CHARACTERISTIC STRENGTH OF MINING PILLARS IN THE PRESENCE OF SIZE AND SHAPE EFFECTS

Youcef Cheikhaoui[1, 2], Olivier Deck[2], Kamel Omraci[1], Hamza Cheniti[1]*
[1] National School of Mining and Metallurgy Annaba, Annaba, Algeria
[2]University of Lorraine, CNRS, CREGU, Georessources Laboratory, Nancy, France
[*]Corresponding author: youcef.cheikhaoui@univ-lorraine.fr
youcef.cheikhaoui@ensmm-annaba.dz

Abstract. The aim of this study is to evaluate the characteristic strength of pillars in mining contexts, taking into account the effects of scale and shape. The characteristic strength is estimated in terms of the probability of exceeding a certain value of pressure, when the probability of activation of fault in the rock reaches a certain value, the rupture is magnified. An analytical formula has been developed linking the approach, which takes into account the effects of scale with the notion of probability of failure in the evaluation of the risk of failure. It also takes into consideration the effect of shape and volume in order to assess the condition of pillars in the mine without resorting to experiments at the pillar level (simple compression tests at the pillar level); this would save effort and money. In this paper, we will use a data set from underground mine (samples of zinc and lead) in Setif-Algeria.

Key words: Strength, Pillar, Size effect, Shape effect, Failure.

ХАРАКТЕРНА МІЦНІСТЬ ПОРОДНИХ ЦІЛИКІВ В ЗАЛЕЖНОСТІ ВІД РОЗМІРУ ТА ФОРМИ

Юсеф Шейхауі[1, 2], Олів'є Дек[2], Камель Омрачі[1], Хамза Ченіті[1]*
[1] Національна школа гірничої справи та металургії Аннаба, Аннаба, Алжир
[2]Університет Лотарингії, CNRS, CREGU, Лабораторія георесурсів, Нансі, Франція
[*]Відповідальна особа: youcef.cheikhaoui@univ-lorraine.fr
youcef.cheikhaoui@ensmm-annaba.dz

Анотація. Метою даного дослідження є оцінка характерної міцності стовпів у гірничих умовах з урахуванням їхніх розмірів та форми. Характерна міцність розуміється як вірогідність перевищення певного значення тиску, коли ймовірність активації розлому в породі досягає певного значення, розрив збільшується. Розроблена аналітична формула, яка пов'язує підхід, що враховує вплив масштабного ефекту з поняттям ймовірності руйнування при оцінці ризику руйнування. Також враховується вплив форми та об'єму для розробки оцінки стану стовпів у шахті. Даний підхід дозволить виключити необхідність проведення експериментів на рівні стовпів у шахті і проводити прості випробування на стиск у лабораторії, що є менш коштовним. Дані отримані на прикладі зразків шахт (цинку та свинцю) в Сетіф-Алжирі.

Ключові слова: Міцність, Стовп, Ефект розміру, Ефект форми, Руйнування.

1. Introduction

The influence of scale on evaluating the mechanical properties of a massif is critical for mining structures, the management of risks linked to the exploitation of resources, particularly in mines, quarries, shafts, tunnels, etc.

In this study, we particularly focused on the compressive strength of the pillars of underground mines (mines or quarries) which are exploited in fractured rocks that is vital to analyze the stability of fractured rocks and to evaluate their mechanical behavior.

The influence of the scale effect observed experimentally in solid mechanics can be attributed to the presence of defects in the material with increasing volume, and from this it can be concluded that when the probability of seeing an important defect increases, mechanical properties (strength and hardness) decrease with increasing size. Therefore, random laws and laws of probability govern the scale effect. This approach was studied by Weibull (1939), who used the concept of probability of failure to assess the risk of failure. Weibull introduced a failure criterion, which does not make it possible to deduce the strength of a pillar, which is practically related to the size; but also of the form (the ratio l / h is the width / height of the pillar). Hudson et al. (1972) reported that the difference in the length / diameter ratio of a sample has a significant effect on the compressive strength of rocks, as well as on the shape of the stress-strain curve in the post-peak segment. This is also evident in the work of Galvin et al. (1996) where he developed an analytical equation which takes into account the effect of the geometry of a pillar (by the w / h ratio) and the volume.

Our work aims to develop an approach to estimate the strength of pillars via grouped model equations:

(1) Galvin (which allows to consider the shape properties and the size of the rock mass);

(2) Weibull that makes it possible to integrate the concept of probability of default in the assessment of the risk of default.

