebra of logic

Unary logical operations

x	$g_1 (\neg)$	$g_2 (=)$	$g_3 (1)$	$g_4 (0)$
0	1	0	1	0
1	0	1	1	0

Binary logical operations

x	y	$f_1 (\wedge)$	$f_2 (\vee)$	$f_3 (\equiv)$ $x_1 \approx x_2$	$f_4 (\oplus)$	$f_5 (\subset)$	$f_6 (\supset)$ $x_1 \to x_2$	$f_7 (\downarrow)$	$f_8 (\mid)$
0	0	0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	1	0	1
1	0	0	1	0	1	1	0	0	1
1	1	1	1	1	0	1	1	0	0
x	y	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}
0	0	0	0	1	1	0	0	1	0
0	1	0	1	1	0	0	1	1	0
1	0	1	0	0	1	1	0	1	0
1	1	0	0	0	0	1	1	1	0

Properties of logical operations

1. Communicative: $xy = yx$ $\circ \in \{\&, \vee, \oplus, \sim, |, \downarrow\}$.
2. Identity: $xx = x$ $\circ \in \{\&, \vee\}$.
3. Associativity: $(xy)\circ z = x\circ(yz)$ $\circ \in \{\&, \vee, \oplus, \sim\}$.
4. Distribution of conjunctions and disjunctions

Regarding disjunction, conjuncture and modulus amount two respectively:

- $x\wedge(y\vee z) = (x\wedge y)\vee(x\wedge z)$,
- $x\vee(y\wedge z) = (x\vee y)\wedge(x\vee z)$,
- $x\wedge(y\oplus z) = (x\wedge y)\oplus(x\wedge z)$.

4. The laws of de Morgan:

- $\neg(x\wedge y) = (\neg x)\vee(\neg y)$,
- $\neg(x\vee y) = (\neg x)\wedge(\neg y)$.

5. Laws of absorption:

- $x\wedge(xy) = x$
- $x\vee(xy) = x$.

6. Others (1):

- $x\wedge(\neg x) = x\wedge 0 = x\oplus x = 0$.
- $x\vee(\neg x) = x\vee 1 = x\sim x = x\rightarrow x = 1$.
- $x\vee x = x\wedge x = x\wedge 1 = x\vee 0 = x\oplus 0 = x$.
- $x\oplus 1 = x\rightarrow 0 = x\sim 0 = x|x = x\downarrow x = \neg x$.
- $\bar{\bar{x}} = x$.

7. Others (2):

- $x \oplus y = (x \wedge \bar{y}) \vee (\bar{x} \wedge y) = (x \vee y) \wedge (\bar{x} \vee \bar{y})$.
- $x \sim y = \overline{x \oplus y} = (x \wedge y) \vee (\bar{x} \wedge \bar{y}) = (x \vee \bar{y}) \wedge (\bar{x} \vee y)$
- $x \rightarrow y = \bar{x} \vee y = ((x \wedge y) \oplus x) \oplus 1$.

8. Others (3) (Addition of de Morgan's laws):

- $x \mid y = \neg(x \wedge y) = \neg x \vee \neg y$.
- $x \downarrow y = \neg(x \vee y) = \neg x \wedge \neg y$.

9. The laws of gluing:

$$(a \vee \bar{b}) \wedge (a \vee b) = a$$

$$(a \wedge \bar{b}) \vee (a \wedge b) = a$$

10. Two more obvious laws

$$x \vee \bar{x} = 1; \qquad x \wedge \bar{x} = 0.$$

In algebra of logic, logical connections and their corresponding logical operations have special names and are denoted as follows:

Logical connection	The name of the logical operation	Marking
Not	Denial, inversion	$\neg$, $\rceil$
And, but	Conjunction, logical multiplication	$\&$, $\bullet$, $\wedge$
Or	Disjunction, logical addition	$\vee$, $+$
If...then	Implication, follow up	$\Rightarrow$, $\rightarrow$
Then and only when	Equivalence	$\Leftrightarrow$, $\sim$, $\equiv$, $\leftrightarrow$

Fuzzy models

The function of belonging $A = \{x/mA(x)>0\}$.

Types of belonging functions

Triangular

$$MF(x)=\begin{cases}1-\dfrac{b-x}{b-a}, & a\le x\le b\\ 1-\dfrac{x-c}{c-b}, & b\le x\le c\\ 0, & \text{in other cases}\end{cases}$$

Trapezoid

$$MF(x)=\begin{cases}1-\dfrac{b-x}{b-a}, & a\le x\le b\\ 1, & b\le x\le c\\ 1-\dfrac{x-c}{d-c}, & c\le x\le d\\ 0, & \text{in other cases}\end{cases}$$

Gaussian

$$MF(x)=\exp\left[-\left(\frac{x-c}{\sigma}\right)^2\right]$$

Circular

$$\mu(x)=\frac{1}{1+\dfrac{(x-a)^2}{b}}/x$$

$$\mu(x)=a\sqrt{\frac{x}{x_{max}}\left(1-\frac{x}{x_{max}}\right)}$$

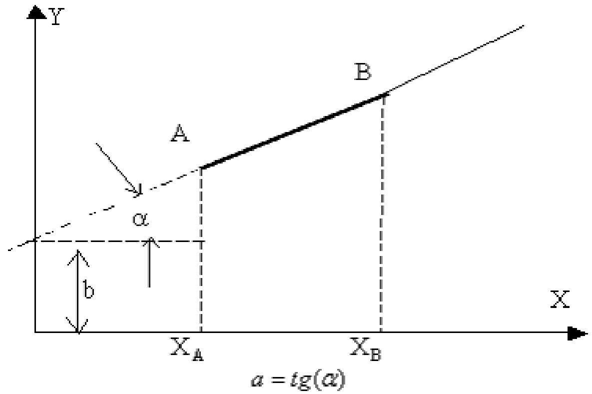
An example of the appearance of membership functions

Triangular Trapezoidal Gaussian

The structure of the fuzzy control system

IF x_1 it A_1 . AND . x_2 it A_2, THEN y it B.

