MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
DNIPRO UNIVERSITY OF TECHNOLOGY

M.M. TRYPUTEN, V.V. KUZNETSOV, V.V. NADTOCHYI

THEORY OF THE AUTOMATED CONTROL
(LINERAL AND SPECIFIC SYSTEMS)

Methodical recommendations for laboratory works for students for specialty 151
«Automation and Computer-Integrated Technologies»

Dnipro
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ELECTRICAL ENGINEERING FACULTY
Department of Cyberphysical and Information and Measuring Systems

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Затверджено до видання навчально-методичним відділом (протокол № 3 від 02.03.2021) за поданням методичної комісії зі спеціальності 151 «Автоматизація та комп'ютерно-інтегровані технології» (протокол № 8 від 24.11.2020).

Methodological recommendations have been submitted as for the laboratory research in the theory of the automated control for students for specialty 151 "Automation and Computer-Integrated Technologies".

Методичні рекомендації до виконання лабораторних робіт спрямовано на поглиблення знань студентів, які навчаються англійською мовою за спеціальністю 151 «Автоматизація та комп'ютерно-інтегровані технології» при вивченні розділів дисципліни «Теорія автоматичного керування». Лабораторні роботи являють собою закінчені дослідження із спеціальних питань, при виконанні яких студенти мають застосовувати засвоєні в теоретичному циклі знання. Результатами виконаних досліджень є підтвердження або спростування теоретичних тверджень щодо процесів, які відбуваються в системах автоматичного керування.

Лабораторні роботи можуть бути використані в навчальному процесі Національній металургійній академії України студентами спеціальності 141 «Електроенергетика, електротехніка і електромеханіка».

Відповідальний за випуск завідувач кафедри кіберфізичних та інформаційно-вимірювальних систем, д-р. техн. наук, проф. В.В. Ткачов

LABORATORY RESEARCH 1

Studying of typical dynamic elements

1.1 **Objective** is to deepen students' knowledge while studying the Chapter "Characteristics of dynamic elements of the automated control systems".

In the process of the activities, students should be able to:

– calculate transition process as well as logarithmic amplitude and phase-frequency responses;

– acquire practical skills to study the automated systems using a computer;

– use a computer to identify the basic characteristics of the typical dynamic elements (i.e. net delay; oscillating; integrating; and inertial differential); and

– identify effect of parameters of transfer functions of the units on their characteristics.

1.2 Intput data to carry out the research are as follows:

– structural patterns and numerical parameters of the studied dynamic elements (Fig.1.1, Table 1.1);

– application packages MATLAB and MathCAD to simulate the automated control systems (ACSs) and perform computer-based mathematical calculations.

1.3 Operating procedures

Following order is recommended:

– apply the MathCAD application package to calculate transient processes, amplitude-phase, logarithmic-amplitude frequency and phase frequency responses of dynamic elements;

– half gradually and then double gradually net delay time, and the elements intensification coefficient with delay; use the MATLAB application package to evaluate effect of the parameters on the transient process;

– half gradually and then double gradually intensification coefficient as well as a time constant of inertial differential element; use the MATLAB application package to evaluate effect of the parameters on the transient process;

– half gradually and then double gradually intensification coefficient of integrating element; use the MATLAB application package to evaluate effect of the parameters on the transient process;

– half gradually and then double gradually intensification coefficient as well as a time constant of oscillating; use the MATLAB application package to evaluate the effect of the parameters on the transient process; and

– identify successively damping coefficient of the oscillating element $d < 0.707$; $0.707 < d < 1$ and $d > 1$; use the MATLAB application package to evaluate effect of the parameters on the transient process.

