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МОДЕЛИРОВАНИЕ ЗАГРЯЗНЕНИЯ ПОДЗЕМНЫХ ВОД ВОЗЛЕ ПРУДОВ – НАКОПИТЕЛЕЙ ШАХТНЫХ ВОД

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SIMULATION OF GROUND WATERS POLLUTION NEAR POOLS WITH MINE WASTEWATERS

Разработана численная модель для быстрого расчета процесса загрязнения подземных вод возле прудов – накопителей шахтных вод. Построенная модель, основана на уравнениях фильтрации и геомиграции. Для численного решения моделирующих уравнений используются разностные схемы. Разработан пакет программ на алгоритмическом языке для решения задач геомиграции. Разработанная модель может быть использована для оценки антропогенной нагрузки на подземные воды в местах размещения прудов-отстойников.

Розроблено чисельну модель для швидкого розрахунку процесу забруднення підземних вод біля ставків-накопичувачів шахтних вод. Побудована модель, заснована на рівняннях фільтрації і геоміграції. Для чисельного розв'язання моделюючих рівнянь використовуються різницеві схеми. Розроблено пакет програм на алгоритмічній мові для вирішення задач геоміграції. Розроблена модель може бути використана для оцінки антропогенного навантаження на підземні води в місцях розміщення ставків-відстійників.

Introduction. Ground waters pollution takes place near pools with mine wastewaters. This pollution is caused by wastewaters filtration through the bottom of the pool. This results in ground waters rise and their pollution. It is important to develop quick computing methods to predict the dynamics of this pollution. These theoretical methods can be used to support the engineer decisions which design.

Literature review. As a rule to solve the problem of ground waters contamination analytical or empirical models are used [3, 5]. But these models are restricted enough. Numerical models represent more effective tool and help in predicting the process of ground waters contamination [1, 2, 4]. For a quick evaluation of spatial and temporal contamination of ground waters it is important to develop not time consuming numerical models.

Purpose. The purpose of this work is development of 2-D numerical model to simulate ground waters pollution near pools with mine wastewaters.

Mathematical model. To simulate the ground waters pollution near pools with mine wastewaters we use equation of filtration [1, 4, 5]

$$\mu \frac{\partial h}{\partial \mathrm{t}} = kh_{cp}\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) + W\delta(x - x_i)\delta(y - y_i),$$

where
h – is depth of ground waters flow;
k – is coefficient of filtration;
μ – is lack of saturation;
W – is intensity of waste waters infiltration from pool;
h_{cp} – is averaged depth;
$\delta(x - x_i(t))\delta(y - y_i(t))$ – is Dirac delta function.

Process of ground waters pollution near a pool is simulated using 2-D transport equation [1-5]:

$$n\frac{\partial C}{\partial t} + \frac{\partial uC}{\partial x} + \frac{\partial vC}{\partial y} + \sigma C = \frac{\partial}{\partial x}\left(\mu_x \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu_y \frac{\partial C}{\partial y}\right) + \\ + \sum q_i(t)\delta(x - x_i(t))\delta(y - y_i(t)), \qquad (1)$$

where
C – is concentration of pollutant in ground waters;
u, v – are the velocity components in x and y directions;
σ – is the reaction rate constant its also takes into account sorption;
μ_x, μ_y – are dispersion coefficients;
$q_i(t)$ – is source term;
$\delta(x - x_i(t))\delta(y - y_i(t))$ – is Dirac delta function;
x_i, y_i – are the coordinates of the point source of pollution.

This equation is numerically integrated using the following boundary conditions:

– at the entrance boundaries we use the boundary condition:

$$C|_{boundary} = C_{etr},$$

where C_{etr} is known concentration (for example $C_{etr} = 0$);

– at the exit boundaries so called “mild boundary condition” is used. For example, in the numerical model it can be written as following:

$$C(i+1, j) = C(i, j),$$

where $(i+1, j)$ is the last computational cell and (i, j) is the previous computational cell.

The initial condition (at time $t = 0$) can be written as $C = 0$ in the whole computational region or $C = C_0$, where C_0 is the known concentration in ground waters under the spillage took place. In this case $C = 0$ in the other part of the computational region.

Numerical model. To solve equation of filtration we use the implicit scheme of conditional approximation. The difference equations in this case are as following

$$\frac{h_{i,j}^{n+\frac{1}{2}} - h_{i,j}^{n}}{\Delta t} = a\left[\frac{-h_{i,j}^{n+\frac{1}{2}} + h_{i-1,j}^{n+\frac{1}{2}}}{\Delta x^2}\right] + a\left[\frac{-h_{i,j}^{n+\frac{1}{2}} + h_{i,j-1}^{n+\frac{1}{2}}}{\Delta y^2}\right],$$

$$\frac{h_{i,j}^{n+1} - h_{i,j}^{n+\frac{1}{2}}}{\Delta t} + a\left[\frac{h_{i+1,j}^{n+1} - h_{i,j}^{n+1}}{\Delta x^2}\right] + a\left[\frac{h_{i,j+1}^{n+1} - h_{i,j}^{n+1}}{\Delta y^2}\right].$$

where $a = \dfrac{h_{cp} \cdot k}{\mu}$.

