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# Solution of the problem to optimize two-stage allocation of the material flows

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**Abstract**

**Purpose** is to elaborate innovative and computationally efficient algorithm to solve a problem of two-stage allocation of the resource occupying continuously the specified area as well as to demonstrate the behaviour of the corresponding software developed with the application of advanced geoinformation resources.

**Methods.** The paper involves mathematical models of continuous problems of optimal set partitioning with additional connections to describe two-stage problems of the material resource location-allocation. Methodological approach to the solution of such problems is based on the idea of their reducing to the problem of infinite-dimensional mathematical programming for which it is possible to obtain optimal solution in the analytical form with the help of the duality theory apparatus.

**Findings.** Mathematical and algorithmic apparatus to solve continuous problems applied for the fuel and energy complex enterprises has been developed making it possible to obtain partitioning of the deposit area into the zones, which are allocated to the first-stage enterprises exclusively. The algorithm operation is demonstrated in terms of the model problem solution. It has been defined that the benefit of such an approach is in the reducing of the infinite-dimensional programming problem to the problem of finite-dimensional nonsmoth optimization since the obtained computational formulas contain the parameters which determination requires solving the auxiliary problem of the nondifferentiable function optimization.

**Originality.** Contrary to the previously developed one, the proposed algorithm does not stipulate solution of the linear programming problem of transport type at each step of the iteration process. Such a problem is solved only once to find the volumes of product transportation between the first-stage and second-stage enterprises after defining all the optimal solution components.

**Practical implications.** Software implementation of the algorithm on the basis of the advanced geoinformation technologies and resources, in terms of the solution of raw material flow allocation, makes it possible to reduce total costs for the management of material flows and their accompanying service flows throughout the whole logistic chain beginning from the flow origin up to its arrival to the end user.

***Keywords:*** *multistage problems, set partitioning, geoinformation technologies, location-allocation, nonsmooth optimization*

## 1. Introduction

Previously, paper [1] considered a two-stage process of material flow allocation in transportation and logistic system, which structural elements were represented by mines (first-stage centres) extracting coal being continuously distributed within a certain area, and enterprises consuming or processing that coal (second-stage centres). In this context, it was stipulated that a specific mining territory (zone) was allocated to each mine. First, raw material flow moves within the system from each point of the zones under consideration immediately to the point of its accumulation at the corresponding mine; then, the assorted resources are sent in the specified amounts to the enterprises acting as the consumers – thermal power stations (TPS), concentrating plants, and other objects. The mentioned paper show mathematical models of such multistage problems to allocate resources being distributed continuously within the defined area.

For the first time, those problems were analysed in [2]. Their continuity was stipulated not only by the possibility to locate fist-stage centres at any point of the specified continual set but also by the necessity to partition the set itself (or its share) into several zones. A method to solve the formulated problems is based on the idea of their reducing to the problems of infinite mathematical programming; optimal solution for those problems is possible to be obtained in analytical form by applying the duality theory apparatus. Though the obtained computational formulas contain the parameters subject to the determination concerning the fact why we need to solve auxiliary problem of the nonsmooth function, benefit of the described approach is in the idea that target function of



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the resulting optimization problem depends only upon the finite number of variables. That makes it possible to apply any methods of finite-dimensional nondifferential optimization to solve the problem. For instance, that can be ellipsoid methods [3], algorithms based on the Monte-Carlo method [4] or *r*-algorithms used traditionally while solving continuous problems of optimal set partitioning [5].

The paper is focused on the numerical algorithm of solving two-stage allocation of the resource occupying continuously the specified area based on the mathematical apparatus described above. Contrary to the algorithm developed to solve similar problems in paper [6], that very algorithm does not involve the solution of problem of linear programming of transportation type at each stage of the iteration process. Such a problem is solved only once to find the volumes of product transportation between the first-stage and second-stage enterprises after the defining of the rest of the optimal solution components. Software implementation of the algorithm is performed involving modern geoinformation technologies and resources. The algorithm operation is demonstrated in terms of the solution of a model problem of the optimization of two-stage material flow allocation, which may take place at fuel and energy complex enterprises, without being stick to any available industrial objects.

Owing to the formulation and solution of the corresponding continuous problems of optimal set partitioning with the additional connections (OSPAC), it is possible to reduce transportation costs at fuel and energy complex enterprises involved in coal mining within the specified territory and coal transportation to the warehouses of the corresponding mines with further product supply to the end users [1].

Objective of the paper is to develop innovative and computationally efficient algorithm of problem solving and to demonstrate the behaviour of corresponding software developed on the basis of integration of the methods to solve OSPAC problems and geoinformation systems and resources.

Topicality of such scientific studies is stipulated by current development of geographically distributed multilevel companies including dozens of large enterprises dealing with full production cycles from the raw material extraction with its complex use and processing up to its transportation to the end users.

Consider in brief the statement and mathematical model of the problem to optimize two-stage material flow allocation in terms of coal (as the allocated resource) mined within the newly developed deposit.

### 1.1. Statement of the problem

Assume that there is a certain territory with the known evaluation of the coal reserves and enterprises requiring specific amount of that coal. We should determine the areas to locate new mines within the specified territory and allocate the mining area to each mine with minimum costs for coal accumulation and transportation to the mine warehouses and end user. At the same time, it is necessary to define both which mine should supply coal to a second-stage enterprise and what amount of coal is required to satisfy completely the needs in that resource.

A mathematical model will apply following notations: $\Omega$ is area within which the resource is allocated and where first-stage centres may be located, m$^2$; $\rho(x)$ is amount of resource at point $x$ of set $\Omega$, t/m$^2$; $N$ is number of first-stage centres; $M$ is number of second-stage centres; $S$ is total amount of the resource within the specified area, t; $\tau_i^r$ is coordinates of the $i^{th}$ centre of the $r^{th}$ stage; $b_i^r$ is production facilities the $i^{th}$ centre of the $r^{th}$ stage, $r = I, II$, t; $c_i^I(x, \tau_i^I)$ is cost of the resource unit delivery from point $x \in \Omega$ to centre $\tau_i^I$, UAH/t; $c_{ij}^{II}(\tau_i^I, \tau_j^{II})$ is cost of the resource unit delivery from centre $\tau_i^I$ to centre $\tau_j^{II}$, UAH/t; $a_i$ is cost of the resource sorting out and shipping at enterprise $\tau_i^I$ calculated per resource unit, UAH/t; $v_{ij}$ is amount of the resource delivered from centre $\tau_i^I$ to enterprise $\tau_j^{II}$, t.