Table 1.1

Input data

	Dynamic element	K_1	K_2	K_3	K_4	τ, c	T_1, c	T_2, c	d
1	oscillating of net delay	0.5	1.0	3.2	4.0	0.5	1.0	1.5	0.1
2	integrating differential	1.7	3.7	4.3	0.8	1.5	2.0	2.5	0.2
3	oscillating of net delay	5.0	2.2	0.9	3.1	2.5	3.0	3.5	0.3
4	differential integrating	3.4	4.4	0.8	2.8	4.5	5.0	1.5	0.4
5	oscillating of net delay	1.5	4.0	2.7	0.9	1.0	2.5	2.0	0.5
6	integrating differential	0.8	3.1	1.7	0.6	3.0	1.5	4.0	0.6
7	oscillating of net delay	2.7	5.0	3.6	4.4	5.0	0.5	1.0	0.7
8	differential integrating	4.3	2.4	4.0	3.7	2.0	3.5	3.0	0.8
9	oscillating of net delay	1.9	1.9	0.5	5.0	0.5	2.0	4.5	0.9
10	integrating differential	0.6	0.9	5.0	4.5	1.5	1.5	2.5	1.0
11	oscillating of net delay	3.5	1.3	2.5	0.9	2.5	2.5	1.0	1.1
12	differential integrating	1.5	3.7	3.9	2.5	3.5	3.5	3.0	1.2
13	oscillating of net delay	2.0	3.2	0.8	0.7	4.5	4.5	0.5	1.3
14	integrating differential	3.0	0.5	1.9	5.0	1.0	2.0	4.0	1.4
15	oscillating of net delay	1.0	4.3	2.0	3.5	2.0	3.5	4.5	1.5
16	differential integrating	4.2	2.5	3.5	0.8	3.0	1.5	0.5	0.3
17	oscillating of net delay	0.6	4.2	3.7	2.5	0.5	4.0	2.5	0.8
18	integrating differential	3.5	0.5	2.5	0.7	1.0	2.0	4.0	1.0
19	oscillating of net delay	2.1	2.6	1.4	3.8	0.3	4.2	1.7	0.6
20	differential integrating	4.1	0.8	2.5	1.6	3.9	2.4	1.9	0.1

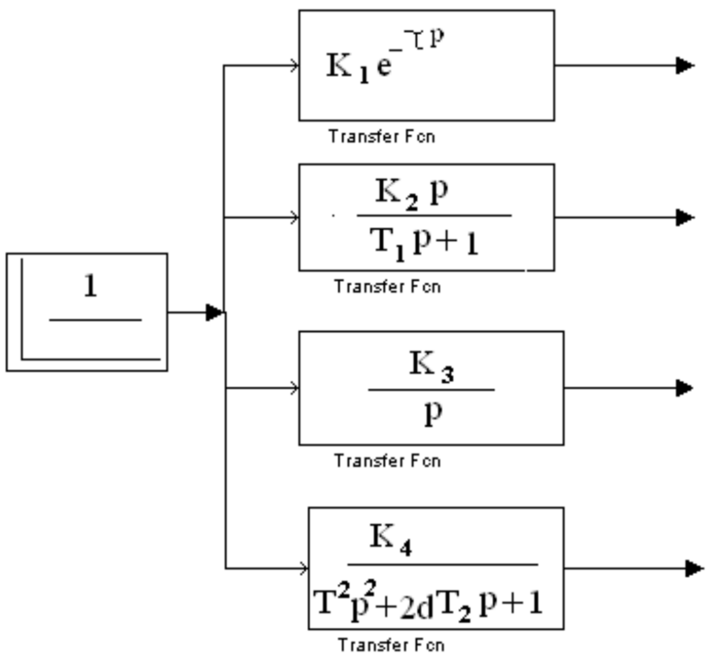

Fig. 1.1 Schematic structure for the studied dynamic elements

1.4 Methodological explanations

Any element of the automated control, being considered from the viewpoint of its dynamic characteristics, is a dynamic element. The dynamic elements, described by differential first- and second-order mathematical expressions, integral mathematical first-order expression or mathematical expression with a delay argument, are considered as typical ones.

Theory of the automated control records differential ACSs equations as well as their elements in operator form:

$$(a_n p^n + a_{n-1} p^{n-1} + \dots + a_1 p + 1)\bar{y} =$$

$$= (b_m p^m + b_{m-1} p^{m-1} + \dots + b_1 p + 1)\bar{x}, \tag{1.1}$$

where $\overline{x}, \overline{y}$ are the representation of input value and output value according to Laplace; a_i, b_j are the coefficients; n, m is the polynomial order; and p is the complex variable.

Physical implementation of the automated control system should involve the fulfilment of $n \geq m$ condition.

In terms of zero initial conditions, a ratio between Laplace representation of output value and Laplace representation of input value is a transfer characteristic $W(p)$:

$$W(p) = \frac{\bar{y}}{\bar{x}} = \frac{b_m p^m + b_{m-1} p^{m-1} + \dots + b_1 p + 1}{a_n p^n + a_{n-1} p^{n-1} + \dots + a_1 p + 1}. \tag{1.2}$$

Like differential equation (1.2) expression is an equation of dynamic balance between output and input values if the input value varies in terms of any law.