In this work, the basic methodology of this composite approach has been identified, and has been validated by the estimation of the compressive strength of the pillars in the case of coal. We used the data from Australian coal case study by Galvin et al. (1996) to allow comparison of results that gave very similar results. Furthermore, this contribution is above all a confirmation of the validity of the basic model of the approach proposed in the study of other rocks, zinc and lead.

2. Methodology

The main proposition supposes that the strength R_p of a pillar of volume is explained by the strength k (of the intact rock), by corrective functions of shape, volume and probability of survival (Eq. 1).

$$\text{Suggestion: } R_p = K.\mathrm{H}(\mathrm{ps}).\mathrm{F}(\mathrm{f}).\mathrm{G}(\mathrm{v}) \tag{1}$$

F (shape): It depends on the geometry of the pillar

G (volume): It is a function of the pillar volume

H (probability of survival): It is the probability of survival associated with an applied constraint such as the probability of survival (Ps) = 1 - probability of failure (Pf).

K: it is the strength of specimen.

By analogy between the power formulas of resistance given by Weibull (1939) and Galvin *et al.* (1996), we can rewrite a new resistance formula by the combination of the two approaches (Back Analysis - Probabilistic Aspect of Failure) as follows:

$$\text{Galvin : } R_{p\,(Ps0)} = R_0\,.\,(\mathrm{w} / \mathrm{h})^b\,.V^a \tag{2}$$

Where:

R_0 : It is the resistance of a 1 m^3 specimen according to Galvin;

a and b: are parameters related to the material, they are determined by the Back-analysis method.

$$\text{Weibull : } R_{p\,(Ps)} = \sigma_0(-ln(P_s))^{1/m}.(V_0)^{1/m}.V^{-1/m} \tag{3}$$

Or:

m: is a material parameter characterizing the dispersion of the defects within the material;

σ_0 : is the solicitation (constraint) associated with a probability of survival of 37%. Weibull defined from its law (Eq.3). When the applied stress $\sigma = \sigma_0$, Ln (Ln (1/ps)) = 0 which implies ps = 0.37.

The comparison of Eq. 2 to 3 allows to suggest the expressions using the Weibull parameters for the coefficients a and b, we will explain the method below. These coefficients a and b are estimated empirically in Galvin's law through tests on pillars (Back-Analysis); this will limit the use of this law because we have to repeat tests on pillars every time whenever the material changes.

Furthermore, if there is a set of N values of compressive strength measured experimentally on test pieces of the same material, of volume V_0 and of slenderness W_{ep}/H_{ep}, it is then possible to obtain the parameters of the Weibull's law m and σ_0.

Equations 2 and 3 make it possible to give a physical meaning to the coefficients a and b which would be linked to the parameter m of Weibull's law. Such as: $a = -\frac{1}{m}$ and $b = \frac{\ln(V_0^{\frac{1}{m}})}{\ln(W_{ep}/H_{ep})}$

These two coefficients, which characterize the pillar strength weakening functions:

$$G\ (volume) = V^a = V^{-\frac{1}{m}} \tag{4}$$

$$F\ (shape) := (w/h)^b = \left(\frac{w}{h}\right)^{\frac{\ln(V_0^{\frac{1}{m}})}{\ln(W_{ep}/H_{ep})}} \tag{5}$$

In addition, from equations 2 and 3:

$$R_{0,(ps0)} = R_0 = \sigma_0(-\ln(Ps0))^{1/m} \tag{6}$$

As ps0 represents the probability of survival of samples with an average strength R_p of 1m3 proposed by Galvin. By extension, it is proposed that :

$$R_{0,(ps)} = \sigma_0(-\ln(Ps))^{1/m} \tag{7}$$

Suppose now that the Galvin parameters are known. Equation 6 is necessarily associated with a value of probability of survival P_{s0} that can reasonably be taken equal to 0.5 if R_p is the mean value observed by Galvin. The equalization of Eq. 1 and 2, taking into account Eq. 3, 4 and 5 then allow to obtain:

$$R_{p\ (Ps)} = \sigma_0 . \ln(1/P_s)^{1/m} . (w/h)^{\frac{\ln(V_0^{\frac{1}{m}})}{\ln(W_{ep}/H_{ep})}} . V^{-1/m} \tag{8}$$

That is to say, in general form: $R_{p,P_s} = \sigma_0 . H(P_s) . G(volume) . F(forme)$ (9)

3. Results and discussion

After having obtained the formula which takes into account both the scale and the shape of the shape effects (Eq. 9). a validation of this approach is necessary, the study is conducted by Galvin *et al.* 1996, Australian coal is used where the application of our model results in the values of the power law constants very close to those obtained by the back-analysis in the works of Galvin *et al.* 1996 (see the table 3).