It is required to find such partitioning of set $\Omega$ into $N$ of disjoint subsets $\bar{\omega} = \{\Omega_1, \Omega_2, ..., \Omega_N\}$ (among which there can be empty ones) as well as to determine coordinates $\tau_1^I, \ldots, \tau_N^I$ of those subsets and such transportation volumes $v_{11}, \ldots, v_{NM}$, in terms of which functional:

$$F\left(\bar{\omega}, \tau^I, v\right) = \sum_{i=1}^{N} \int_{\Omega_i} c_i^I\left(x, \tau_i^I\right) \rho(x)\, dx + \sum_{i=1}^{N} \sum_{j=1}^{M} \left(c_{ij}^{II}\left(\tau_i^I, \tau_j^{II}\right) + a_i\right) v_{ij}, \quad (1)$$

would reach minimal value, and following conditions would be met:

$$\int_{\Omega_i} \rho(x)\, dx = \sum_{j=1}^{M} v_{ij},\ i = \overline{1, N}; \quad (2)$$

$$\sum_{i=1}^{N} v_{ij} = b_j^{II},\ j = \overline{1, M}; \quad (3)$$

$$\bar{\omega} \in \Sigma_{\Omega}^{N},\ v \in R_{NM}^{+},\ \tau^I \in \Omega^N; \quad (4)$$

$$\Sigma_{\Omega}^{N} = \left\{ \begin{array}{l} \bar{\omega} = \{\Omega_1, \Omega_2, ..., \Omega_N\}: \\ \bigcup_{i=1}^{N} \Omega_i = \Omega,\ mes\left(\Omega_i \cap \Omega_j\right) = 0,\ i \neq j,\ i, j = \overline{1, N} \end{array} \right\}, \quad (5)$$

where $\Sigma_{\Omega}^{N}$ is class of all the possible cases of the partitioning of set $\Omega$ into $N$ of disjoint subsets; $\bar{\omega} = \{\Omega_1, \Omega_2, ..., \Omega_N\}$ is element of class $\Sigma_{\Omega}^{N}$; $R_{NM}^{+}$ is $NM$-dimensional space of non-negative real numbers.

It should be remembered that problem (1)-(5) is called a continuous problem of optimal set partitioning with additional connections. Further, we will consider it in terms of two variants – with the location of first-stage centres and with their fixed coordinates.

### 1.2. Literature review

Since the paper is the consistent continuation of the scientific studies represented in [1], we are not going to focus on scientific publications dealing with the problems of optimization of supply volumes and development of the mechanisms to reduce total costs for stocks preservation at mining enterprises. Consider only the papers involving up-to-date GIS-technologies to solve similar problems.

Geoinformation systems have been widely used in different fields of practical activities owing to their advantages in unifying diversified data on the basis of geographical information. Nowadays, GIS is often applied in town-planning activities, management of natural resources and many other fields helping users solve the problems of spatial analysis, planning, and forecasting [7]-[12].

GIS stores the information about real worlds in the form of set of thematic layers united on the basis of geographical location. That simple but very flexible approach has proved its benefits during the solution of numerous important problems. As a rule, electronic maps contain full information concerning the length of roads, their connections, and their traffic conditions (speed rate, prohibited ramps, available transfer points etc.).

Thus, basing on multispectral satellite data and accompanying information on geology, geomorphology, topography, settlements, traffic intersections, forest cover, hydrology, and climate, paper [13] elaborates a platform to select the appropriate site to locate a thermal power station along the mining region in India. After its processing, digital satellite information was used to study the lignite seam reserves and to analyse the use of land and plant cover. Other thematic information such as geology, geomorphology, coal basin boundaries and infrastructure, administrative units, canals and large settlements were digitized using GSI maps and SOI topographic data respectively. All the thematic vector layers were united and represented for their overlapping and prioritizing. Site selection for a thermal power station was based on four basic criteria (land, water, coal mine, and environment) and two secondary ones (cost and location access). Index of the site suitability was calculated involving the spatial analysis tool in ArcGIS 9.3.

Paper [14] deals with the problems of optimal planning of allocated power biomass-based systems to satisfy electric power needs of rural areas in India. The proposed approach is based on the use of intellectual analysis of data and modern GIS along with the clusterization algorithmic k-method to divide the whole region into clusters and find the systems to generate biomass energy. Optimal value k is defined iteratively taking into consideration the demand and supply; it is selected appropriately to minimize total costs for system installation, costs for biomass transportation, and costs for its transfer and distribution. Clusterization results are represented on the regional GIS map.

Paper [15] represents modification of classic p-meridian problem which takes into account spatial distribution of supply resources and competition for them on the part of potential objects. It proposes simplified study making it possible to determine optimally anaerobic reactors (AD) of the community scale within the area of East-Midlands in Great Britain.

Paper [16] represents new GIS enhancement called ArcMine developed to support melioration planning within the abandoned mining regions. ArcMine gives four tools to (a) evaluate hazards of mine subsidence; (b) to assess erosions of mine wastes, (c) to analyse ways of mine water flow to the surface, and (d) to determine the appropriate species of wood for mine reafforestation. Spatial database also includes topographic and geological maps as well as a map concerning mine demolition data and information about the well. Facts of ArcMine implementation and application to analyse abandoned mining regions in Korea tell about the benefits of applying ArcMine information concerning the land mine threats.

Paper [17] considers modern techniques based on geoinformation technologies and their application to simulate and evaluate risks connected with mining activities, i.e. soil contamination, soil erosion, water pollution, and deforestation.