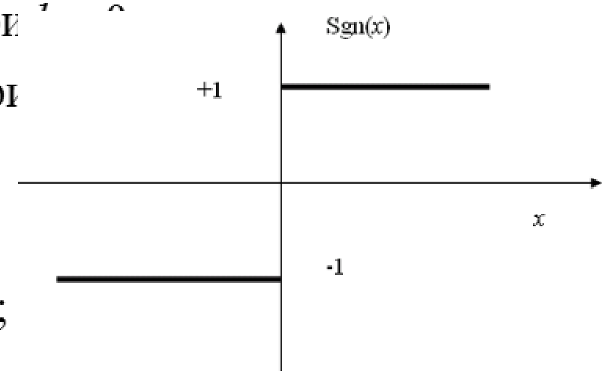
SELECTIVE FUNCTIONS



Example description of segment AB

$$y = Si(x, x_A, x_B)\cdot(a\cdot x + b)$$

$$sign(a) = \begin{cases} 1, & \text{при } b > 0; \\ 0, & \text{при} \ldots \\ -1, & \text{при} \ldots \end{cases}$$

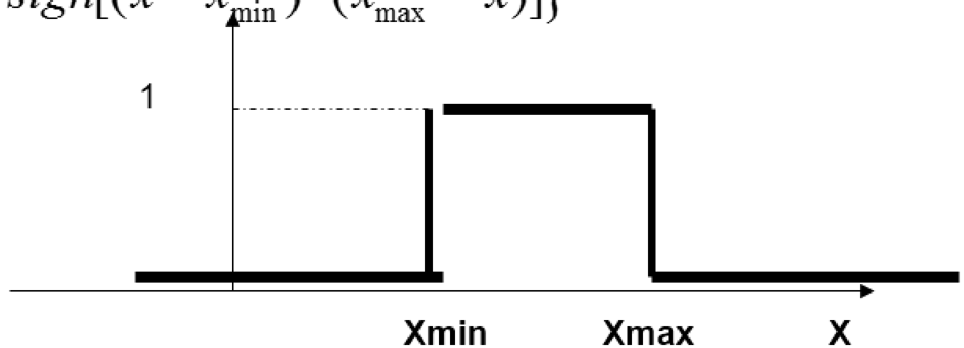


The look of the function sgn (x)

$$\operatorname{sgn}(a) = \begin{cases} 1, & \text{при } b \geq 0; \\ -1, & \text{при } b < 0. \end{cases}$$

$$Si(x, x_{\min}, x_{\max}) = \frac{1}{2}\cdot\{1 + \operatorname{sgn}[(x - x_{\min})\cdot(x_{\max} - x)]\}$$

$$Si(x, x_{\min}, x_{\max}) = sign\{1 + sign[(x - x_{\min})\cdot(x_{\max} - x)]\}$$



Economic and mathematical models make it possible to make a choice of a set of numbers x_i (variables in equations) that provide the extremum of some function Z (target function or quality indicator of an approved solution) under the constraints defined by the operating conditions of the system. The model is written as follows: calculate the vector that rotates at the maximum or minimum $\overline{X} = x_1, x_2, x_3, \ldots, x_j, \ldots, x_n,$ target function

$$z = f(\overline{X}) = f(x_1, x_2, x_3, \ldots, x_j, \ldots, x_n) \quad (1)$$

for condition

$$\left.\begin{array}{l} q_i(x_1, x_2, x_3, \ldots, x_j, \ldots, x_n) = 0 \ (i = 1, 2, 3, \ldots, m_1); \\ q_i(x_1, x_2, x_3, \ldots, x_j, \ldots, x_n) \lesseqgtr 0 \ (i = m_1 + 1, m_1 + 2, \ldots, m); \end{array}\right\} \quad (2)$$

$$x_j \geqslant 0 \ (j = 1, 2, 3, \ldots, n).$$

Expressions of type (2) are called constraints that define the region of solutions of the objective function (1). It is the target function, partly, and is a model of the socio-economic system. It is worth noting that the type of functions f and qi can be both linear and nonlinear.

EXAMPLE OF CONSTRUCTION OF THE REAL TASK

Price of commodity products: TV - 180 USD, video recorder - 260 USD, music center - 420 USD

The cost consists of the cost of renting an area - 4000 USD, the cost of components - 100, 170, 315 respectively for the release of one TV, VCR and music center; other expenses - 4500 y. at.

The required salary of engineers is 400, the director - 600, the workers on the conveyor - 350. For the enterprise of our scale, there are 3 directors, regardless of issue and 1 per 150 units of production. It is necessary 1-2 engineers for every 150 production units. The wage fund is limited due to tax problems at 260.5 million USD.

In total there are 230 seats, each of which occupies 7.4 square meters, taking into account passages to them. It is also necessary to deploy at least 10 jobs for engineers, each of which occupies 8.5 square meters. The total area of production space is 2000 square meters.