Also, the transfer characteristic is represented as follows:

$$W(p) = \frac{\bar{y}}{\bar{x}} = \frac{KN(p)}{L(p)}, \tag{1.3}$$

where K is the total intensification coefficient; $N(p), L(p)$ are the polynoms of numerator and denominator of the transfer characteristic.

Transition function $y(t)$ is a dynamic element response to a single step signal. A transition function is inverse Laplace transformation of output value in terms of zero initial conditions. It is

$$y(t) = L^{-1}(\bar{y}), \tag{1.4}$$

where L^{-1} is the operator of inverse Laplace transformation.

A single step signal is determined with the help of the mathematical expression:

$$x(t) = \begin{cases} 1 \text{ if} t \geq 0 \\ 0 \text{ if} t < 0. \end{cases} \quad (1.5)$$

Table 1.2 demonstrates the most commonly used Laplace functions.

Table 1.2

Representation of certain functions according to Laplace

	$\bar{y} = \int_0^\infty e^{-pt} y(t)dt$	$y(t)$
1	$\frac{1}{p}$	1
2	$\frac{\alpha}{p^2 + \alpha^2}$	$\sin(\alpha t)$
3	$\frac{p}{p^2 + \alpha^2}$	$\cos(\alpha t)$
4	$\frac{1}{p + \alpha}$	$e^{-\alpha t}$
5	$\frac{\alpha}{p^2 - \alpha^2}$	$sh(\alpha t)$
6	$\frac{p}{p^2 - \alpha^2}$	$ch(\alpha t)$
7	$\frac{\alpha}{(p + \alpha)^2 + \alpha^2}$	$e^{-\alpha t} \sin(\alpha t)$
8	$\frac{p + \alpha}{(p + \alpha)^2 + \alpha^2}$	$e^{-\alpha t} \cos(\alpha t)$
9	$\frac{n!}{p^{n+1}}$	t^n
10	$\frac{2p\alpha}{(p^2 + \alpha^2)^2}$	$t\sin(\alpha t)$
11	$\frac{p^2 - \alpha^2}{(p^2 + \alpha^2)^2}$	$t\cos(\alpha t))$
12	$\frac{1}{(p + \alpha)^2}$	$te^{-\alpha t}$
13	$\frac{1}{(p^2 + \alpha^2)^2}$	$\frac{\sin(\alpha t) - \alpha t\cos(\alpha t)}{2\alpha^3}$
14	$(-1)^n \frac{d^n}{dp^n} \bar{y}$	$t^n y(t)$

If a sine wave signal with ω frequency and amplitude, being equal to a unit $x(t) = \sin\omega t = e^{j\omega t}$, is applied to the input then output sine wave with similar ω frequency but other A(ω amplitude and φ(ω) phase will be available after the transient process terminates:

$$y(t) = A(\omega)\sin\big(\omega t + \varphi(t)\big) = A(\omega)e^{j(\omega t + \varphi(\omega))}. \quad (1.6)$$

According to (1.6), both input and output sine waves are described by means of the complex expressions. Ratio of the expressions is a complex intensification coefficient $W(j\omega)$ (or a frequency transfer function):

$$W(j\omega) = \frac{y(j\omega)}{x(j\omega)} = \frac{b_m(j\omega)^m + b_{m-1}(j\omega)^{m-1} + \cdots + b_1(j\omega) + 1}{a_n(j\omega)^n + a_{n-1}(j\omega)^{n-1} + \cdots + a_1(j\omega) + 1}\cdot \quad (1.7)$$

Formulas and graphs, characterizing response of element on a sine wave input signal, are called characteristics. Graphic representation of the complex intensification coefficient within the complex plane in terms of $P(\omega)$ and $jQ(\omega)$ coordinates is the amplitude and phase frequency response (APFR).

Graphic representation of changes in A(ω) amplitude or φ(ω) phase of output signal depending on ω frequency as an element response to an input sine wave with ω frequency and amplitude, being equal to a unit, are called amplitude and phase response (APR) or phase and frequency response (PFR). In practice, APRs are represented usually in terms of $L_m(\omega) = 20lgA(\omega); lg\omega$ coordinates; as for the PFRs, $\varphi(\omega); lg\omega$ coordinates are the most popular ones. The responses, constructed in such a way, are logarithmic amplitude and frequency response (LAFR) and logarithmic phase and frequency response (LPFR) respectively.

While analyzing and synthesizing, ACSs also apply dependence graphs of real P(ω) part and imaginary Q(ω) part of the complex intensification coefficient of ω frequency called real frequency response (RFR) and imaginary frequency response (IFR) respectively.