To solve transport equation (1) the implicit change – triangle scheme is used. The main features of this scheme are shown below. To build the scheme we perform the splitting of the transport equation at the differential level.

$$\frac{\partial C}{\partial t} + \frac{\partial u C}{\partial x} + \frac{\partial v C}{\partial x} + \sigma c = 0,$$

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(\mu \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial C}{\partial y}\right),$$

$$\frac{\partial C}{\partial t} = \sum q_i(t)\delta(r - r_i).$$

where $r_i = (x_i, y_i)$.

After that the approximation of time dependent derivative is used:

$$\frac{\partial C}{\partial t} \approx \frac{C_{ijk}^{n+1} - C_{ijk}^{n}}{\Delta t}.$$

At the next step the convective derivatives are represented as follows:

$$\frac{\partial uC}{\partial x} = \frac{\partial u^{+}C}{\partial x} + \frac{\partial u^{-}C}{\partial x},$$

$$\frac{\partial vC}{\partial y} = \frac{\partial v^{+}C}{\partial y} + \frac{\partial v^{-}C}{\partial y},$$

where $u^{+}=\frac{u+|u|}{2}$, $u^{-}=\frac{u-|u|}{2}$, $v^{+}=\frac{v+|v|}{2}$, $v^{-}=\frac{v-|v|}{2}$.

At the second step the convective derivatives are approximated as following:

$$\frac{\partial u^{+}C}{\partial x}\approx\frac{u^{+}_{i+1,j}C^{n+1}_{i,j}-u^{+}_{i,j}C^{n+1}_{i-1,j}}{\Delta x}=L^{+}_{x}C^{n+1},$$

$$\frac{\partial u^{-}C}{\partial x}\approx\frac{u^{-}_{i+1,j}C^{n+1}_{i+1,j}-u^{-}_{i,j}C^{n+1}_{i,j}}{\Delta x}=L^{-}_{x}C^{n+1},$$

$$\frac{\partial v^{+}C}{\partial y}\approx\frac{v^{+}_{i,j+1}C_{i,j}-v^{+}_{i,j}C_{i,j-1}}{\Delta y}=L^{+}_{y}C^{n+1},$$

$$\frac{\partial v^{-}C}{\partial y}\approx\frac{v^{-}_{i,j+1}C_{i,j+1}-v^{-}_{i,j}C_{i,j}}{\Delta y}=L^{-}_{y}C^{n+1}.$$

The second order derivatives are approximated as following:

$$\frac{\partial}{\partial x}\left(\mu_x\frac{\partial C}{\partial x}\right)\approx\mu_x\frac{C^{n+1}_{i+1,j}-C^{n+1}_{i,j}}{\Delta x^2}-\mu_x\frac{C^{n+1}_{i,j}-C^{n+1}_{i-1,j}}{\Delta x^2}=$$
$$=M^{-}_{xx}C^{n+1}+M^{+}_{xx}C^{n+1},$$

$$\frac{\partial}{\partial y}\left(\mu_y\frac{\partial C}{\partial y}\right)\approx\mu_y\frac{C^{n+1}_{i,j+1}-C^{n+1}_{i,j}}{\Delta y^2}-\mu_y\frac{C^{n+1}_{i,j}-C^{n+1}_{i,j-1}}{\Delta y^2}=$$
$$=M^{-}_{yy}C^{n+1}+M^{+}_{yy}C^{n+1}.$$

In these expressions L^{+}_{x},L^{-}_{x}, L^{+}_{y},L^{-}_{y}, M^{+}_{xx},M^{-}_{xx} are the difference operators.

At the next step we write the finite difference scheme of splitting:

– at the first step k=1/4:

$$\frac{C^{n+k}_{ij}-C^{n}_{ij}}{\Delta t}+\frac{1}{2}\left(L^{+}_{x}C^{k}+L^{+}_{y}C^{k}\right)+\frac{\sigma}{2}C^{n}_{ij}=0;$$

– at the second step k=1/2, c=n+1/4:

$$\frac{C^{k}_{ij}-C^{c}_{ij}}{\Delta t}+\frac{1}{2}\left(L^{-}_{x}C^{k}+L^{-}_{y}C^{k}\right)+\frac{\sigma}{2}C^{k}_{ij}=0;$$