The area necessary for placing the finished product of each type is:

0,3 - for TVs that are issued for a month;

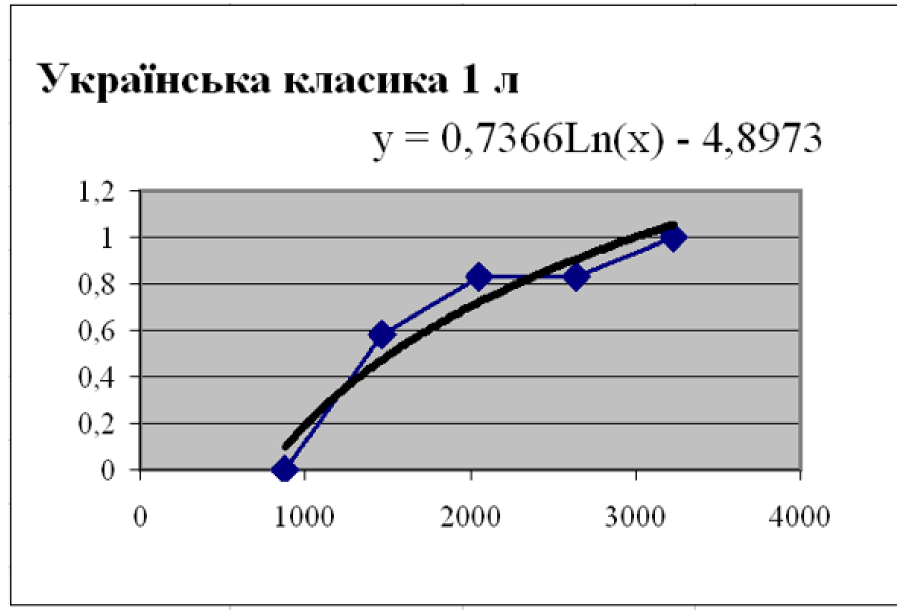
0.35- for VCRs issued per month;

0,4 - for musical centers, let out for a month.

EXAMPLE OF SOLVING NONLINEAR MATHEMATIC PROGRAMMING

FUNCTIONAL

$$\frac{\sum_{i=1}^{19} k_{1i} x_i}{\sum_{i=1}^{19}\left[\frac{k_{1i}}{k_{1i}+k_{2i}} - (a + b\ln x_i)\right] + 1} \to \max$$



Limitation

- on the warehouse area and the height of the boxes

$$\sum_{i=1}^{n} s_i x_i \le S * 5$$

- on the upper and lower limits of trade turnover

$$Q_{1i} \le xi \le Q_{2i}$$

- the type of variables

$$x_i \ge 0$$

$$(1 \le i \le n)$$

Coefficients of exponential smoothing of functions demand distribution on certain types of goods.

№		А	в
1	Вина кріплені	0,25	-0,74
2	Вина сухі	0,38	-1,27
3	Вина СК	0,08	-0,012
4	ДАР 0,2	0,83	-6,6
5	ДАР 1	1,24	-10,57
6	ДАР 1,5	1	-7,55
7	Сандорік 0,2	0,83	-6,29
8	Садочок 0,2л	0,58	-4,7
9	Садочок 0,5л	2,38	-16,9
10	Садочок 1л	0,93	-8,21
11	Садочок 1,5л	0,65	-4,81
12	Соки "Українська класнка"1л	-0,73	-4,89
13	Соки "Фрукти світу" 1л	0,19	-6,79
14	Соки Класик 1л	1,35	-10,31
15	Соки Gold 1,5л	0,77	-5,72
16	Соки Gold 1л	1,12	-10,07
17	Соки Gold 0,25л	1,02	-7,88
18	Напої 0,2	0,22	-0,49
19	Напої 1	0,24	-0,61

Frequency analysis of consumption of products

x_i мин	*x_i макс*	*Теоретична кількість діапазоні*		*Крок*	*Розрахунок правої межі інтервалів*								
7	328	29		40	47	88	128	168	208	248	288	328
29	406	34		47	76	124	171	218	265	312	359	406
0	656	60		82	82	164	246	328	410	492	574	656
2520	10642	740		1015	3535	4550	5566	6581	7596	8611	9626	10642
4820	11337	593		815	5635	6449	7264	8079	8893	9708	10523	11337
1809	5028	293		402	2211	2613	3016	3418	3821	4223	4626	5028
1471	5044	325		447	1918	2364	2811	3257	3704	4151	4597	5044
3415	11575	743		1020	4435	5455	6475	7495	8515	9535	10555	11575
0	1812	165		226	226	453	679	906	1132	1359	1585	1812
9507	31597	2012		2761	12268	15029	17790	20552	23313	26074	28835	31597
1031	5200	380		521	1552	2074	2595	3116	3637	4158	4679	5200
0	3231	294		404	404	808	1212	1615	2019	2423	2827	3231
0	994	91		124	124	248	373	497	621	745	870	994
3066	6415	305		419	3485	3903	4322	4740	5159	5578	5996	6415
1000	3981	271		373	1373	1745	2118	2490	2863	3236	3608	3981

Wilson's formula

Enter the following notation: A - the cost of placing and fulfilling the order; S - annual resource requirement; q - the size of a single delivery; r - interest rate on storage of resources (discount rate); p - price of unit of purchased resources, Ссв - total expenses for a certain period of time (for simplification of calculations, the period of time is usually taken equal to one year); Ср - the cost of placing an order; Сх - expenses on storage of resources, Сз - expenses for purchase of resources.

The total cost of the material flow is determined by the following known formula: $C_{св} = C_р + C_х + C_з$.

