

# STUDY OF THE CHARACTERISTICS OF MINERAL SUSPENSIONS FOR THE DEVELOPMENT OF EQUIPMENT FOR THE SEPARATION OF FINELY DISPERSED ROCK MASS

*V.P. Nadutiy<sup>1</sup>, V.V Chelyshkina.<sup>1</sup>, V.S. Kurilov<sup>1\*</sup>*

*<sup>1</sup>Institute of Geotechnical Mechanics named by N. Poljakov of NAS of Ukraine, Dnipro, Ukraine*

*\*Corresponding author: [papuycv@gmail.com](mailto:papuycv@gmail.com)*

**Abstract.** Calculation of hydraulic devices and apparatuses for extracting valuable components from mineral suspensions is based on determining the speed of the constrained motion of particles. For calculation of the speed, it is necessary to know the characteristics of the suspension - viscosity, porosity, density. A method for analytical calculation of these characteristics has been developed. Its peculiarity is that all characteristics are related only to the bulk density of the medium, which is easily determined in practice by weighing. The results of analytical calculations of the characteristics are given using the example of two suspensions - with a solid particle density of 2.65 g/cm<sup>3</sup> (amber sands) and 2.0 g/cm<sup>3</sup> (crushed tuff) with a medium density of up to 1.8 g/cm<sup>3</sup>. The described method can also be used for multicomponent mixtures. It allows to predict the change in the characteristics of the slurry from the operating parameters of the equipment. The method is used to assess technological modes and determine structural indicators in the design of hydraulic devices and apparatuses.

**Key words:** mineral suspension, viscosity, porosity, density

## ДОСЛІДЖЕННЯ ХАРАКТЕРИСТИК МІНЕРАЛЬНИХ СУСПЕНЗІЙ ДЛЯ РОЗРОБКИ ОБЛАДНАННЯ ДЛЯ ПОДІЛУ ТОНКОДИСПЕРСНОЇ ГІРСЬКОЇ МАСИ

*В.П. Надутий<sup>1</sup>, В.В. Челишкіна<sup>1</sup>, В.С. Курілов<sup>1\*</sup>*

*<sup>1</sup>Інститут геотехнічної механіки ім. М.С. Полякова НАН України, Дніпро, Україна*

*\* Відповідальний автор: [papuycv@gmail.com](mailto:papuycv@gmail.com)*

**Анотація.** Розрахунок гідравлічних пристроїв і апаратів для вилучення цінних компонентів з мінеральних суспензій заснований на визначенні швидкості туго руху частинок. Для розрахунку швидкості потрібно знати характеристики суспензії - в'язкість, пористість, щільність. Розроблено метод аналітичного розрахунку зазначених показників. Його особливістю є те, що всі характеристики пов'язані лише з об'ємною щільністю середовища, яка легко визначається на практиці методом зважування. Наведено результати аналітичних розрахунків характеристик на прикладі двох суспензій – з щільністю твердих частинок 2,65 г/см<sup>3</sup> (янтароносні піски) і 2,0 г/см<sup>3</sup> (подрібнений туф) при щільності середовища до 1,8 г/см<sup>3</sup>. Викладений метод також можна використовувати для багатокомпонентних сумішей. Він дозволяє прогнозувати зміну характеристик пульпи від режимних параметрів обладнання. Метод використовується для оцінки технологічних режимів і визначення конструкторських показників при проектуванні гідравлічних пристроїв і апаратів.

**Ключові слова:** мінеральна суспензія, в'язкість, пористість, щільність

**Introduction.** Gravitational or hydraulic separation of mineral suspensions by particle density is used in hydraulic classification and separation, jigging, clarification of circulating water in sedimentation tanks and other processes. The IGTM NAS of Ukraine is working to improve the technology and develop devices for these processes [1]. The calculation of technological indicators and the design of the corresponding apparatuses is based on the determination of the rate of hindered settling of particles. All known formulas for determining the rate of hindered settling include the characteristics of the suspension, such as viscosity, porosity or transparency of the granular layer, weight or bulk density [2]. The determination of these characteristics, as a rule, is carried out

experimentally. This requires a large number of measurements, given that the characteristics of the slurry in the working area change depending on the operating mode of the apparatus.