Other examples of GIS application in different spheres of human activity while solving the location-allocation problems are given in papers [18]-[20].

While solving continuous multistage location-allocation problems, up-to-date GIS-technologies may be used as follows:

1) to evaluate mineral reserves at the territory;

2) to consider terrain and other features of the area where new industrial objects may be located;

3) to define the shortest way and calculate its length between the objects etc.

In terms of the paper, we are going to use cross-platform open-code geoinformation system QGIS, which helps apply instrument of analysis, sampling, geoprocessing, geometry control, and database control. QGIS includes a functional to search for the shortest way required to solve practical tasks of optimal location-allocation. Information concerning mineral deposits at the territory of Ukraine, their reserves and extraction as well as the available Ukrainian mining enterprises can be found on the information resource "Mineral resources of Ukraine" [20] created to meet the users with current state of mineral and raw material base of Ukraine.

## 2. Algorithm of the problem solution

The required and sufficient conditions of optimality for continuous problem of optimal set partitioning with additional connections of (1)-(5) type are obtained in [1] in the assumption that function $\rho(x) \geq 0$ for almost all $x \in \Omega$, and balance conditions are met:

$$\sum_{j=1}^{M} b_j^{II} = \int_{\Omega} \rho(x)\,dx\,. \tag{6}$$

Optimal partitioning of set $\Omega$ is represented in the form of characteristic functions of subsets making it up: for $i = 1,\ldots, N$ and almost all $x \in \Omega$:

$$\lambda_i^*(x) = \begin{cases} 1,\, if\;\; c_i^I\left(x, \tau_{*i}^I\right) + \psi_i^* \leq c_k^I\left(x, \tau_{*k}^I\right) + \psi_k^*, \\ i \neq k,\;\, a.e.\;\, for\;\, x \in \Omega,\, i,k = 1,...,N,\, then\;\, x \in \Omega_{*i}, \\ 0\, in\ the\ other\ cases. \end{cases} \tag{7}$$

where $\tau^I{}_{*i}$, $\psi_i{}^*$, $i = 1, \ldots, N$ is solution of following problem:

$$G(\psi) \to \max_{\psi \in R^N},\;\; G(\psi) = \min_{\tau^I \in \Omega^N} G_1\left(\tau^I, \psi\right); \tag{8}$$

$$G_1\left(\tau^I, \psi\right) = \int_{\Omega} \min_{k=\overline{1,N}} \left(c_k^I\left(\pi x, \tau_k^I\right) + \psi_k\right)\rho(x)\,dx + \\ + \sum_{j=1}^{M} b_j \max_{i=\overline{1,N}} \left(\psi_i - c_{ij}^{II}\left(\tau_i^I, \tau_j^{II}\right) - a_i\right). \tag{9}$$

Components $v_{ij}^*,\, i = \overline{1,N},\; j = \overline{1,M}$ are the constituents of the optimal solution of transport problem of such a type:

$$\sum_{i=1}^{N}\sum_{j=1}^{M}\left(c_{ij}^{II}\left(\tau_i^I, \tau_j^{II}\right) + a_i\right)v_{ij}^* \to \min\,; \tag{10}$$

$$\sum_{j=1}^{M} v_{ij} = b_i^{I*},\;\; b_i^{I*} = \int_{\Omega} \rho(x)\lambda_i^*(x)\,dx,\; i = \overline{1,N}\,; \tag{11}$$

$$\sum_{i=1}^{N} v_{ij} = b_j^{II},\; j = \overline{1,M}\,; \tag{12}$$

$$v_{ij} \geq 0,\, i = 1,...,N;\;\, j = 1,...,M. \tag{13}$$

Note that function $G_1(\tau_i^I, \psi)$, being in its essence the refined dual functional of problem (1)-(5) does not depend on variable $v$ (contrary to the dual problem shown in paper [6]). That has become possible owing to the fact that in terms of any admissible fixed first-stage centres $\tau_i^I, \ldots, \tau_N^I$ (including optimal ones), optimality criterion of pair $\left\{\lambda^*(\cdot), \nu^*\right\}$ is represented by the availability of such real constants as $\psi_i^*$, $i = 1, \ldots, N$ and $\eta_j^*$, $j = 1, \ldots, M$, in terms of which following ratios are correct (apart from conditions (7)):

$$\begin{cases} c_{ij}^{II}\left(\tau_i^I, \tau_j^{II}\right) + a_i = \psi_i^* + \eta_j^*, \; if \; \nu_{ij}^* > 0, \\ c_{ij}^{II}\left(\tau_i^I, \tau_j^{II}\right) + a_i > \psi_i^* + \eta_j^*, \; if \; \nu_{ij}^* = 0, \end{cases} \quad (14)$$

for all $i = 1, \ldots, N$ and $j = 1, \ldots, M$.

Owing to the mentioned differences, the represented algorithm to solve problem (1)-(5) differs from the iteration algorithm proposed in [5]. It stipulates that at first, there is problem solution (8) to find optimal coordinates of the first-stage centres and corresponding partitioning of set $\Omega$; then it means taking the obtained data into consideration to calculate the production facilities of the first-stage centres and solve problem (10)-(13). However, like in the aforementioned iteration algorithm, $r(\alpha)$-algorithm is the key component here with the constant coefficient of space expansion $\alpha$ and adaptive method to regulate step factor [5].

Limitations $\tau^I \in \Omega^N$ may be taken into account with the help of projection operator $P_{\Omega^N}$, if set $\Omega$ is convex, closed, and has rather simple structure, which does not require solving auxiliary problem of conditional optimization to define the point projection on the set. If structure of set $\Omega$ is rather complex, then its shape should be described with the help of limitations-inequalities; to take them into consideration, it may be possible to use nonsmooth penalty functions.