In the expanded form, the formula will be as follows:

$$C_{св} = \frac{AS}{q} + \frac{rpq}{2} + Sp.$$

The minimum value is at the point of its extremum. If we take the derivative in q and equate to zero, then the result will be:

$$-\frac{AS}{q^2} + \frac{rp}{2} = 0.$$

Accordingly, the point of the extremum of the function, the minimum cost and the optimal size of delivery will be at the point of q_{opt}. Solving the equation for *q*, we obtain:

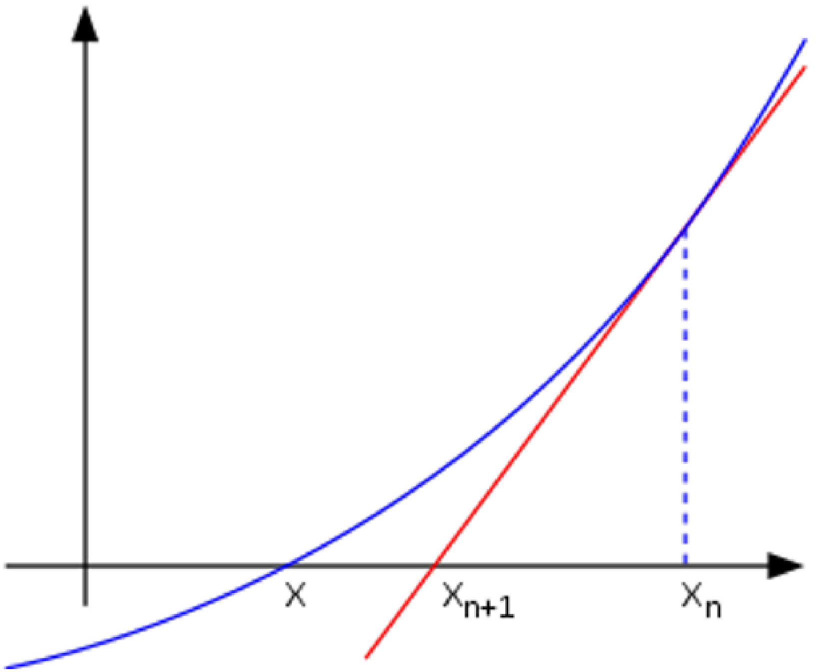
$$q_{опт} = \sqrt{\frac{2AS}{rp}}.$$

Numerical methods for finding the optimal solution for statistical models:

Newton's method

$$f'(x_n) = \operatorname{tg}\alpha = \frac{\Delta y}{\Delta x} = \frac{f(x_n) - 0}{x_n - x_{n+1}} = \frac{0 - f(x_n)}{x_{n+1} - x_n},$$

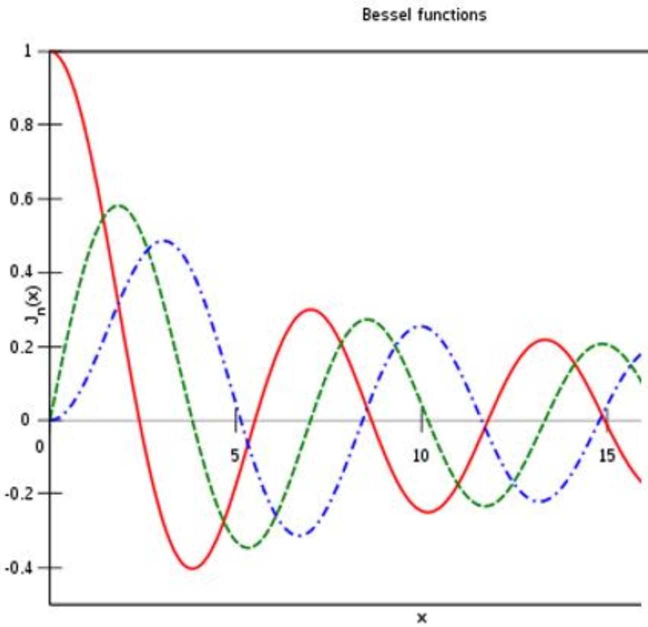
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

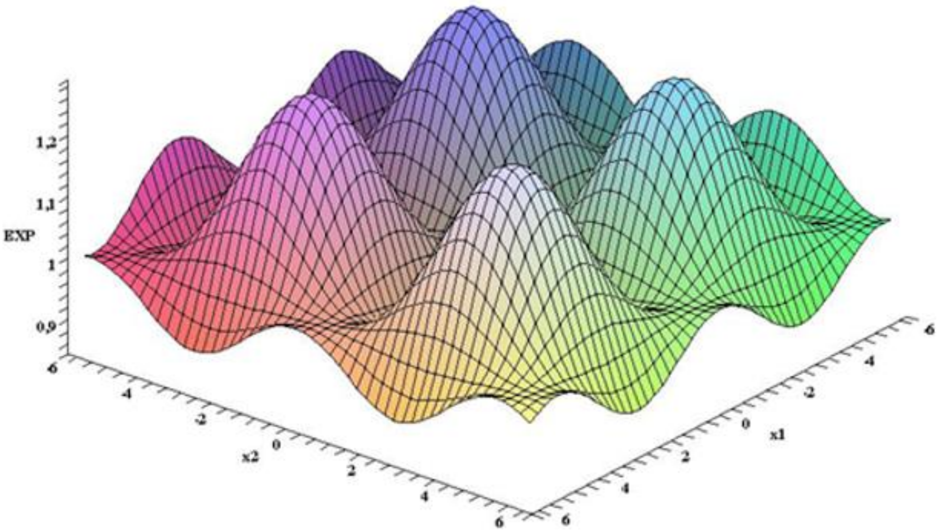


Geometric interpretation of Newton's method

Functions of Bessel in mathematics are a family of functions that are canonical solutions of the Bessel differential equation:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2) y = 0,$$