Theoretical studies of viscosity, porosity, and density are most often based on modeling a suspension in the form of a cellular discrete structure. This makes it possible, for example, to determine the indicators for cubic or rhombohedral packing of particles [3]. Thus, this model describes a densely packed granular layer, but does not allow evaluating the characteristics for dilute mineral suspensions. For dilute suspensions, empirical and semi-empirical formulas for determining porosity and viscosity are known. However, in general, they cover a narrow range of solid concentration and are applicable only for specific suspensions. Therefore, the development of methods for the analytical assessment of the characteristics of suspensions is very important.

**The aim** of the work is to establish analytical relationships for determining the characteristics of a mineral suspension - porosity, viscosity, weight density. The idea of the work is to determine all characteristics depending on only one indicator - on the bulk density of the suspension  $\rho_c$ . The indicated dependencies are required to calculate the speed of free and constrained motion of particles, which is the basis for calculations in the design of hydraulic devices.

**Methodology.** The characteristics of the suspension will be determined as functions of one argument - the bulk density  $\rho_c$ . In practice, it is easily measured by weighing a sample of the suspension in a 1 liter measuring cup. We will get the calculated functions using the known definitions of the parameters.

The weight density (hereinafter referred to as the solid percentage)  $\theta$  and the density of the suspension  $\rho_c$  are determined by the formulas [4]:

$$\theta = \frac{V_m \cdot \rho_m}{V_m \cdot \rho_m + V_{жс} \cdot \rho_{жс}} \cdot 100, \% \quad (1)$$

$$\rho_c = \frac{1}{1 - \frac{\theta}{100} \left(1 - \frac{1}{\rho_m}\right)}, \text{ г/см}^3 \quad (2)$$

where  $V_m$ ,  $V_{жс}$ , и  $\rho_m$ ,  $\rho_{жс}$  are the volume and density of solid and liquid phases, respectively.

The porosity or volume of gaps between particles is defined as the relative amount of liquid between particles [2, 3]:

$$\varepsilon = \frac{V_{жс}}{V_{жс} + V_m}, \text{ units} \quad (3)$$

From formula (2) it follows that:

$$\theta = 100 \frac{\rho_m (\rho_c - 1)}{\rho_c (\rho_m - 1)}, \% \quad (4)$$

From formulas (1) - (3) we get:

$$\varepsilon = 1 - \frac{\rho_c}{\rho_m} \cdot \frac{\theta}{100} \quad (5)$$

To calculate the kinematic viscosity, we use the Wend formula, since it covers the widest range of changes in the porosity of the suspension  $\varepsilon$  [2]:

$$\nu = \nu_0 \exp \frac{2,5\beta + 0,675\beta^2}{1 - 0,609\beta} \quad (6)$$

where  $\beta = 1 - \varepsilon$  is the volume fraction of a solid or the coefficient of the volume concentration of a solid in a suspension,  $\nu_0 = 0.01 \text{ cm}^2 / \text{s}$  is the kinematic viscosity of water at  $20^\circ$ .

The procedure for determining the analytical dependences of the characteristics  $\varepsilon$ ,  $\nu$ ,  $\theta$  is as follows.

For a specific suspension, we set the density of its constituent solid particles  $\rho_m$  and the density of the fluidizing agent  $\rho_{жс}$ , usually this is the density of water,  $\rho_{жс} = 1 \text{ g/cm}^3$ . In the practice of operating hydraulic devices, the density of the suspension  $\rho_c$  can be from 1.2 to 1.7  $\text{g/cm}^3$ . Let's take a range of 1.1 - 1.8 in which we will change  $\rho_c$  with a step of 0.1. For each value of  $\rho_c$ , we calculate  $\theta$ ,  $\varepsilon$ ,  $\nu$  according to the corresponding formulas (4) - (6). Thus, a database is compiled.

Its analysis allows one to obtain approximation equations describing the dependence of these properties on the density  $\rho_c$ . Note that formulas (4) - (6) include the density of solid particles  $\rho_m$ , the density of the fluidizing agent is included only in formula (1), that is, it is taken into account indirectly. It is advisable to supplement and expand the resulting database by taking into account the performance of the device. To do this, each current value of  $\rho_c$  is associated with the load for solid (productivity)  $P_m = V_m \cdot \rho_m$  and for liquid -  $V_{жс}$ . In this case, one of these two parameters,  $P_m$  or  $V_{жс}$ , is set fixed, the second is calculated by the formula (1), where the value of  $\theta$  for the current value of  $\rho_c$  has already been determined by formula (4).