## 3. Initialization

Put area $\Omega$ within rectangular parallelepiped $\Pi$, which sides are parallel to the Cartesian axes; assume that $\rho(x) = 0$, if $x \in \Pi/\Omega$. Cover parallelepiped with the rectangular grid and specify the initial approximation of vector of variables $G_1(\tau, \psi)$ (further, it will be defined as $(u = (\tau^I, \psi)$: $u^{(0)} = (\tau^{I(0)}, \psi^{(0)})$. In the following, we will omit upper index $I$ of the unknown coordinates of the first-stage centres to simplify the formula. Specify parameters $\alpha, q_1, q_2, n_k, \varepsilon$ of the modification of $r(\alpha)$-algorithm.

Assume that $k = 0$.

### 3.1. Step 1

Calculate the values of vector-function $\lambda^{(k)}(x) = (\lambda_1^{(k)}(x), \lambda_2^{(k)}(x), \ldots, \lambda_N^{(k)}(x))$ at the nodes of grid $x \in \Pi$ using formulas:

$$\lambda_i^{(k)}(x) = \begin{cases} 1, \; if \; c_i^I\left(x, \tau_i^{(k)}\right) + \psi_i^{(k)} \le c_k^I\left(x, \tau_m^{(k)}\right) + \\ + \psi_m^{(k)}, \; m \ne i, \; m = 1, \ldots, N, \\ 0 \; \; in \; the \; other \; cases, \; i = 1, \ldots, N. \end{cases} \quad (15)$$

Calculate the values of function $G_1(\tau, \psi)$ and vector-pseudogradient of that function:

$$g(u) = g(\tau, \psi) = \left(g^{\tau}(\tau, \psi), -g^{\psi}(\tau, \psi)\right) =$$
$$\left(g^{\tau_1}(\tau, \psi), g^{\tau_2}(\tau, \psi), \ldots, g^{\tau_N}(\tau, \psi), -\right.$$
$$\left. -g^{\psi_1}(\tau, \psi), -g^{\psi_2}(\tau, \psi), \ldots, -g^{\psi_N}(\tau, \psi)\right),$$

at the grid nodes, if $\tau = \tau^{(k)}$, $\psi = \psi^{(k)}$, $\lambda(x) = \lambda^{(k)}$.

Select the initial exploratory step $h_0 > 0$, assume that $B_0 = I_{3N}$ is square matrix of $3N \times 3N$ dimension and define $u^{(1)} = u^{(0)} - h_0 g(u^{(0)})$.

If $\tau^{(1)} \notin \Pi^N$, then $\tau^{(1)} := P_{\Pi}^{N(\tau(1))}$, where $P_{\Pi}^N(\cdot)$ is projection operator on Cartesian product $\Pi^N$.

### 3.2. Step

Assume that as the calculation result after $k$, $k = 1, 2, \ldots$ algorithm steps, values $\tau^{(k)}$ $\psi^{(k)}$ are obtained, in terms of the grid nodes, matrix $B_k$.

Describe the $(k + 1)^{\text{th}}$ step.

1. Calculate values $\lambda^{(k)}(x)$ at the grid nodes according to formula (15) if $\tau = \tau^{(k)}$, $\psi = \psi^{(k)}$.
2. Calculate values of the component of vector-pseudogradient $g(u^{(k)})$.
3. Perform recurrent iteration of $r(\alpha)$-algorithm, which computational formula is as follows:

$$u^{(k+1)} = u^{(k)} - h_k B_{k+1} \frac{B_{k+1}^T g\left(u^{(k)}\right)}{\left\| B_{k+1}^T g\left(u^{(k)}\right) \right\|}.$$

In this context, $B_{k+1}$ is mapping operator of the transformed space into the basic one with coefficient of expansion $\alpha$, which is recalculated according to formula:

$$B_{k+1} = B_k \left( I + \left( \frac{1}{\alpha} - 1 \right) \theta_k \left( \theta_k \right)^T \right),$$

where:

$I$ – unit matrix of the corresponding dimension, $\theta_k$ is normalized vector of the difference of two sequential pseudogradients within the transformed space, i.e.:

$$\theta_k = \frac{B_{k+1}^T \left( g\left(u^{(k)}\right) - g\left(u^{(k-1)}\right) \right)}{\left\| B_{k+1}^T \left( g\left(u^{(k)}\right) - g\left(u^{(k-1)}\right) \right) \right\|},$$

under condition that $\left\| B_{k+1}^T \left( g\left(u^{(k)}\right) - g\left(u^{(k-1)}\right) \right) \right\| \ge \varepsilon_0$ and $\theta_k = 0$ in other cases, in this context $\varepsilon_0$ is accuracy of the machine zero representation in a computing device. Length of step factor $h_k$ is regulated adaptively involving parameters $q_1, q_2, n_k$.

If $\tau^{(k+1)} \notin \Pi^N$, then $\tau^{(\pi k+1)} := P_{\Pi^N}\left(\tau^{(k+1)}\right)$.

4. If condition:

$$\left\| u^{(k)} - u^{(k-1)} \right\| \le \varepsilon, \; \varepsilon > 0, \quad (16)$$

is not met, then move on to $(k + 2)^{\text{th}}$ step of the algorithm; in other case, move on to point 5.

5. Assume that $\tau^* = \tau^{(s)}$, $\psi^* = \psi^{(s)}$, $\lambda^*(x) = \lambda^{(s)}$, where $s$ is number of the iteration at which condition (16) is met.

6. Calculate values $b_i^I = \int_\Omega \rho(x)\lambda_i^*(x)dx$, $i=\overline{1,N}$ with the help of any cubature formula.

7. Determine values $v_{ij}^*$, $i=\overline{1,N}$, $j=\overline{1,M}$ while solving the problem of linear programming of transport type (10)-(13).

8. Calculate the value of target functional, if $\tau^*$, $\lambda^*$, $v^*$. End of the algorithm.

Algorithm is described.