We consider a set of 14 tests uni-axial compression to samples of zinc-lead (see the table 1) with V0 (m3) = 2.159×10-4 and the length to diameter ration l/d=0.5 allowing the different parameters of Weibull's law to be calculated. The results are summarized in Table 2.

Table1 : uni-axial compression strength (Mpa) of *zinc-lead* samples Setif-Algeria

sample number	uni-axial compression strength (Mpa)	sample number	uni-axial compression strength (Mpa)
1	47,4	8	43
2	82,6	9	90.5
3	159,9	10	102.3
4	92,5	11	61.1
5	40,4	12	112

6	108	13	155.5
7	80,9	14	93.2

Table 2: Weibull parameters of *zinc-lead* samples

Weibull Parameters	Value	Geometric parameters	Value
m	2.36	W_{Ep}/H_{Ep} (l/d ratio)	0.5
σ_0(Mpa)	103.61	$V_0(m^3)$	2.159×10^{-4}

The strength formula of a coal pillar found using Weibull parameters is written as follows:

$$R_{p\ (Ps)} = 103.61\ ln(1/P_s)^{0.42}(w/h)^{5.15}\ V^{-0.42} \quad (10)$$

Table 3: Power law coefficients defined by equation 2 from Salamon *et al*. (1996). The table is modified.

Author	R_0 [Mpa]	*a*	*b*	Comment	Rock type
Zern (1928)	-	0	0,5	-	-
Greenweld (1939)	19.3	-0,11	0,72	Large-scale in situ testing	'Coal'
Galvin *et al.* (1996)	7.2	-0.067	0.59	As reported by Salamon and Munro (1966)	'Coal'
Cheikhaoui *et al.* (2019)	-	-0.086	0.45	Laboratory tests. As reported by Galvin *and al. (1996)*	'Coal'
Holland et Goddy (1957)	-	-0,166	0,83	Laboratory tests	'Coal'
Cheikhaoui *et al.* (2019)	-	-0.42	5.15	Laboratory tests	*'zinc-lead'*

In this table the values of Galvin *et al*. (1996), are almost identical with those of Cheikhaoui *et al.* (2019) in the case of Australian coal because we were taken the data base of uni-axial compression strength used by Galvin to obtained the coefficients of power law a and b using the weibull parameters . However, in all cases the coefficient a takes negative value. On the other hand, for b coefficient we have a positive value. We also note that if a = 0 then the strength no longer depends on the volume in these formulas.

The curve in (Fig. 1) shows a decrease in strength with an increase in volume. This phenomenon represents the scale effect, a result that was confirmed in the works of (Zhang *et al*., 2011 and others).

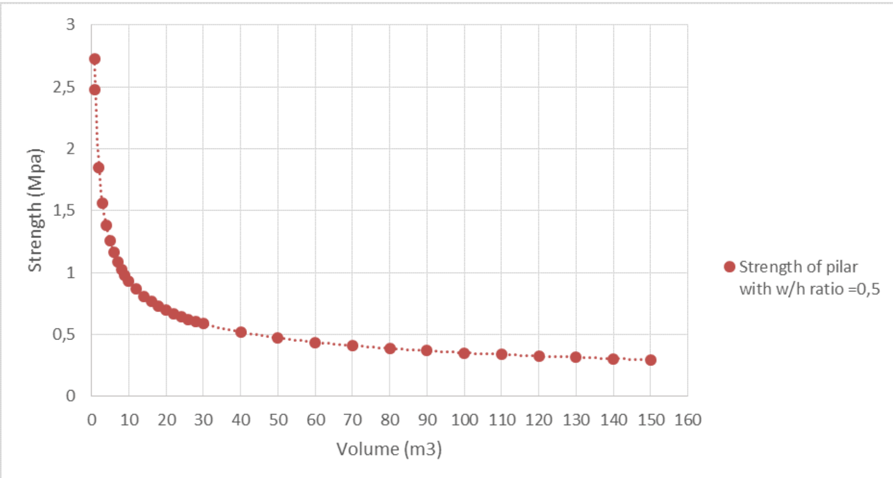

Fig. 1. The effect of volume on the strength of a zinc-lead pillar with (w / h = 0.5) according to

probability of survival Ps = 50 %. The formula developed using Weibull parameters