For example, we set the performance of the device  $P_m = V_m \cdot \rho_m$ , then for the current value of  $\rho_c$  we calculate the percentage of solid  $\theta$  according to formula (4), then, using formula (1), we calculate the water consumption  $V_{жс}$ , which will provide these indicators  $\rho_c$  and  $\theta$ . For a given productivity  $P_m$ , the obtained values of water consumption  $V_{жс}$  are entered into the database in a separate column.

Indicators  $P_m$ ,  $V_{жс}$ ,  $\theta$  are usually given on a water-sludge diagram characterizing the operation of the device. They are recorded separately for each of the products - supply, sands, drain. According to the above procedure, it is possible to evaluate the characteristics of the suspension that constitutes these products. To do this, we calculate the density of the suspension  $\rho_c$  for each of the products using formula (2), then we calculate the corresponding values of  $\varepsilon$  and  $\nu$  using formulas (4), (6). Note that the density of the solids that make up the products should be defined as the weighted average density of the main components. In this case, for example,  $\rho_m$  of sands will be slightly higher than  $\rho_m$  of supply, and  $\rho_m$  of drain will be lower. That is, the weighted average density of the concentrate particles is higher than that of waste and supply, which is taken into account according to the data of the chemical analysis of products.

**Results and discussion.** According to the described method, the database was obtained for two suspensions with the density of solid particles  $\rho_m = 2.65$  and  $2 \text{ g/cm}^3$ . The first suspension is a mixture of sand with water, which reflects the process of hydraulic enrichment of amber sands, the second is a mixture of crushed tuff with water. Using the database, the dependences of the characteristics  $\varepsilon$ ,  $\theta$ ,  $\nu$  on the density  $\rho_c$  were constructed and the corresponding correlation equations were obtained. The dependence  $\varepsilon = f(\rho_c)$  is shown in (Fig. 1).