Example 1.1. Calculate transient process, amplitude and phase frequency response, logarithmic amplitude frequency response, and logarithmic phase frequency response of the first-order aperiodic element. Intensification coefficient and time constant of the aperiodic element are $K = 5$ and $T = 2\ s$ respectively.

Transfer function of the first-order aperiodic element is as follows:

$$W(p) = \frac{\bar{y}}{\bar{x}} = \frac{K}{Tp + 1}\cdot \quad (1.8)$$

Apply intensification coefficient value and time constant value to (1.8):

$$W(p) = \frac{\bar{y}}{\bar{x}} = \frac{5}{2p + 1}\cdot \quad (1.9)$$

Using (1.9) equation, determine Laplace representation of output value:

$$\bar{y} = \frac{5}{2p + 1}\bar{x}\cdot \quad (1.10)$$

Representation of input single step signal is

$$\bar{x} = \frac{1}{\bar{p}} \cdot \tag{1.11}$$

Then

$$\bar{y} = \frac{5}{(2p+1)p} \cdot \tag{1.12}$$

Perform inverse Laplace transformation. For the purpose, represent (1.12) denominator as ($p_1 = 0;\ p_2 = -2^{-1}$ are the denominator roots):

$$\frac{5}{(2p+1)p} = \frac{5}{2\left(p+\frac{1}{2}\right)p} = \frac{2\frac{1}{2}}{\left(p+\frac{1}{2}\right)p} \cdot \tag{1.13}$$

Represent a fraction in the right-hand side of mathematical expression (1.13) as the total of the simplest fractions with the unknown coefficients:

$$\frac{2\frac{1}{2}}{\left(p+\frac{1}{2}\right)p} = \frac{A}{p} + \frac{B}{(p+\frac{1}{2})} \cdot \tag{1.14}$$

Add the fractions:

$$\frac{A}{p} + \frac{B}{(p+\frac{1}{2})} = \frac{A\left(p+\frac{1}{2}\right) + Bp}{p\left(p+\frac{1}{2}\right)} \cdot \tag{1.15}$$

Fractions of (1.14) and (1.15) mathematical expressions are equal in value. Since denominator of fraction of the left-hand side of (1.14) is equal to a fraction of the right-hand side of (1.15), numerators of the fractions should also be equal. Derive the equation:

$$2\frac{1}{2} = A\left(p+\frac{1}{2}\right) + Bp. \tag{1.16}$$

Transform the right-hand side of (1.16):

$$2\frac{1}{2} = Ap + \frac{1}{2}A + Bp, \tag{1.17}$$

$$2\frac{1}{2} = (A+B)p + \frac{1}{2}A. \tag{1.18}$$

Equate coefficients in terms of p^1 and p^0 (free term), and obtain equation system to identify A and B:

$$\begin{cases} 2\frac{1}{2} = \frac{1}{2}A \\ A + B = 0. \end{cases} \tag{1.19}$$

Using (1.19) system, determine $A = 5$ and $B = -5$. The expression (1.14) is represented as follows:

$$\frac{2\frac{1}{2}}{\left(p + \frac{1}{2}\right)p} = \frac{5}{p} + \frac{-5}{(p + \frac{1}{2})}. \tag{1.20}$$

Hence,

$$\bar{y} = \frac{5}{p} + \frac{-5}{(p + \frac{1}{2})}. \tag{1.21}$$

Define transition function, using 1 and 4 formulas from Table 1.2:

$$y(t) = 5 - 5e^{-\frac{t}{2}} = 5\left(1 - e^{-\frac{t}{2}}\right). \tag{1.22}$$

Fig. 1.2 demonstrates a graph of the transient process calculated with the help of MathCAD APs using (1.22) formula.

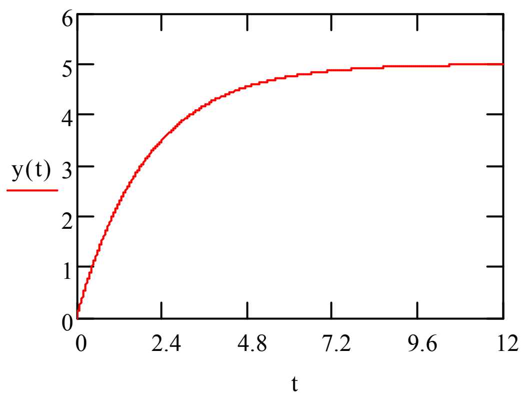

Fig. 1.2 The transient process graph

Fig.1.3 shows a structural pattern to simulate aperiodic elements under the SIMULINK MATLAB application package; Fig.1.4 demonstrates graph of transient process obtained as a modelling result.