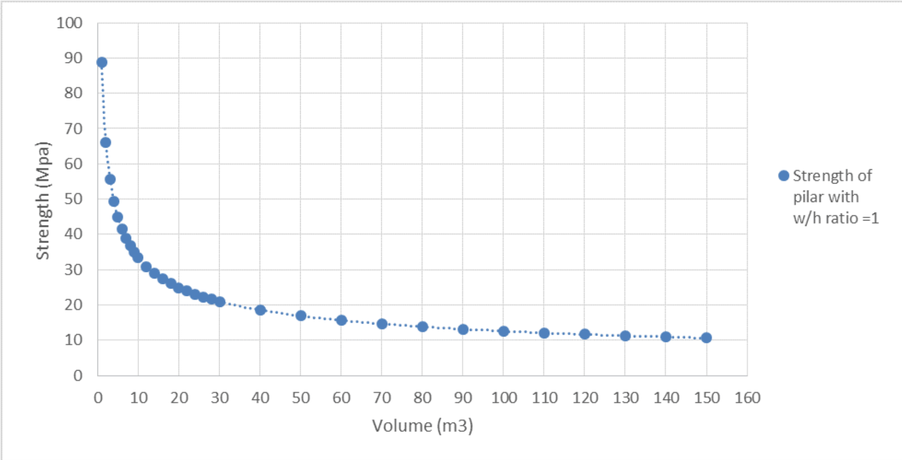

Fig. 2. The effect of volume on the strength of a zinc-lead pillar with (w / h = 1) according to probability of survival Ps = 50 %. The formula developed using Weibull parameters

Looking at curves in (Fig. 1, 2), the square pillar with (w / h ratio = 1) resists better than a less slender pillar (w / h ratio = 0.5).

(Fig. 3) shows that there is a shape effect where the strength increases with the increase in the w /h ratio of the pillar, which is consistent with the results of (Hudson *et al.* 1972, Martin and Maybee, 2000)

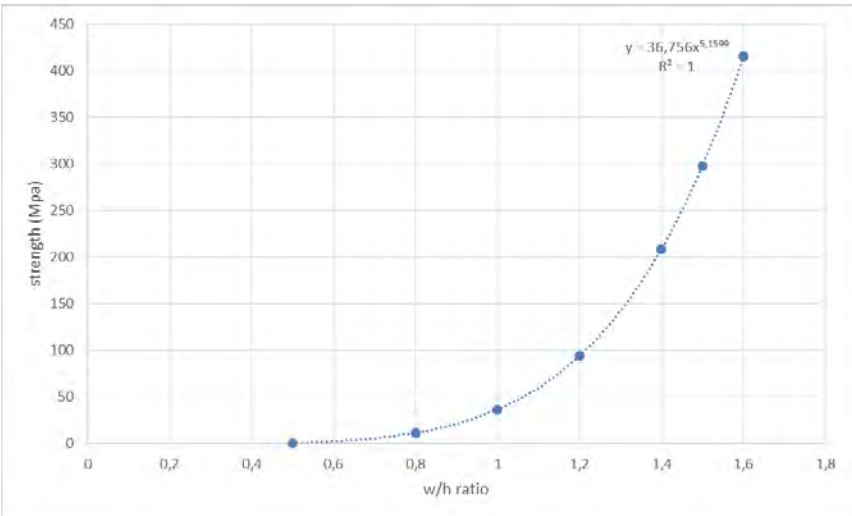

Fig. 3. The shape effect on the strength of a zinc-lead pillar of a volume (10 m3) according to probability of survival Ps = 50 %. The formula developed using Weibull parameters

4. Conclusion

The formula developed by the combination of two formulas (Weibull and Galvin) explicitly reproduces the effect of volume and shape. It allows us to give an interpretation of the influence of each of the two on the resistance of a pillar, based on Weibull parameters, derived from an approach of similarity with the formula of Galvin *et al.* (1996).

An analysis of this formula indicates that the pillar strength decreases with increasing volume and it increases with increasing width (a chunky pillar is more resistant than a slender pillar). However, the influence of discontinuity in this basic formula is to be considered in our next contributions.

One of the advantages of the probabilistic resistance measurement is its functional relation with the deformation at the level of the pillars and the progress of the works of the mining sites. That they have the greatest impact on the overall strength of the mine. That is to say, it is necessary to choose an optimal critical size of the pillars to ensure good operation and good safety.

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