### 3.3. Note 1

In terms of the represented algorithm, constraint satisfaction $\tau^I \in \Omega^N$ is implemented with the help of projection operator. Thus, algorithm 2 may be applied to solve OSPAC continuous problem only if set $\Omega$ is convex, closed, and has rather simple structure. Practical tasks may assume location of the first-stage centres within the specified area being often nonconvex and sometimes unconnected and having restricted areas (e.g. if there are some water bodies, industrial zones, rivers etc. at the territory). Then, there arises the necessity to solve the problem how not to allow locating the centre within the restricted area. Following approach is proposed to take into consideration the situation of not going beyond the limits of the admissible (partitioned) area of the located centres: to check the centre coordinates at each iteration step in terms of their belonging to the admissible area; if some centre appears to be within the restricted area, to find its “pseudoprojection” on the set under partitioning. In this context, any point $w \in \Omega$ being at the shortest distance from point $z$ will be considered as the presudoprojection of point $z \in E_2$ on closed set $\Omega \subset E_2$.

### 3.4. Note 2

Computational efficiency of the algorithm depends upon the coefficient of space expansion $\alpha$ and parameters of adaptive step adjustment $q_1$, $q_2$, $n_k$ from $r(\alpha)$-algorithm. In terms of nonsmooth functions, it is expedient to select those parameters as follows: $\alpha = 2 \div 3$, $h_0 = 1$, $q_1 = 1$, $q_2 = 1.1 \div 1.2$, $n_k = 2 \div 3$ [5]. Parameter $q_1$ is coefficient of step reduction, if condition of relaxation of the iteration process along the current descent direction is met per one step; $q_2$ is coefficient of step increase; in this context, natural number $n_k$ ($n_k > 1$) specifies the number of one-dimensional descent steps after which the step will be increased by $q_2$ times.

### 3.5. Note 3

The algorithm is easy to be modified in case of fixed coordinates of the first-stage centres. In this case, to find components of characteristic functions of subsets making up optimal partitioning of set $\Omega$, problem of unconditional maximization of nonsmooth function is solved instead of problem (8)-(9):

$$G_2(\psi) \to \max_{\psi \in R^N}, \tag{17}$$

$$G_2(\psi) = \int_\Omega \min_{k=\overline{1,N}} \left( c_k^I\left(x,\tau_k^I\right) + \psi_k \right) \rho(x)dx + \sum_{j=1}^{M} b_j \max_{i=\overline{1,N}} \left( \psi_i - c_{ij}^{II}\left(\tau_i^I, \tau_j^{II}\right) - a_i \right). \tag{18}$$

### 4. Examples of implementation of the problem-solving algorithms

We are going to demonstrate the operating results of the developed software to solve model two-stage OSPAC problems. Consider the bituminous coal deposits of Dnipropetrovsk Region as the area within which the resources are pooled; data concerning the deposit are represented by interactive map [21] (Fig. 1). Locations of the second-stage enterprises will be defined arbitrarily within the region territory without their connection to any specific objects.

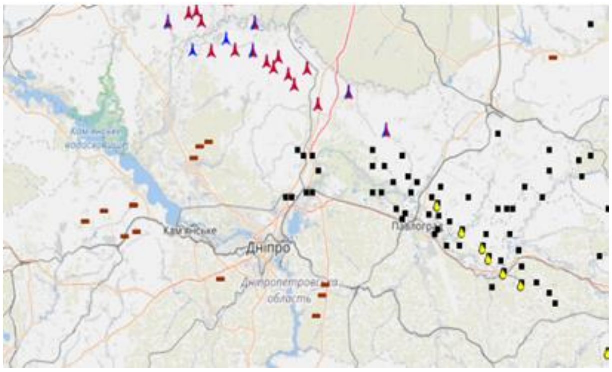



***Figure 1. Map of mineral deposits within the territory of Dnipropetrovsk region***

Preparatory stage of the electronic map processing involves elimination of some places, which do not belong to the regional territory, from the map with the help of graphic editor. Introduce the conditional (relative) rectangular coordinate systems, having taken some arbitrary point as the beginning, so that the area under consideration would be completely inside rectangle $\Pi = \{(x_1, x_2): 0 \le x_1 \le 12; 0 \le x_2 \le 12\}$. The territory corresponding to the deposit on the map is the set under partitioning $\Omega$. In terms of numerical implementation of the algorithms, discretization of the specified area is performed. To calculate multiple integrals, the developed software applies cubature trapezoid formula; problem (10)-(13) is solved by means of the potential method. All the calculations were carried out in terms of following values of errors and parameters of $r(\alpha)$-algorithm: $\varepsilon_{1,2} = 0.0001$; $\alpha = 3$, $\beta = 0.9$, $\varepsilon = 0.0001$. All the problems represented below mean that total needs in the resources for the second-stage enterprises are 1 conditional unit. Consequently, according to condition (6), total amount of the resource pooled in terms of $\Omega$ by all the first-stage enterprises is also equal to 1 conditional unit. To calculate functions $c_i^I(x, \tau_i)$ and $c_{ij}^{II}(\tau_i, \tau_j)$, Minkowskian metric $c(x, y) = ((x_1 - y_1)^p + (x_2 - y_2)^p)^{1/p}$ is applied in terms of specific parameter setting $p$. Function $\rho(x) = 1$ is for all the points of area $\Omega$.

### 4.1. Problem 1. Continuous OSPAC problem with fixed centres

Initial data: $N = 4$, $M = 7$; $\tau_1^I = (6.094;\ 3.52)$, $\tau_2^I = (9.35;\ 3.828)$, $\tau_3^I = (9.702;\ 5.082)$, $\tau_4^I = (7.832;\ 4.356)$;

$\tau_1^{II} = (10.098;\ 4.202)$, $\tau_2^{II} = (8.91;\ 6.446)$, $\tau_3^{II} = (7.788;\ 2.574)$, $\tau_4^{II} = (4.466;\ 2.398)$, $\tau_5^{II} = (2.2;\ 4.422)$, $\tau_6^{II} = (2.464;\ 6.644)$, $\tau_7^{II} = (6.292;\ 5.39)$, $b^{II} = (0.174;\ 0.06;\ 0.127;\ 0.154;\ 0.158;\ 0.192;\ 0.131)$. Figure 2 shows optimal partitioning of set $\Omega$ and indicates connections of the first- and second-stage enterprises in two variants:

1) parameters of Minkowskian metric $p = 1$ and for functions $c_i^I(x, \tau_i)$;

2) for functions $c_{ij}^{II}(\tau_i, \tau_j)$, $a_i = 0$, $i = \overline{1,4}$;

3) $p = 10$ for functions $c_i^I(x, \tau_i)$, $p = 1$ for functions $c_{ij}^{II}(\tau_i, \tau_j)$, a = (0.5; 0.62; 0.36; 0.45).