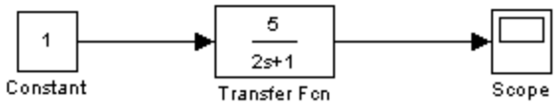

Fig. 1.3 Structural pattern of aperiodic element modeling

Amplitude and phase frequency response of the aperiodic element is assumed by means of the mathematical expression:

$$W(j\omega) = \frac{K}{\omega^2 T^2 + 1} - j\frac{K\omega}{\omega^2 T^2 + 1}. \quad (1.23)$$

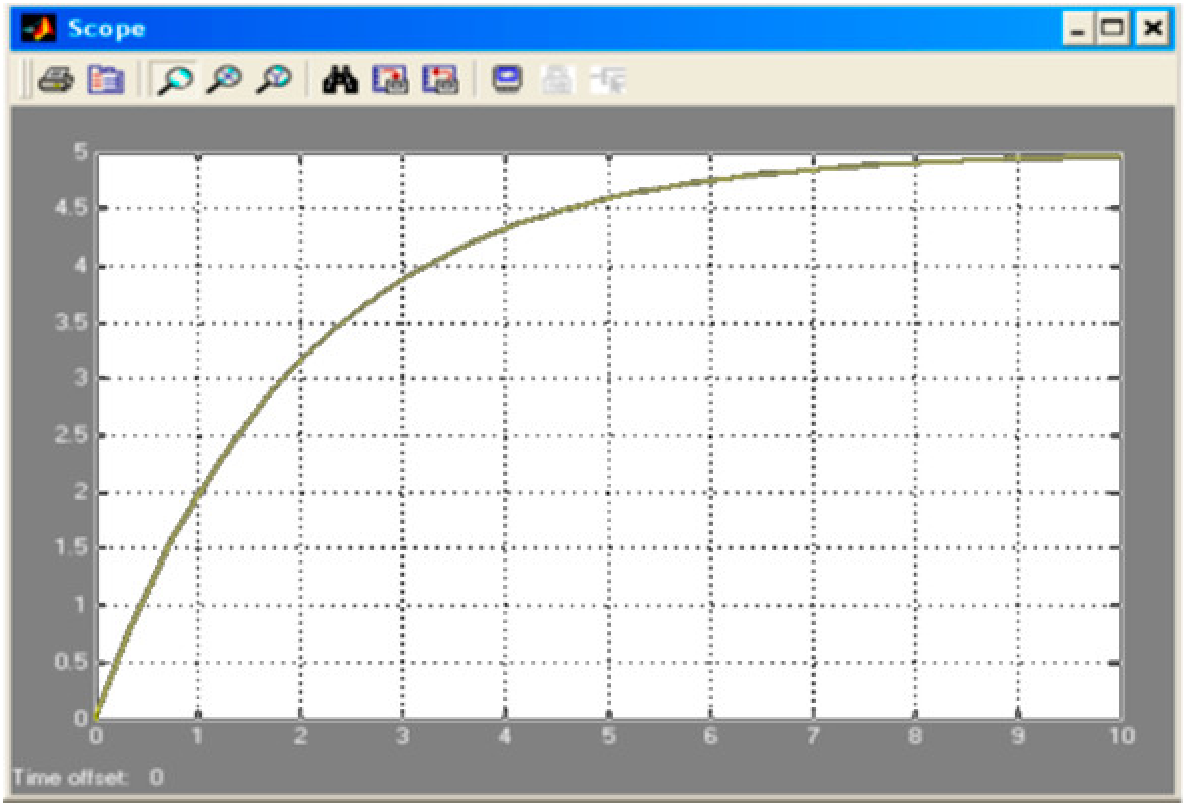

Fig. 1.4 Transient process graph

Substitute values for K and T:

$$W(j\omega) = \frac{5}{4\omega^2 + 1} - j\frac{5\omega}{4\omega^2 + 1}. \quad (1.24)$$

Fig. 1.5 explains graph of amplitude and phase frequency response.