– at the third step k=3/4, c=n+1/2:

$$\frac{C_{ij}^{k} - C_{ij}^{c}}{\Delta t} = \frac{1}{2}\left(M_{xx}^{-} C^{c} + M_{xx}^{+} C^{k} + M_{yy}^{-} C^{c} + M_{yy}^{+} C^{k} \right);$$

– at the forth step k=1, c=n+3/4:

$$\frac{C_{ij}^{k} - C_{ij}^{c}}{\Delta t} = \frac{1}{2}\left(M_{xx}^{-} C^{k} + M_{xx}^{+} C^{c} + M_{yy}^{-} C^{k} + M_{yy}^{+} C^{c} \right);$$

– at the fifth step:

$$\frac{\overset{5}{C}{}_{i,j}^{n+1} - \overset{5}{C}{}_{i,j}^{n}}{\Delta t} = \sum_{l=1}^{N} \frac{q_l(t^{n+1/2})}{\Delta x \Delta y} \delta_l .$$

Function δ_l is equal to zero in all cells accept the cells where the source of emission is situated.

This difference scheme is implicit and absolutely steady but the unknown concentration C is calculated using the explicit formulae at each step (so called “method of running calculation”), where $\overset{1}{C}, \overset{k}{C}, \overset{5}{C}$ – are concentrations at each time step.

FORTRAN language was used to code the developed numerical model.

Results. The developed numerical model was used to solve the following problem. We consider the ground waters pollution near the pool with mine waste waters which is situated near Kryvyi Rih City. Parameters of filtration and admixture mass transfer are known. The goal of computing is to obtain the dynamics of ground watres pollution.

Results of numerical simulation which were obtained are shown in Figures 1-4.

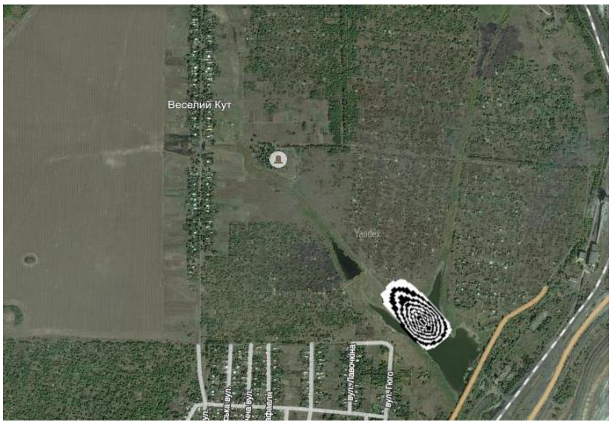

Fig. 1. Contamination zone in ground waters, t=2.1 (time is dimensionless)

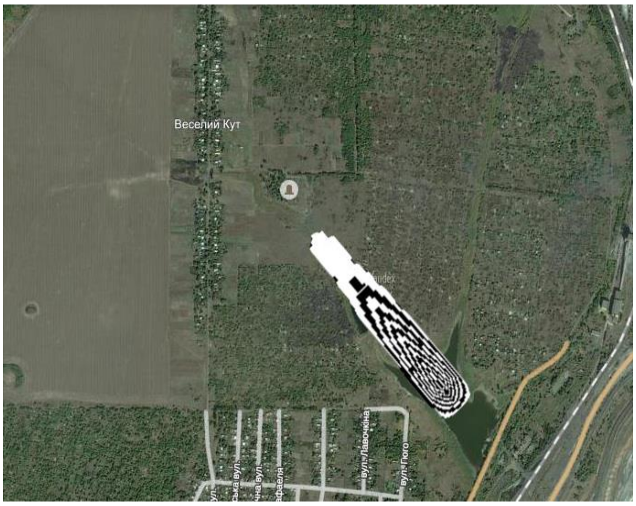

Fig. 2. Contamination zone in ground waters, t=5.6 (time is dimensionless)

Fig. 3. Contamination zone in ground waters, t=8.7 (time is dimensionless)

Fig. 4. Contamination zone in ground waters, t=13.5 (time is dimensionless)

These Figures illustrate the contamination area at different times. As we can see from these Figures the contaminated area is enlarging during time. The contaminated zone has the "plume" form. It takes 2sec to obtain numerical result.

Conclusions. A numerical model to compute ground waters pollution near pools with mine waste waters was developed. The model is based on the 2-D mass transport equation and equation of filtration. The future work in this field will be connected with development of 3D filtration model to simulate ground waters pollution near pools.

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ABSTRACT

The results of the study, dedicated to the process of ground waters pollution near pools.

The purpose of the study is development of numerical model to compute quickly ground waters pollution near pools with mine waste waters.

The method of the research is CFD simulation.

Findings. New numerical model is proposed to compute the process ground waters pollution near pools.

The originality. New model was developed for 2D computing of ground waters pollution.

Practical implications. Developed model allows quick computing of ground waters pollution near pools with waste waters.

Keywords: pools, mine waste waters, numerical modeling