In this context, amount of the resources pooled by the corresponding first-stage enterprises is as follows (with the accuracy of 0.001):

a) $b^I = (0.307;\ 0.219;\ 0.178;\ 0.295)$;

b) $b^I = (0.29;\ 0.194;\ 0.201;\ 0.313)$.

(a) (b)

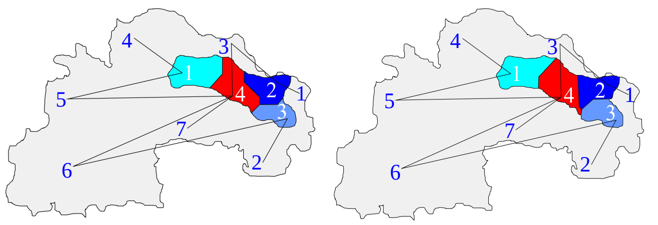


***Figure 2. Optimal set partitioning and resource transportation scheme between the first- and second-stage enterprises***

According to the figure, the available connections between the first- and second-stage enterprises are identical; there is slight difference only in the amount of the transported resources shown in Table 1.

***Table 1. Amount of the resource transported between the enterprises of two stages in problem 1***

| Number of the first-stage centre $i$ | Number of the second-stage centre $j$ | Amount of the transported resource $v_{ij}$ in the problem | |
|---|---|---|---|
| | | *a* | *b* |
| 2 | 1 | 0.1748 | 0.1748 |
| 3 | 2 | 0.0606 | 0.0606 |
| 2 | 3 | 0.044 | 0.0199 |
| 4 | 3 | 0.083 | 0.1075 |
| 1 | 4 | 0.155 | 0.1548 |
| 1 | 5 | 0.152 | 0.1352 |
| 4 | 5 | 0.0062 | 0.0233 |
| 3 | 6 | 0.1177 | 0.1407 |
| 4 | 6 | 0.0746 | 0.0517 |
| 4 | 7 | 0.1314 | 0.1314 |

### 4.2. Problem 2. Continuous OSPAC problem with the location of the first-stage centres

The problem was solved in terms of different initial data.

Problem 2.1. Initial conditions: $N = 4$, $M = 7$; $\tau_1^I = (8.8;\ 6.4)$, $\tau_2^{II} = (10.1;\ 4.36)$, $\tau_3^{II} = (5.5;\ 1.8)$, $\tau_4^{II} = (4.5;\ 2.8)$, $\tau_5^{II} = (2.6;\ 3.6)$, $\tau_6^{II} = (2.3;\ 5.6)$, $\tau_7^{II} = (4.95;5.37)$; $b^{II} = (0.14;\ 0.173;\ 0.134;\ 0.16;\ 0.133;\ 0.076;\ 0.181)$; $a_i = 0$, $i = \overline{1,4}$, $p^{I,II} = 2$.

Figure 3a demonstrates the initial location of the first-stage centres. It also shows its corresponding set partitioning and scheme of additional connections as the solution for OSPAC problem with the fixed centres. It should be noted that the value of target functional (1) of the problem with such data is equal to 4.4322 conv. un. While solving OSPAC problem with the centres location, following components of optimal solution were obtained: $\tau_1^I = (7.93;\ 3.92)$, $\tau_2^I = (7.01;\ 3.45)$, $\tau_3^I = (9.13;\ 4.78)$, $\tau_4^I = (9.07;\ 4.0)$, $v_{11} = 0.14$, $v_{12} = 0.08$, $v_{22} = 0.09$, $v_{23} = 0.134$, $v_{24} = 0.127$, $v_{34} = 0.033$, $v_{35} = 0.136$, $v_{36} = 0.072$, $v_{46} = 0.004$, $v_{47} = 0.182$. Optimal partitioning and additional connections are shown in Figure 3b. Optimal functional value is 3.5011 conv. un. being much lower comparing to the abovementioned value.

(a) (b)

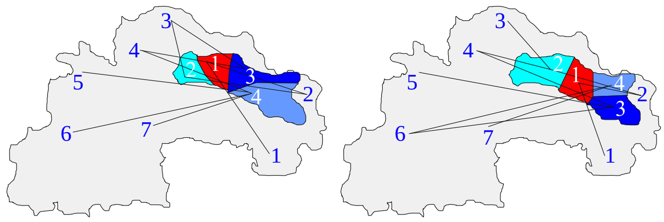


***Figure 3. Optimal set partitioning and resource transportation scheme between the first- and second-stage enterprises in problem 2.1***

Problem 2.2. Initial conditions: $N = 6$, $M = 3$; $\tau_1^{II} = (4.246;\ 2.53)$, $\tau_2^{II} = (1.54;\ 6.424)$, $\tau_3^{II} = (8.7;\ 6.16)$, $b^{II} = (0.337;\ 0.161;\ 0.502)$; $a^I = (0.12,\ 0.15,\ 0.18,\ 0.10,\ 0.13,\ 0.17)$, $a_i^{II} = 0$, $i = \overline{1,3}$, $p^{I,II} = 1$.