Multicriteria tasks

$$L^r(x) = \sum_{j=1}^{n} c_j^r \cdot x_j + c_0^r \rightarrow \max_x, \quad r = \overline{1;R},$$

$$D_x \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \geq -b_i, \Rightarrow \varepsilon_i = \sum_{j=1}^{n} a_{ij} x_j + b_i, i = \overline{1;m}, \\ x_j \geq 0, \quad j = \overline{1;n}, \end{cases}$$

The formal definition of the □-optimality of the solution x is written as a requirement for the absence of such a solution x $\in$ *Dx,* at which conditions the conditions were fulfilled

$$L^r(x) \geq L^r(x^{'}), \forall r,$$

and at least one of them - strictly (with a sign >).

Summary to the problem of mathematical programming

$$\delta^r = L^r(x) - L^r(x') = \sum_{j=1}^{n} c_j^r x_j - \sum c_j x_j', \forall \delta^r,$$

$$\Delta = \Delta(x) = \sum_{r=1}^{R} \delta^r \to \max_x, \quad x \in D_x, \forall x_j \geq 0.$$

Guaranteed result method

$$\varphi = \varphi(x) = \min_{1 \leq r \leq R} L^r(x) \to \max_x$$

Method of convolution of partial criteria

$$L(x) = \sum_{r=1}^{R} \mu_{\hat{e}} L^r(x),$$

$$\frac{L^r(x) - L^r_{O\Pi T}(x)}{L^r_{O\Pi T}(x)}$$

$$L^r_{MAX}(x) = \frac{1}{L^r_{MIN}(x) + 1}$$

$$L(x) = \frac{\sum_{r=1}^{R} \mu_{ђ} L^r_{MIN}(x) + 1}{\sum_{r=1}^{R} \mu_{ђ} L^r_{MAX}(x)},$$

TRANSPORT TASK

Matrix statement of transport task:

Let there be a number of points of consumption and suppliers of some products, where

A_i - resource of the i-th supplier (stock of products or plan of shipment from the current production).

V_j - the requirements for the same product in points j.

C_{ij} - distance or cost of transportation from i to j.

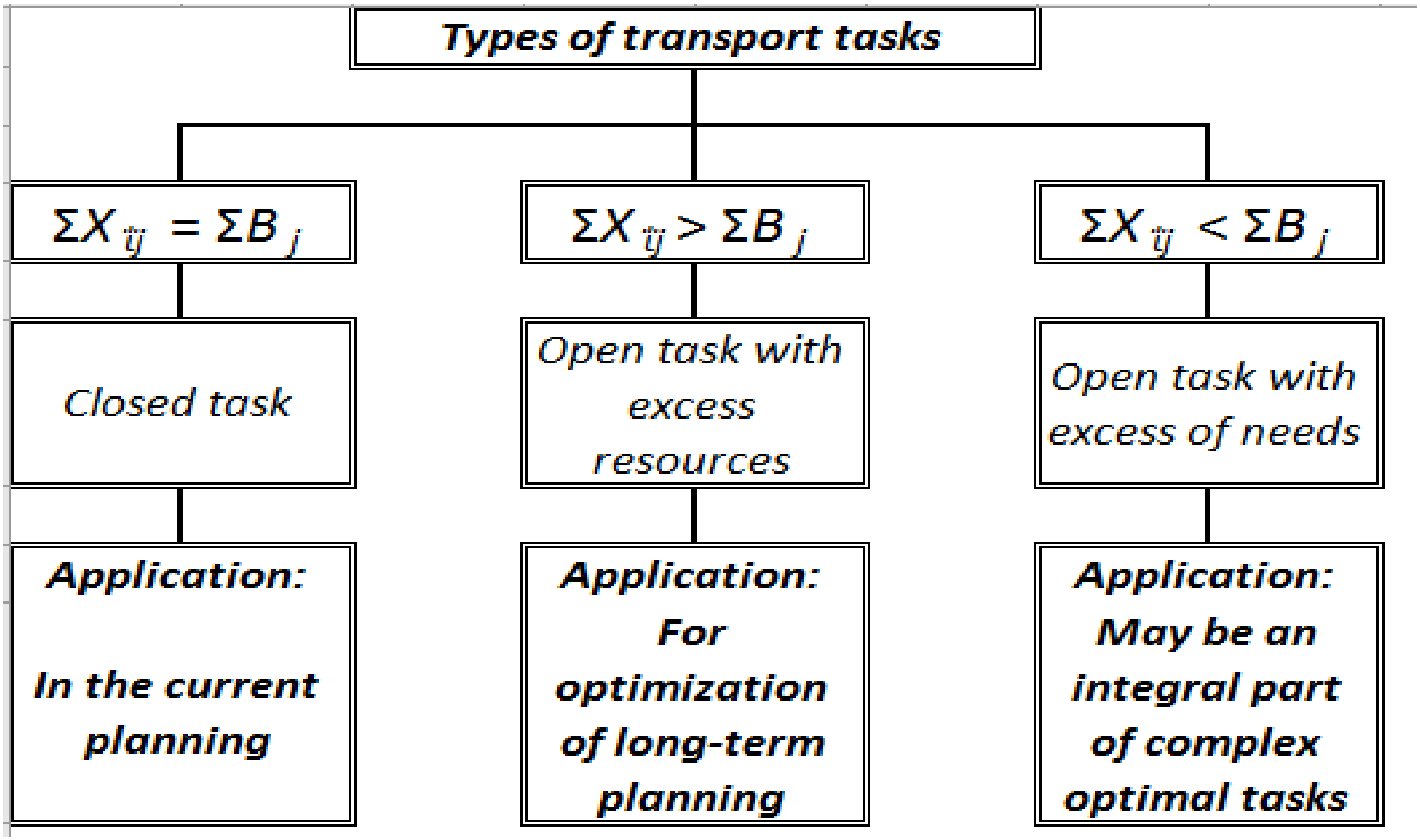
It is required to find such deliveries from each vendor to each customer, Xij (task variables), in which the total cost or total mileage will be minimal.

