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THE STOCK OPTIMIZATION OF AUTOMATED WAREHOUSE

Automated warehouses play an important role in the functioning of production. One of the tasks of managing the warehouse system is to select the optimal size of the stock that is stored in the warehouse. In real production systems, certain delays are possible and common in the arrival of the next batch of goods of volume Q. Therefore, in the general case, the process of their receipt is random. One can compensate the deviations from the delivery schedule by introducing a safety stock of qstr, which allows organizing of the stable operation of the entire system. The sequence diagram of the receipt and consumption of stocks of goods for the system with safety stock is shown in fig. 1.

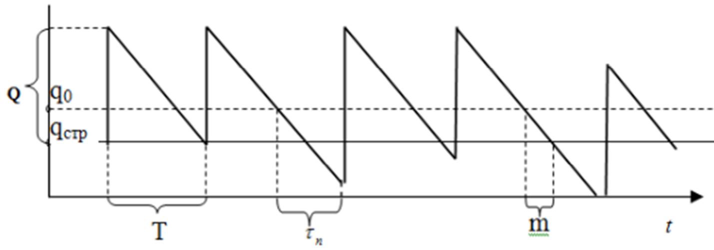

Figure 1 – Cyclorama of receipt and consumption of stocks of goods for the system with safety stock

The expenses attributable to a unit of a batch of goods when using safety stock are of the form

$$z = \frac{c}{Q} + \frac{Q}{2v}s + \frac{q_{str}s}{v}, \tag{1}$$

where c is overhead costs, s is the cost of storing one unit of a consignment of goods, v is the rate of stock consumption. Differentiating (1) on Q and equating the resulting expression to zero, we find the optimal value of Q, where the costs per unit of a batch of goods have a minimum value.

$$\frac{dz}{dQ} = --\frac{c}{Q^2} + \frac{s}{2v} \Rightarrow Q_{opt} = \sqrt{\frac{2cv}{s}} . \tag{2}$$

This is Wilson's famous formula. Let's calculate the value of the safety stock qstr. In the general case the arrival time of the next batch of goods τ is random. It lies in the interval (0, ∞) and is described by a distribution function with mathematical expectation m and standard deviation σ. These characteristics can be obtained by statistical processing of the array of arrival times τ of consignments of goods. If

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$\tau > \frac{q_0}{v}$, then there will be a downtime of the production system, which is maintained by the inventory management system. The probability of this event is $P\left\{\tau > \frac{q_0}{v}\right\} = \alpha$. Complementary event probability is $P\left\{\tau < \frac{q_0}{v}\right\} = 1 - \alpha$. But this is the distribution function F(t), so $F\left\{\frac{q_0}{v}\right\} = 1 - \alpha$. Usually, the average time m is chosen for the planned delivery time. In time m mv units of stock will be spent. Therefore, $q_0 = q_{str} + mv$. Substituting q0 into F(x), we obtain

$$F\left(\frac{q_{str}}{v} + m\right) = 1 - \alpha \quad (3)$$

Substituting expressions for specific distribution functions into (3), we can obtain specific equations in order to determine qstr. Let the arrival times be distributed according to the exponential law with the distribution function F, mathematical expectation m and standard deviation also equal to m. Substituting the function argument into (3), we obtain

$$1 - e^{-\frac{\frac{q_{str}}{v} + m}{m}} = 1 - \alpha \Rightarrow e^{-\left(\frac{q_{str}}{vm} + 1\right)} = \alpha .$$

Let us find logarithm of both parts of the last expression. We get $-\left(\frac{q_{str}}{vm} + 1\right) = \ln\alpha$, whence we obtain the final expression for qstr:

$$q_{str} = -mv(1 + \ln\alpha) . \quad (4)$$

Expression (4) has a minus sign on the right. But since the probability α is chosen small enough (about 0.05), then $\ln\alpha < -1$ and the general expression will also have a positive sign.**Conclusion**

The process of receipt and consumption of stocks of goods in a batch replenishment system in a warehouse is characterized by stochasticity. In order to compensate it, one should) use the safety stock. Author derived an expression, that determines the size of safety stock), which makes it possible to ensure the stable functioning of the warehouse system at a given minimum probability of downtime.

References

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