Figure 4a shows the initial location of the first-stage centres as well as the corresponding set partitioning and scheme of additional connections obtained as the solution of OSPAC problem with the fixed centres. Value of target functional (1) of the problem in terms of such data is equal to 7.13 conv. un. While solving OSPAC problem with the centres location, following components of optimal solutions were obtained: $\tau_1^I = (6.7;\ 3.6)$, $\tau_2^I = (7.39;\ 3.3)$, $\tau_3^I = (7.7;\ 3.9)$, $\tau_4^I = (8.35;\ 4.13)$, $\tau_5^I = (9.2;\ 3.96)$, $\tau_6^I = (9.57;\ 4.75)$, $v_{11} = 0.214$, $v_{21} = 0.122$, $v_{12} = 0.021$, $v_{32} = 0.13$, $v_{42} = 0.01$, $v_{43} = 0.171$, $v_{53} = 0.143$, $v_{63} = 0.188$. In this context, with the accuracy of 0.001, amount of the resource pooled by the corresponding first-stage enterprises is $b^I = (0.236;\ 0.122;\ 0.129;\ 0.181;\ 0.142;\ 0.188)$. Optimal partitioning and additional connections are shown in Figure 4b. Functional value is 7.06 conv. un.

(a) (b)

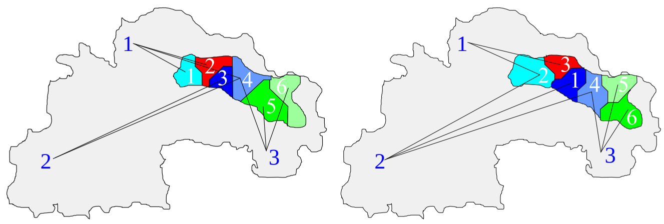
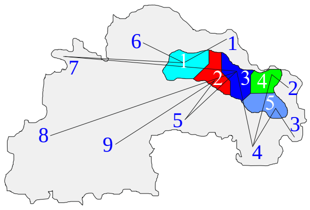
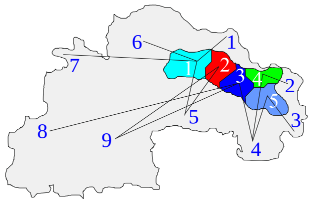
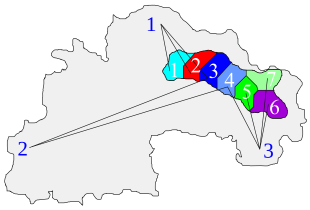
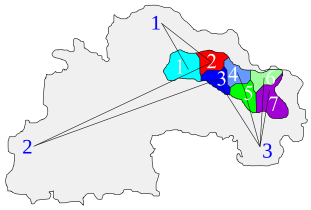
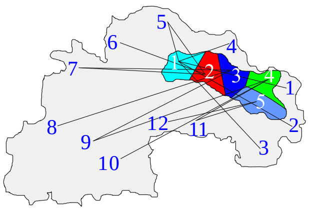
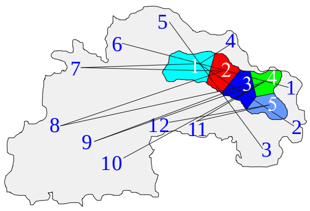


***Figure 4. Optimal set partitioning and resource transportation scheme between the first- and second-stage enterprises in problem 2.2***

Table 2 represents only optimal partitioning in terms of fixed first-stage centres (in their initial approximation) for short demonstration of the results of other problems solution; it also shows optimal location of the first-stage centres with the representation of their specified zones and additional connections with the second-stage enterprises. Note that in terms of all the experiments dealing with optimal centres location, 10-30% reduction of target functional is observed comparing to its value in the initial approximation.

***Table 2. Solution results of the OSPAC continuous problems with the first-stage centres location***

| No | Optimal partitioning of set $\Omega$ and transportation scheme: Initial location of the first-stage centres | Optimal partitioning of set $\Omega$ and transportation scheme: Optimal location of the first-stage centres |
|---|---|---|
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |

To compare, the first line of the table contains the results of partitioning involving Manhattan metric; the second line uses Minkowskian metric, $p = 10$; and the third line involves Euclidean metric.

Figure 5 illustrates the area partitioning into six zones taking into consideration the fact that two of the second-stage enterprises are within the territory under partitioning. Figure 5a corresponds to the fixed coordinates of the first-stage centres. Figure 5b shows the case of their optimal location. It is clear that both variants demonstrate the partitioning when the second-stage centres turn to be at the boundary between some zones, and first-stage centres are the vertexes of a polygon with the gravitational centres at points $\tau_j^{II}$.

(a) (b)

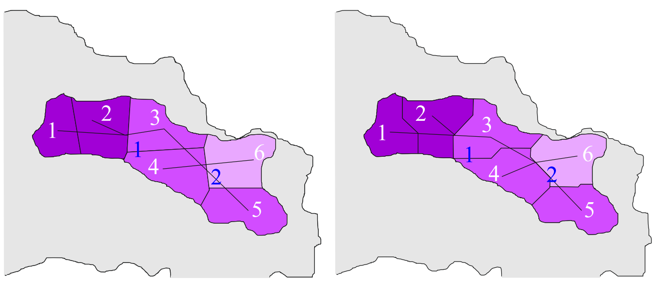


***Figure 5. Optimal set partitioning and resource transportation scheme. Second-stage enterprises are within the territory under partitioning***

It should be noted that while solving concrete practical two-stage OSPAC problems with the fixed centres to find the shortest way between any two first- and second-stage centres taking into account traffic diagram, we can use Google Maps Distance Matrix API library. To reduce the amount of library calls and, consequently, the amount of the involved computing resources such as the amount of random access memory and the time required to run either one query or all the queries in general, it is possible to solve OSPAC problems with centres location in two stages. Stage one deals with the problem solution with distance functions equal to one of the known metrics – Manhattan, Euclidian, Minkowskian, Chebyshev. The metric is selected by preliminary comparison of the results of the specific area partitioning basing upon the distance between the points calculated both theoretically and with the help of GIS. Stage two added GIS to search actual distance between the determined centres and regional points; this stage deals with the definition of optimal partitioning of the specified region and costs for the resource transportation between the first- and second-stage enterprises.

## 5. Conclusion

The paper has considered the issues of optimal arrangement of two-stage processes of material flow allocation in transportation and logistic chain, which structural elements are represented by the mines extracting coal within the certain territory and enterprises consuming or processing that resource.