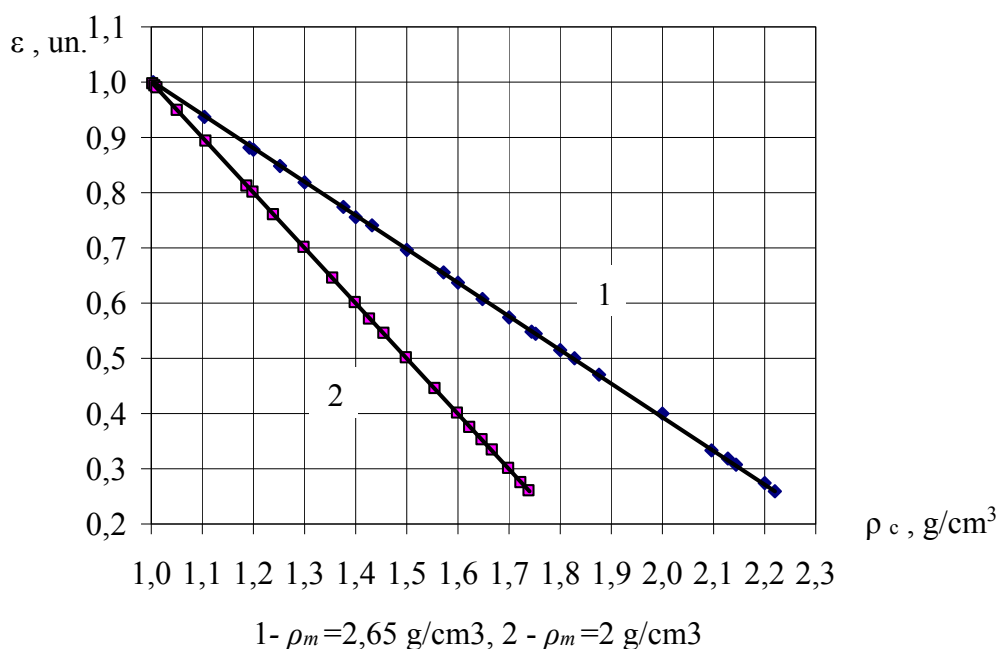


Figure 1. – Dependence of porosity on the density of suspensions

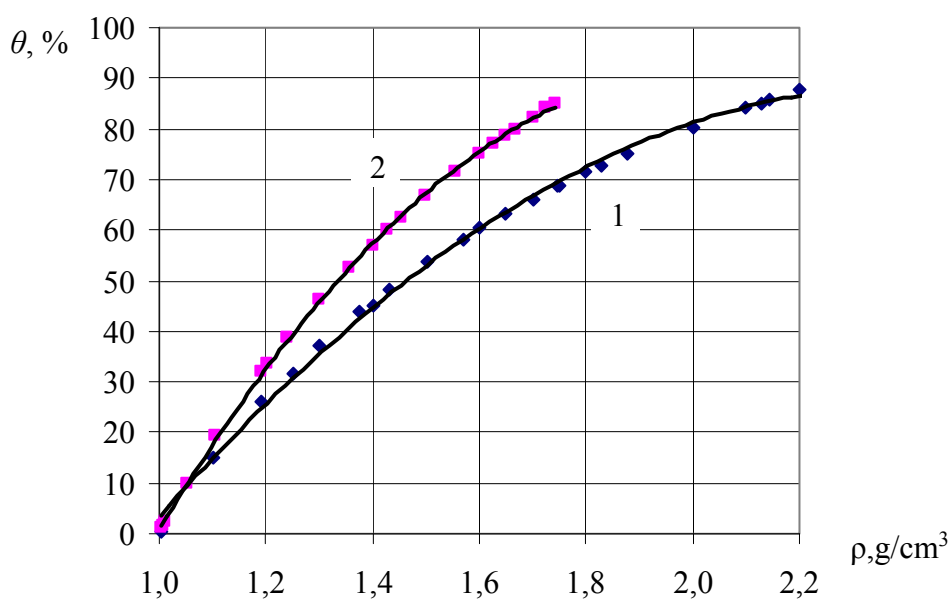
The dependences in Fig. 1 are limited from below by the value  $\varepsilon = 0.259$ , which corresponds to the most dense rhombohedral packing of particles (packing of equal spheres according to Gaussian). The upper boundary value  $\varepsilon$  only tends to 1, but does not reach it, since the equality  $\varepsilon = 1$  means that instead of a suspension there is water without solid particles. For example, with  $\rho_c = 1.002 \text{ g/cm}^3$  according to formulas (4), (5)  $\varepsilon = 0.9986$  and  $\theta = 0.3\%$ . A low, but nonzero, value of  $\theta$  indicates that at such a high porosity, there are still particles in the suspension.

Fig. 1 shows that at the same suspension density, the porosity in a suspension with lighter particles is lower than in a suspension with heavier ones. If two suspensions have the same porosity, then for heavier particles the density of the suspension will be higher.

The approximating equations for the dependence of porosity and density are the following:

$$\begin{aligned}
 - \rho_m = 2,65 \text{ g/cm}^3: & \quad \varepsilon = 1,6073 - 0,6069\rho_c, \quad R^2 = 0,999, & (7) \\
 - \rho_m = 2,0 \text{ g/cm}^3: & \quad \varepsilon = 2 - \rho_c, \quad R^2 = 0,999
 \end{aligned}$$

Fig. 2 shows the dependences  $\theta = f(\rho_c)$ .



1-  $\rho_m = 2,65 \text{ g/cm}^3$ , 2 -  $\rho_m = 2 \text{ g/cm}^3$

Figure 2. – Dependence of the weight percent of solids in suspension on density

For the dependences in Fig. 2, the minimum porosity  $\varepsilon = 0.259$  also served as the upper limit. With  $\varepsilon = 0.259$  according to formula (5), the percentage of solids in the slurry is 88.2%, with  $\rho_m = 2.65 \text{ g/cm}^3$  and 85.1% with  $\rho_m = 2 \text{ g/cm}^3$ . For the accepted cellular model, a higher percentage of solids in the suspension is not realized. This is sufficient for evaluating suspensions in hydraulic devices, since such a density is not observed there, for example, it is 70-75% in the sands of spiral classifiers  $\theta$ . The approximation of the obtained data showed that the dependences of  $\theta$  on  $\rho_c$  are parabolic:

$$\begin{aligned}
 - \rho_m = 2,65 \text{ g/cm}^3: & \quad \theta = 43,628 \cdot (4,784\rho_c - \rho_c^2 - 3,712), \quad R^2 = 0,998, & (8) \\
 - \rho_m = 2,0 \text{ g/cm}^3: & \quad \theta = 84,448 \cdot (4,07\rho_c - \rho_c^2 - 3,06), \quad R^2 = 0,9996
 \end{aligned}$$

When analyzing the dependences  $v = f(\rho_c)$  (Fig. 3, a), it was found that the accuracy of their approximation by simple functions is not sufficient. Therefore, the following reduced variables were used  $(x, y) = [(100-1/v), \rho_c]$ . This made it possible to increase the correlation coefficient  $R^2$  to 0.99 when approximating the dependences by a polynomial of the second degree (Fig. 3, b).