	Need	Recipients				
Stocks		B_1	B_2	B_3	....	B_m
Warehouses	A_1	C_{11}	C_{12}	C_{13}	...	C_{1m}
	A_2	C_{21}	C_{22}	C_{23}	...	C_{2m}
	A_3	C_{31}	C_{32}	C_{33}	...	C_{3m}
	...					
	A_n	C_{n1}	C_{n2}	C_{n3}	...	C_{nm}

Transport task

$\Sigma X_{ij} = B_i$, (j=1,2, … n – number of suppliers),

$\Sigma X_{ij} = A_i$, (i= 1,2, … m – number of consumers);



In an open task with an excess of resources, fewer exports can be removed

$$\Sigma X_{ij} < A_i,$$

In the open task of overcoming the needs of possible supplies less availability

$$\Sigma X_{ij} < \Sigma B_j,$$

The criterion for optimality of the solution is a minimum of total transport or mileage costs in ton-kilometers (car-kilometers) for all planned departures. If the cost of transportation (distance) from i to j - denote as C_{ij} the target function will be determined in this way

$$\Sigma \Sigma C_{ij} X_{ij} \rightarrow \min,$$

OPTIMAL TRANSFER OF MANUFACTURING OBLIGATIONS OF EMPLOYEES SERVICE ENTERPRISE

Criterion for minimizing transport costs

$$F(X)=\sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij}\cdot X_{ij}\cdot D_i\cdot P_j \rightarrow \min .$$

Limitation

$$\sum_{i=1}^{N} X_{ij}\cdot 10 \le P_j\ ,\ i = 1..N,$$

$$\sum_{j=1}^{N} X_{ij} = D_i\ ,\ j = 1..N,$$

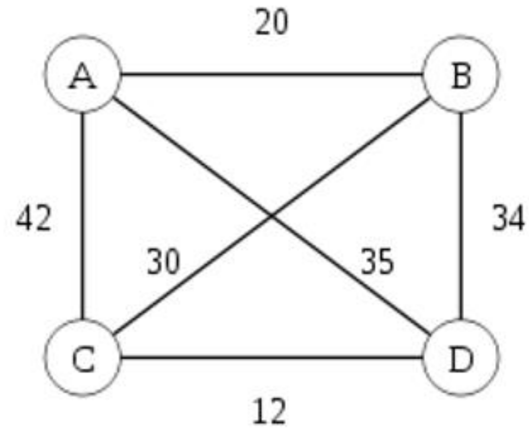
$$X_{ij} = 0 \text{ або } 1 .$$

An interim plan for moving specialists in the middle of a working day

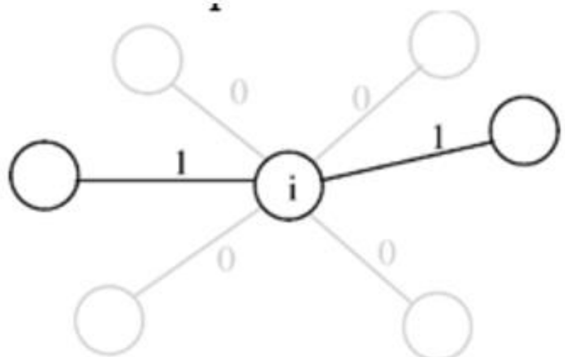
Enterprise from which we move

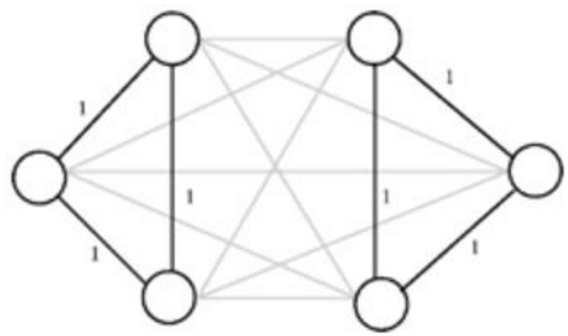
Фір-ми	Enterprise to which we move										
	Л1	Л2	Л3	Л4	Л5	П1	П2	П3	П4	П5	П6
П1	1	1	0	0	0	0	0	0	0	0	0
П2	0	0	0	1	0	0	0	0	0	0	0
П3	0	0	0	0	0	1	0	0	0	0	0
П4	0	0	0	0	1	0	0	0	0	0	0
П5	0	0	1	0	0	0	0	0	0	0	0
П6	0	0	0	0	0	0	1	0	0	0	0
П7	0	0	0	0	0	0	0	1	0	0	0
П8	0	0	0	0	0	0	0	0	1	0	0
П9	0	0	0	0	0	0	0	0	0	1	0
П10	0	0	0	0	0	0	0	0	0	0	1
П11	0	0	0	0	0	0	0	0	0	0	0
П12	0	0	0	0	0	0	0	0	0	0	0
П13	0	0	0	0	0	0	0	0	0	0	0
П14	0	0	0	0	0	0	0	0	0	0	0

The task of traveling salesman



Symmetric TSP for four cities.