Solution of two-stage problems of raw material flow allocation helps demonstrate that it is possible to reduce the total costs due to the management of material and their accompanying service flows throughout the whole logistic chain beginning from the moment of flow origin up to its arrival to the end user.

The developed software implementing the algorithms of OSPAC problems solution involving up-to-date geoinformation resources may be applied for quantitative substantiation and decision-making as for the location of new enterprises connected with mineral extraction taking into account its further transportation to the end users.

The proposed mathematical and algorithmic apparatus to solve OSPAC continuous problems relative to the fuel and energy complex enterprises makes it possible to partition the deposit area into the zones, which are allocated to the first-stage enterprises exclusively. However, according to the interactive map of fossil fuel minerals [21], territories of some operating mines developing one and the same deposit, may be overlapped (due to coal mining in terms of different mines). It is possible to consider such a situation while describing two-stage optimization problems of location-allocation with the help of the models of continuous problems of optimal multiplex set partitioning [22] being the subject for further studies. Moreover, application of other methods to solve problems of nonsmooth functions minimization (e.g. quasi-Newtonian) is of certain interest as well [23], [24]. Taking into consideration the fact that in real problems of optimal material flow allocation it is quite difficult to describe analytically the distance between two arbitrary regional points, it would be useful to study the possibility of applying DFO methods (derivative-free optimization) [25] or uncertain algorithms (represented in [26]) to solve the problems obtained as a result of the OSPAC problems reduction.

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## Вирішення задач оптимізації двоетапного розподілу матеріальних потоків

А. Булат, С. Дзюба, С. Мінєєв, Л. Коряшкіна, С. Ус


**Мета.** Розробка нового, ефективного з обчислювальної точки зору, алгоритму вирішення двоетапної задачі розподілу ресурсу, що безперервно займає задану область, а також демонстрація роботи відповідного програмного забезпечення, створеного із застосуванням сучасних геоінформаційних ресурсів.

**Методика.** У роботі використано математичні моделі безперервних задач оптимального розбиття множин з додатковими зв'язками для опису двоетапних задач розміщення-розподілу матеріальних ресурсів. Методичний підхід вирішення таких задач заснований на ідеї зведення їх до задач нескінченномірного математичного програмування, для яких, в свою чергу, за допомогою застосування апарату теорії подвійності оптимальне рішення вдається отримати в аналітичному вигляді.

**Результати.** Розроблено математичний і алгоритмічний апарати вирішення безперервних задач у застосуванні до підприємств паливно-енергетичного комплексу, що дозволяє отримувати розбиття району родовища на зони, за якими підприємства першого етапу закріплюються монопольно. Робота алгоритму показана на прикладі вирішення модельної задачі. Визначено, що виграшем описаного підходу є зведення задачі нескінченномірного програмування до задачі кінцево-мірної негладкою оптимізації, оскільки отримані розрахункові формули містять параметри, для визначення яких потрібно вирішити допоміжну задачу оптимізації недиференційованої функції.

**Наукова новизна.** Представлений алгоритм, на відміну від раніше розробленого, не передбачає вирішення задачі лінійного програмування транспортного типу на кожному кроці ітераційного процесу. Така задача вирішується лише один раз для відшукання обсягів перевезень продукції між підприємствами першого і другого етапів після того, як знайдені інші компоненти оптимального рішення.

**Практична значимість.** Програмна реалізація алгоритму на основі сучасних геоінформаційних технологій і ресурсів на прикладі сировинних потоків дозволяє зменшити сукупність витрат, пов'язаних з управлінням матеріальними і супутніми їм сервісними потоками по всьому логістичному ланцюгу, від моменту зародження потоку до надходження його кінцевому споживачеві.


***Ключові слова:*** *багатоетапні задачі, розбиття множини, геоінформаційні технології, розміщення-розподіл, негладка оптимізація*

### Решении задач оптимизации двухэтапного распределения материальных потоков

А. Булат, С. Дзюба, С. Минеев, Л. Коряшкина, С. Ус

**Цель.** Разработка нового, эффективного с вычислительной точки зрения, алгоритма решения двухэтапной задачи распределения ресурса, непрерывно занимаемого заданную область, а также демонстрация работы соответствующего программного обеспечения, созданного с применением современных геоинформационных ресурсов.

**Методика.** В работе использованы математические модели непрерывных задач оптимального разбиения множеств с дополнительными связями для описания двухэтапных задач размещения-распределения материальных ресурсов. Методический подход решения таких задач основан на идее сведения их к задачам бесконечномерного математического программирования, для которых, в свою очередь, с помощью применения аппарата теории двойственности оптимальное решение удается получить в аналитическом виде.

**Результаты.** Разработан математический и алгоритмический аппарат решения непрерывных задач в применении к предприятиям топливно-энергетического комплекса, который позволяет получать разбиение района месторождения на зоны, за которыми предприятия первого этапа закрепляются монопольно. Работа алгоритма показана на примере решения модельной задачи. Определено, что выигрышем описанного подхода является сведение задачи бесконечномерного программирования к задаче конечномерной негладкой оптимизации, поскольку полученные расчетные формулы содержат параметры, для определения которых нужно решить вспомогательную задачу оптимизации недифференцируемой функции.

**Научная новизна.** Представленный алгоритм, в отличие от ранее разработанного, не предусматривает решения задачи линейного программирования транспортного типа на каждом шаге итерационного процесса. Такая задача решается лишь один раз для отыскания объемов перевозок продукции между предприятиями первого и второго этапов после того, как найдены остальные компоненты оптимального решения.

**Практическая значимость.** Программная реализация алгоритма на основе современных геоинформационных технологий и ресурсов на примере решения задачи распределения сырьевых потоков позволяет уменьшить совокупность издержек, связанных с управлением материальными и сопутствующими им сервисными потоками по всей логистической цепи, от момента зарождения потока до поступления его конечному потребителю.

***Ключевые слова:*** *многоэтапные задачи, разбиение множества, геоинформационные технологии, размещение-распределение, негладкая оптимизация*

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