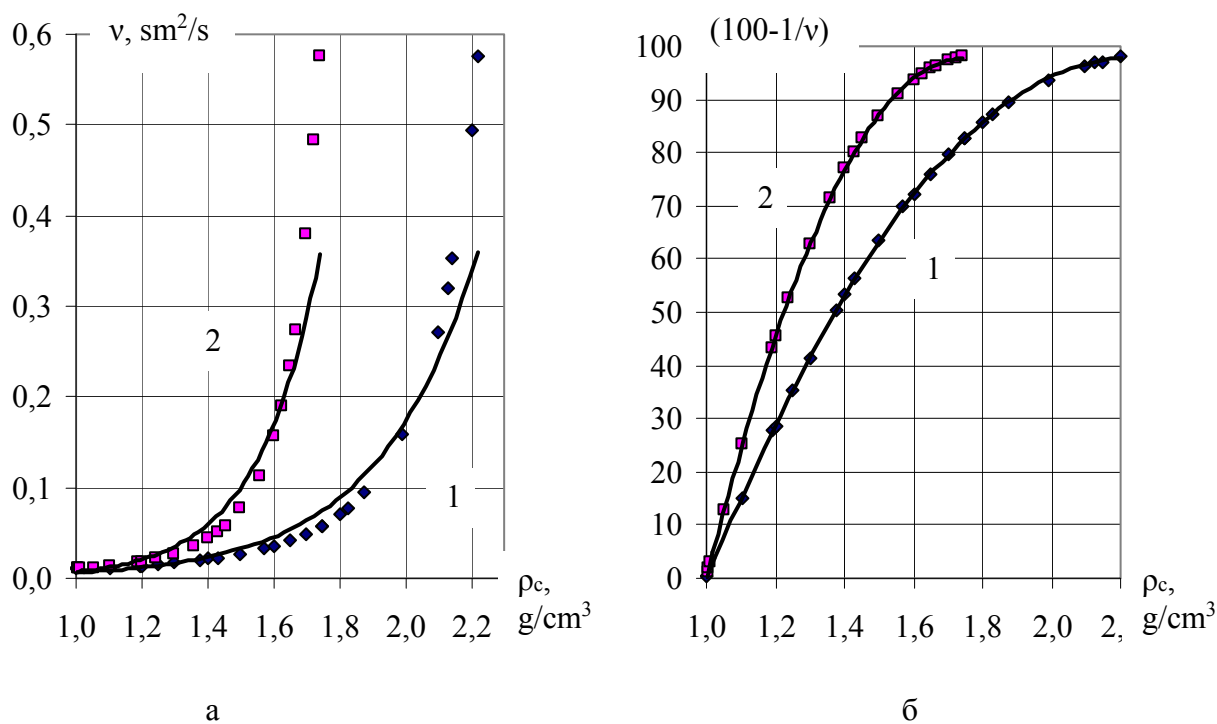


Figure 3.- Dependences of kinematic viscosity on the density of suspensions

$$\text{With: } \rho_m = 2,65 \text{ g/cm}^3: \quad \nu = \frac{1}{64,89 \cdot \rho_c^2 - 289,46 \cdot \rho_c + 325}, \quad R^2 = 0,99 \quad (9)$$

$$\text{with } \rho_m = 2,0 \text{ g/cm}^3: \quad \nu = \frac{1}{175,42 \cdot \rho_c^2 - 612,94 \cdot \rho_c + 537,7}, \quad R^2 = 0,99$$

The average absolute error in calculating  $\nu$  by formulas (9) is 0.00107 cm<sup>2</sup>/s, the average relative error is 2.2%.

In conclusion, we note that, since the already crushed and classified raw materials enter the hydraulic devices, we did not consider the possible dispersion of particles in size and shape. If it is necessary to take these factors into account, one can use, for example, the approach described in [5]. However, taking into account the small scatter of the supply size of the hydraulic devices and the shape of the particles, in the first approximation it can be assumed that the estimation of the characteristics of the investigated suspensions according to formulas (7), (8), (9) is acceptable for further calculations of the velocity of the constrained motion of particles and further equipment parameters.