Condition multiplicity: each vertex must have one input and one starting edge of the route.

Cycles: variables satisfy condition multiplicity, but not determine the rout.

Dynamic programming

The method of dynamic programming is based on the condition of the absence of an aftereffect and the condition of the additivity of the target function.

Condition of absence of aftereffect. State $\bar{x}_k$, in which the system went over in one step, depends on the state $\bar{x}_{k-1}$ and selected managerial actions (MA) $\bar{u}_k$ and does not depend on how the system came into the state $\bar{x}_{k-1}$, $\bar{x}_k = \bar{f}_k(\bar{x}_{k-1}, \bar{u}_k)$.

Similarly, the value of Wk depends on the state $\bar{x}_{k-1}$ and selected MA $\bar{u}_k$

$$W_k = W_k(\bar{x}_{k-1}, \bar{u}_k).,$$

The condition of the additivity of the target function. The total gain for N steps is calculated by the formula $S = \sum_{k=1}^{N} W_k(\bar{x}_{k-1}, \bar{u}_k)$.

Year	The effectiveness of the investment project for one invested hryvnia				
	A	*B*	*C*	*D*	*E*
1	-1,00	0	-1,00	-1,00	0
2	+0,30	-1,00	+1,00	0	0
3	+1,00	+0,30	0	0	-1,00
4	0	+1,00	0	+1,75	+1,40

THEORY OF GAME
Antagonistic game

Гравець 2 / Гравець 1	B_1	B_2	………	B_n	α_i
A_1	a_{11}	a_{12}	………	a_{1n}	α_1
A_2	a_{21}	a_{22}	………	a_{2n}	α_2
………	………	….	………	………	………
A_m	a_{m1}	a_{m2}	………	a_{mn}	α_m
β_j	β_1	β_2	………	β_n	

$$\max_i \min_j \alpha_{ij} = \min_j \max_i \alpha_i = v.$$

$$M(A, \overline{p}, \overline{q}) = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} p_i q_j$$

$$S_1 = \begin{pmatrix} A_1 & A_2, & \dots, & A_m \\ p_1 & p_2, & \dots, & p_m \end{pmatrix},$$

де $\sum_{i=1}^{m} p_i = 1,\ p_i \geq 0$

$$S_2 = \begin{pmatrix} B_1 & B_2, & \dots, & B_n \\ q_1 & q_2, & \dots, & q_n \end{pmatrix},$$

де $\sum_{j=1}^{n} q_j = 1,\ q_j \geq 0$;

Cooperative (bitartic) game

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{pmatrix};\quad B=\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_{m1} & b_{m2} & \cdots & b_{mj} & \cdots & b_{mn} \end{pmatrix}.\quad C=\begin{pmatrix} (a_{11},b_{11}) & (a_{12},b_{12}) & \cdots & (a_{1j},b_{1j}) & \cdots & (a_{1n},b_{1n}) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (a_{i1},b_{i1}) & (a_{i2},b_{i2}) & \cdots & (a_{ij},b_{ij}) & \cdots & (a_{in},b_{in}) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (a_{m1},b_{m1}) & (a_{m2},b_{m2}) & \cdots & (a_{mj},b_{mj}) & \cdots & (a_{mn},b_{mn}) \end{pmatrix}$$

$$M_1^i \le M_1,\ i=\overline{1,m};\qquad M_2^j \le M_2,\ j=\overline{1,n},$$

$$\sum_{j=1}^{n} a_{ij} y_j \le \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} x_i y_j,\ i=\overline{1,m};\quad \sum_{i=1}^{m} b_{ij} x_i \le \sum_{i=1}^{m}\sum_{j=1}^{n} b_{ij} x_i y_j,\ j=\overline{1,n}.$$

$$\begin{cases} \sum_{i=1}^{m} x_i = 1;\ x_i \ge 0,\ i=\overline{1,m}; \\ \sum_{j=1}^{n} y_j = 1;\ y_j \ge 0,\ j=\overline{1,n}. \end{cases}$$

Games with nature

Decision making with known probabilities of the state of nature

$$\alpha = \max_i \sum^n a_{ij} p_j$$

In some cases, the probability of occurrence of certain states of nature is given conditioned that the accuracy of determining these probabilities is less than 100%. More often, for each state of nature, Π_i indicates its accuracy of calculating the probability of its state t_j, which, as well as the probability varies in the range from 0 to 1 (from about 0% to 100%). In this case, the choice of the optimal strategy of the active player is determined by the involvement of the risk matrix r_{ii}

$$\alpha = \max_i \left(\sum_{j=1}^n a_{ij} p_j t_j - \sum_{j=1}^n r_{ij} p_j (1 - t_j) \right)$$

The matrix R can be constructed directly from the conditions of the problem or on the basis of the matrix of prizes A. The risk of the rij player when using the strategy Ai and when the environment of Π_i is called, we will name the difference between the winnings that a player would receive if he knew that the state of the environment would be Π_j, and the winnings the player receives without having this information. That is,

$$R_{ij} = \max_i a_{ij} - a_{ij} .$$

Games with nature

in conditions of complete uncertainty

$$R_{ij} = \max_i a_{ij} - a_{ij}$$

$$A = \left(\begin{array}{c|cccc} & \Pi_1 & \Pi_2 & \dots & \Pi_n \\ \hline A_1 & a_{11} & a_{12} & \dots & a_{1n} \\ A_2 & a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ A_m & a_{m1} & a_{m2} & \dots & a_{mn} \end{array}\right)$$

