СЕКЦІЯ «ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ ТА ТЕЛЕКОМУНІКАЦІЇ»

UDC 517.968.21

Gorev V., Ph.D., Head of the Department of Physics
Shedlovska Y., Associate Professor of the Department of Information Technology and Computer Engineering
Zhuravlyov M., Senior lecturer of the Department of Physics
(*Dnipro University of Technology, Dnipro, Ukraine*)

ON THE USE OF THE CONTINUOUS WIENER FILTER TO GENERALIZED FRACTIONAL GAUSSIAN NOISE

The paper is devoted to the investigation of the continuous Kolmogorov–Wiener filter weight function for forecasting of the process described by the generalized fractional Gaussian noise model. Such a model may describe telecommunication traffic, see [1, 2]. Our recent papers were devoted to the corresponding weight function obtaining for the processes in the power-law structure function model [3–5], in the "usual" fractional Gaussian noise model [6–8] and in the Gaussian fractional sum-difference model [9]; see also a review in [10]. However, the corresponding investigation for the generalized fractional Gaussian noise model is not yet done. In [11] it is shown, in particular, that the continuous Kolmogorov–Wiener filter may be applied to prediction of smoothed heavy-tail processes. So, the problem under consideration is of interest.

The process correlation function in the framework of the generalized fractional Gaussian noise is as follows [1, 2]:

$$R(t)=\frac{\sigma^2}{2}\left(\left(|t|^a+1\right)^{2H}+\left||t|^a-1\right|^{2H}-2|t|^{2aH}\right) \tag{1}$$

where $R(t)$ is the process correlation function, σ^2 is the process variance, $a\in(0,1]$ is a constant and $H\in(0,5;1)$ is the Hurst exponent. In the case where $a=1$ the generalized fractional Gaussian noise coincides with the "usual" fractional Gaussian noise, see [1, 2].

The Kolmogorov–Wiener filter weight function is the solution of the Wiener–Hopf integral equation, see, for example, [10], which is as follows:

$$\int_0^T h(\tau)R(t-\tau)d\tau=R(t+z) \tag{2}$$

where $h(\tau)$ is the unknown weight function which should be derived from (2), T is the length of the time interval which contains the input data and z is the length of the "future" time interval for which the prediction should be made.

The corresponding equation is solved on the basis of the Galerkin method, see [12]. In should be stressed that the Galerkin method is widely used in similar problems; for example, it is used in our previous papers [3–10]. We use the Walsh functions as the orthogonal functions in terms of which the unknown weight function should be expanded:

$$h(\tau)=\sum_{s=1}^{n}g_s f_s(\tau) \tag{3}$$

where $f_s(\tau)$ are the Walsh functions, n is the number of functions in the corresponding approximation and g_s are some unknown numerical coefficients which should be found. The corresponding coefficients may be found on the basis of the linear system of algebraic equations

$$\sum_{s=1}^{n}g_s G_{sk}=B_k \tag{4}$$

where

$$G_{sk} = \int_0^T \int_0^T R(t-\tau) f_s(t) f_k(\tau) dt d\tau \quad (5)$$

and

$$B_k = \int_0^T R(t+z) f_k(t) dt \quad (6)$$

The corresponding coefficients may be found in a matrix form as

$$\begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix} = \begin{pmatrix} G_{11} & G_{21} & \cdots & G_{n1} \\ G_{12} & G_{22} & \cdots & G_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ G_{1n} & G_{2n} & \cdots & G_{nn} \end{pmatrix}^{-1} \cdot \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} \quad (7)$$

The numerical calculation of G_{sk} and B_k and the corresponding matrix calculations are made in the Wolfram Mathematica package.

The following numerical values of the parameters are investigated: $T = 100$, $z = 3$, $H = 0.75$, $a = 0.8$. The approximation of $n = 64$ Walsh functions is investigated. It is shown that in the considered approximation the left-hand side and the right-hand side of the equation (2) are rather close to each other, the corresponding MAPE of misalignment is approximately equal to 0.5%. So, the proposed method may be applied to the problem under consideration.

The generation of the generalized fractional Gaussian noise data and the corresponding forecasting investigation may be plan for the future. Another plan for the future is to investigate the above-mentioned MAPE of misalignment for approximations of more Walsh functions and for different numerical values of the parameters a and H.

References:

1. M. Li, Physica A, 579, 126138 (2021). doi: 10.1016/j.physa.2021.126138.
2. M. Li, Advances in Mathematical Physics, 2013, 315979 (2013). doi: 10.1155/2013/315979.
3. V. N. Gorev, A. Yu.Gusev and V. I. Korniienko, Radio Electronics, Computer Science, Control, 2, 44–52 (2019). doi: 10.15588/1607-3274-2019-2-5.
4. V. N. Gorev, A. Yu.Gusev and V. I. Korniienko, Radio Electronics, Computer Science, Control, 2, 39–47 (2021). doi: 10.15588/1607-3274-2021-2-4.
5. V. Gorev, A. Gusev and V. Korniienko, CEUR Workshop proceedings, 2353, 596-606 (2019). Online: https://ceur-ws.org/Vol-2353/paper47.pdf
6. V. N. Gorev, A. Yu. Gusev V. I. Korniienko and A. A. Safarov, Radio Electronics, Computer Science, Control, 1, 29–35, (2021). doi: 10.15588/1607-3274-2021-1-3.
7. V. Gorev, A. Gusev and V. Korniienko, CEUR Workshop proceedings, 2623, 164-173 (2020). Online: https://ceur-ws.org/Vol-2623/paper16.pdf
8. V. Gorev, A. Gusev and V. Korniienko, CEUR Workshop proceedings, 2853, 389-400 (2021). Online: https://ceur-ws.org/Vol-2853/paper35.pdf
9. V. N. Gorev, A. Yu. Gusev and V. I. Korniienko, Radio Electronics, Computer Science, Control, 3, 31–37 (2022). doi: 10.15588/1607-3274-2022-3-3.
10. V. Gorev, A. Gusev, V. Korniienko and M. Aleksieiev, in: P. Vorobiyenko, M. Ilchenko, I. Strelkovska (Eds.), Lecture Notes in Networks and Systems, vol 212, Springer, 2021, pp. 111–129. doi:10.1007/978-3-030-76343-5_7
11. V. Gorev, A. Gusev, V. Korniienko and Y. Shedlovska, On the use of the Kolmogorov–Wiener filter for heavy-tail process prediction, Journal of Cyber Security and Mobility (2023). In press.
12. A. D. Polyanin and A. V. Manzhirov, Handbook of integral equations. Second edition, New York: Chapman and Hall, 1144 p, (2008). doi: 10.1201/9781420010558