**Conclusions.** For the design of hydraulic devices, it is necessary to calculate the rate of hindered settling and floating of particles. To do this, you first need to determine the characteristics of the suspension, such as viscosity, porosity, weight percent solids. This is usually done through time-consuming experimental research.

The article proposes a method for analytical calculation of the specified characteristics depending on only one parameter - the bulk density of the suspension  $\rho_c$ . This parameter is easily determined in practice by weighing.

The developed method is illustrated by the example of two aqueous suspensions. In the first suspension, solid particles are quartz sand (the main component of the amber-bearing rocks of the Klesovsky quarry), in the second – crushed tuff (waste of basalt mining at the Rafalovsky basalt quarry). For these suspensions, approximation dependences of the characteristics on the density of the medium are obtained. The influence of the size and shape of particles on the characteristics of suspensions, if necessary, is taken into account by introducing correction factors. However, taking

into account that crushed and classified raw materials enter the hydraulic devices, in the first approximation it can be assumed that the obtained approximation dependences are acceptable for practical calculations.

The described method for assessing the characteristics of suspensions can be used for multicomponent mixtures of solid particles and various fluidizing agents, while, for example, the density of solid particles is defined as the weighted average density of the constituent components. Using this method, it is possible to take a sample of the suspension at any point in the hydraulic apparatus, determine the density by weighing, calculate the characteristics of the suspension and, as a result, calculate the velocity field in the entire working zone of the apparatus. Also, using this method, it is possible for each product, which is reflected in the water-sludge scheme of the device, to match the characteristics of the suspension that this product consists of.

The described method for determining the characteristics of suspensions is used in calculating the rate of hindered settling and floating of particles of mineral slurries when determining the technological and design parameters of hydraulic devices.

**Acknowledgements.** Performed on the basis of own research without external financial assistance.

### **Список літератури**

1. Надутий, В. Аналітична презентація поділу щільних суспензій для видобутку бурштину / Надутий В., Корнієнко В., Маланчук З., Челишкіна О. // E3S Web of Conferences 109, 00059 (2019). Нариси гірничої науки та практики DOI: 10.1051 / e3sconf / 201910900059.
2. Хаппель, Дж. Гидродинамика при малых числах Рейнольдса / Дж. Хаппель, Г. Бреннер // Перевод с англ. – М.: Мир, 1976. – 630 с. С. 483-486.
3. Айнштейн, В.Г. Общий курс процессов и аппаратов химической технологии. Кн.1 / В.Г. Айнштейн, М.К. Захаров, Г.А. Носов и др.- М.: Химия, 1999. – С. 213-232.
4. Богданов, О.С. Справочник по обогащению руд / О.С.Богданов, В.А. Олевский, И.К. Акиншин и др. – М. : Недра, 1972. – Т.1.- 447 с.
5. Томаш, А.А. Анализ влияния различных факторов на порозность зернистых материалов / А.А. Томаш, В.П.Тарасов, И.А. Ковалевский / Известия вузов. Чёрная металлургия, 1998.- № 9. - С. 8-12.

### **References**

1. Nadutyi, V. Analytical presentation of the separation of dense suspensions for the extraction of amber / Nadutyi, V., Korniyenko, V., Malanchuk, Z., Chelyshkina, O. // E3S Web of Conferences 109, 00059 (2019). Essays of Mining Science and Practice DOI: 10.1051/e3sconf/201910900059
2. Happel, J., Brenner, H. (1965). Low Reynolds number hydrodynamics, New York University, Prentice-Hall, 630
3. Planovsky, A.N., Ramm, V.M., Kagan, S.Z. (1967). Processes and devices of chemical technology, Moskva: Chemistry, 848
4. Bogdanov, O.S., Olevsky, V.A., et al. (1972). Spravochnik po Obogascheniju rud [Handbook for ore enrichment], Moskva: Nedra, V.1, 447
5. Tomasz, A.A. Analysis of the influence of various factors on the porosity of granular materials / A.A. Tomash, V.P. Tarasov, I.A. Kovalevsky / Izvestiya vuzov. Ferrous metallurgy, 1998.- № 9. - S. 8-12.