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MODELED HEAVY-TAIL PROCESS PREDICTION BASED ON THE CHEBYSHEV POLYNOMIALS OF THE SECOND KIND

Nowadays telecommunication traffic in systems with data packet transfer is considered to be a heavy-tail process. The traffic prediction is an urgent problem for telecommunications, see, for example, [1]. There are many different approaches to traffic prediction. Some recent papers of the author were devoted to the investigation of the Kolmogorov–Wiener filter approach to the investigation if stationary heavy-tail data prediction, see, for example, [2].

In [2] the smoothed modeled data based on the fractional Gaussian noise were generated on the basis of the symmetric moving average approach [3]. In [2] it is also shown that both discrete and continuous Kolmogorov–Wiener filter may be applied to the prediction of the smoothed heavy-tail data. In [2] the Walsh function orthogonal system was used as the basis of the Galerkin method [4], with the help of which the Wiener–Hopf integral equation [5] was solved. In [6] the following question was discussed: may another orthogonal system be used in the framework of the Galerkin method instead of the Walsh function one? In [6] the Chebyshev polynomials of the first kind were investigated and the conclusion was made that the Walsh functions give better results than the Chebyshev polynomial of the first kind.

In this paper the corresponding investigation for the Chebyshev polynomials of the second kind is made. The Galerkin method and the process correlation function are described in detail in [2, 6]. The process with the Hurst exponent *H=0.8* is investigated.

The Wiener–Hopf integral equation

$$\int_0^T h(\tau) R(t-\tau)\,d\tau = R(t+z) \tag{1}$$

is solved with the help of the Galerkin method based on the Chebyshev polynomials of the second kind:

$$h(t) = \sum_{s=0}^{n-1} g_s S_s(t),\quad S_s(t) = U_s\left(\frac{2t}{T}-1\right),\quad U_s(x) = \sum_{k=0}^{[s/2]} C_{s+1}^{2k+1} x^{s-2k}\left(x^2-1\right)^k, \tag{2}$$

$U_s(x)$ are the Chebyshev polynomials of the second kind, the functions $S_s(t)$ are orthogonal on $t \in (0,T)$:

$$\int_0^T S_n(y) S_m(y) \sqrt{1-\left(\frac{2y}{T}-1\right)^2}\,dy = \frac{T\pi}{4}\delta_{mn}. \tag{3}$$

The coefficients g_s are the solutions of the following linear equation set:

$$\sum_{j=0}^{n-1} G_{ij} g_j = B_i,\quad i=\overline{0,n-1},\quad G_{ij} = \int_0^T\int_0^T S_i(\tau) S_j(t) R(t-\tau)\,dt\,d\tau,\quad B_i = \int_0^T S_i(t) R(t+z)\,dt, \tag{4}$$

the calculation of the integrals in (4) is similar to that given in [6].

The process correlation function is given in [6], the following parameters are used: $T=1$, $z=10^{-3}$. The MAPE (mean absolute percentage error) of the corresponding prediction of the process generated in [2] is described in details in [2].

The MAPE results for the Chebyshev polynomials of the second kind are as follows, see

Table 1 and Fig.1.

Table 1. **Average MAPE for different numbers of polynomials**

n	MAPE, %	*n*	MAPE, %	*n*	MAPE, %
1	23.2	9	14.4	17	9.19
2	21.9	10	13.3	18	8.83
3	20.7	11	12.5	19	8.51
4	19.5	12	11.9	20	8.22
5	18.5	13	11.2	21	7.98
6	17.4	14	10.7	22	7.82
7	16.4	15	10.1	23	7.73
8	15.4	16	9.65	24	7.71

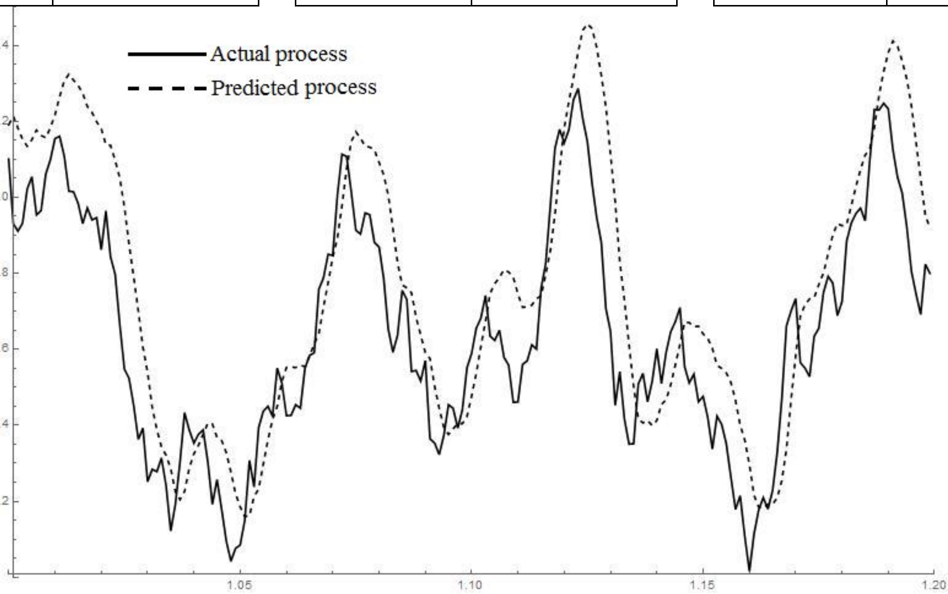

Figure 1. Comparison of the actual and predicted processes

The obtained results are in fact the same as the results for the Chebyshev polynomials of the first kind. So, both polynomial systems give identical results. The Walsh functions give better prediction results than the Chebyshev polynomials for the problem under consideration.

References

1. C. Katris and S. Daskalaki, “Comparing forecasting approaches for Internet traffic”, Expert Systems with Applications, Vol. 42, Issue 21, pp. 8172–8183 (2015). doi: 10.1016/j.eswa.2015.06.029

2. V. Gorev, A. Gusev, V. Korniienko and Y. Shedlovska, “On the use of the Kolmogorov–Wiener filter for heavy-tail process prediction”, Journal of Cyber Security and Mobility, 12(3), pp. 315-338, 2023. doi: 10.13052/jcsm2245-1439.123.4

3. D. Koutsoyiannis, “The Hurst phenomenon and fractional Gaussian noise made easy”, Hydrological Sciences Journal, 47 (2002), 573-595. doi: 10.1080/02626660209492961.

4. A. D. Polyanin and A. V. Manzhirov. “Handbook of integral equations. Second edition”. Boca Raton, Chapman & Hall/CRC Press. Taylor & Francis Group (2008).

5. S. L. Miller and D. Childers. “Probability and Random Processes With Applications to Signal Processing and Communications, 2nd edition”. Elsevier, Academic Press, Amsterdam, 2012, doi: 10.1016/C2010-0-67611-5.

6. V. Gorev, A. Gusev, V. Korniienko and Y. Shedlovska, “Polynomial solutions for the Kolmogorov–Wiener prediction of modeled smoother heavy-tail process”, Computer Science, Software Engineering and Cyber Security, under review.