Maximax criteria

$$M = \max_{1 \le i \le m} \max_{1 \le j \le n} a_{ij}.$$

Maximum Wald criterion

$$W = \max_{1 \le i \le m} \min_{1 \le j \le n} a_{ij}.$$

Minimum risk criterion for Sevgias

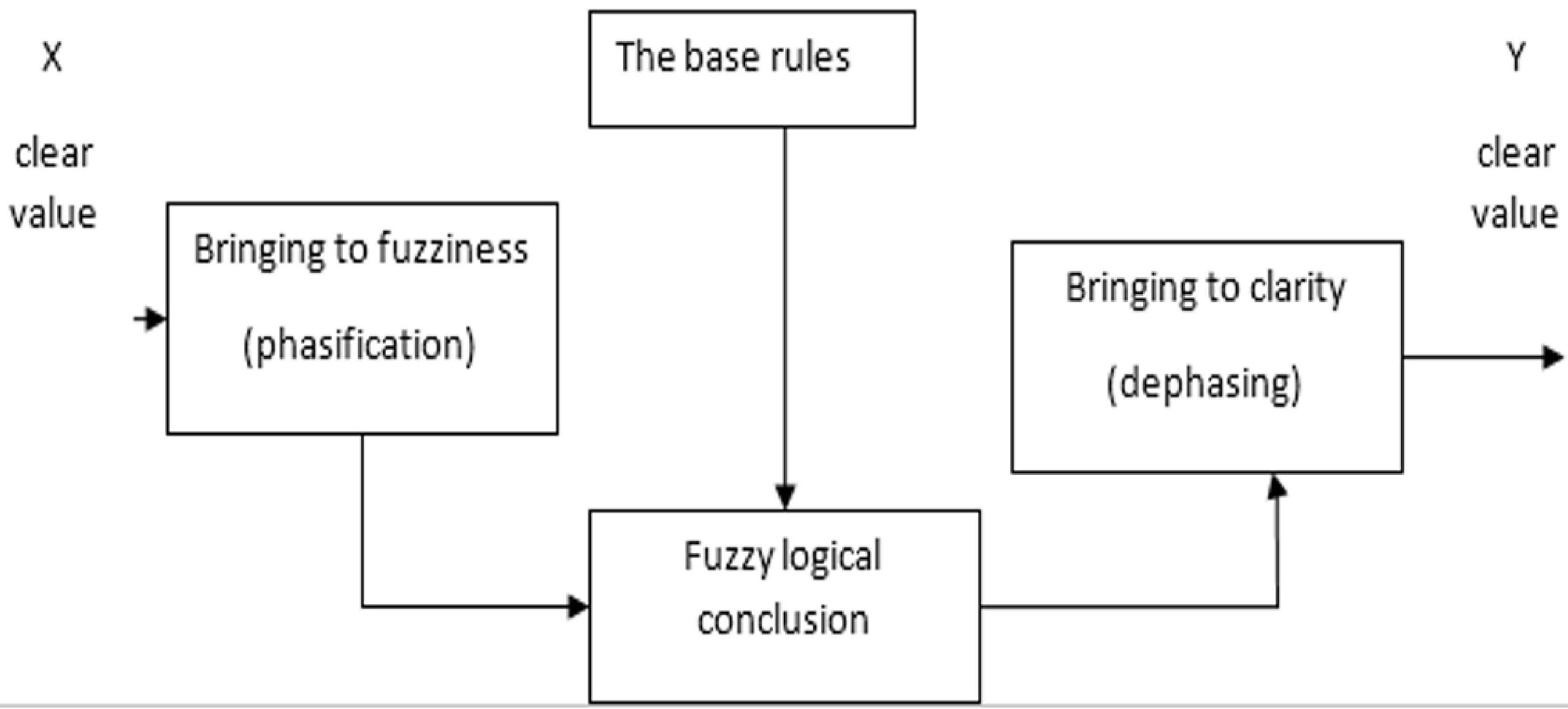
$$S = \min_{1 \le i \le m} \max_{1 \le j \le n} r_{ij}$$

The criterion for pessimism-onmimizm Hurwitz

$$H_A = \max_{1 \le i \le m} \{p \min_{1 \le j \le n} a_{ij} + (1-p) \max_{1 \le j \le n} a_{ij}\},$$

Optimization of the management of the socioeconomic system, given by the fuzzy model

R1: IF x1 is A11. And xn is A1n, the THEN is B1.
Ri: IF x1 is Ai1. And xn is Ain, THEN Bi.
. . . .
Rm: IF x1 is Am1. And xn is Amn, THEN is Bm.



Fuzzy outlet by Mamdani

$$\alpha_i = \min_i(A_{ik}(x_k)) \qquad B_i^*(y) = \min_i(\alpha_i, B_i(y)) \qquad MF(y) = \max_i(B_i^*(y))$$

$$y = \frac{1}{y_{\max} - y_{\min}} \int_{y_{\min}}^{y_{\max}} MF(y)dy = \frac{1}{y_{\max} - y_{\min}} \int_{y_{\min}}^{y_{\max}} \max(B_i^*(y))dy$$