Logarithmic amplitude and phase frequency response of aperiodic element is specified with the help of the mathematical expression:

$$20lgA(\omega) = 20lgK - 20lg\sqrt{T^2\omega^2 + 1}. \quad (1.25)$$

Substitute values for *K* and *T*:

$$20lgA(\omega) = 20lg5 - 20lg\sqrt{4\omega^2 + 1}. \qquad (1.26)$$

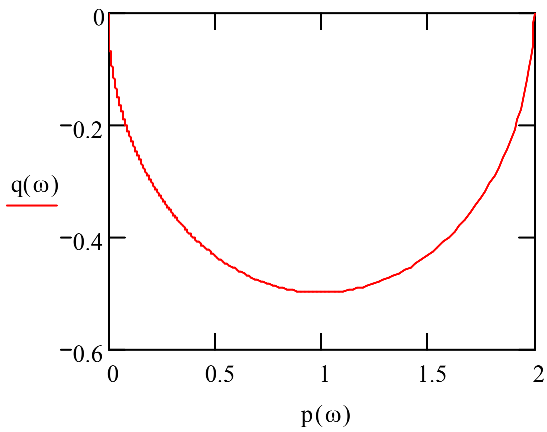

Fig. 1.5 Graph of logarithmic amplitude and phase frequency response

Fig. 1.6 demonstrates graph of logarithmic amplitude and frequency response.

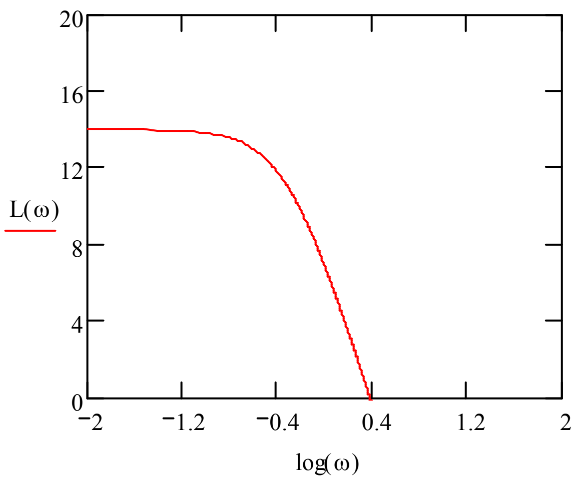

Fig. 1.6 Graph of logarithmic amplitude and frequency response

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Logarithmic phase and frequency response of aperiodic element is assumed using the mathematical expression:

$$\varphi(\omega) = -\text{arctg}T\omega. \qquad (1.27)$$

Substitute values for *K* and *T*:

$$\varphi(\omega) = -\text{arctg}2\omega. \qquad (1.28)$$

Fig. 1.7 demonstrates logarithmic phase and frequency response.

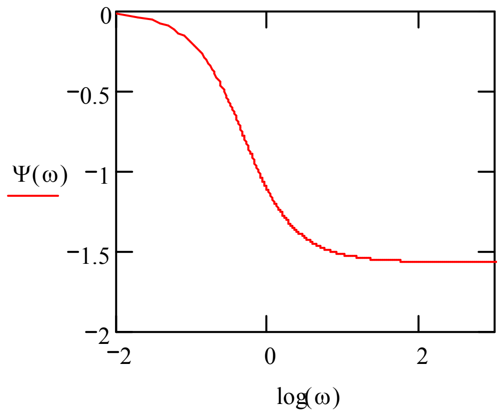


Fig. 1.7 Graph of logarithmic phase and frequency response

1.5 Report contents.

Structural patterns of the analyzed dynamic elements.

Graphs of transient processes, amplitude and phase frequency characteristics, logarithmic amplitude and phase frequency characteristics of the analyzed elements have been calculated.

Graphs of transient processes of the analyzed elements, determined in the process of computer-based modelling, have been constructed.

Graphs of transient processes of the analysed elements, determined in the process of computer-based modelling, have been constructed in terms of changes in their parameters.

1.6 Control questions

What is transfer function?

What is dynamic element?

What typical dynamic elements do you know?

What typical input signal are applied to study ACSs?

What is transient function?

What are frequency responses?

What are the coordinates to construct a logarithmic amplitude frequency response?

What are the coordinates to construct a logarithmic phase frequency response?

What is the nature of net delay of APFR variation if a time constant varies?

What is the nature of a transient process at the output of aperiodic element if intensification coefficient (time component) is increased (decreased)?

What is the nature of LPFR of integrating element variation if its intensification coefficient is decreased (increased)?

What is the nature of LPFR of inertial differential element variation if intensification coefficient (time constant) is decreased (increased)?

What is the difference between graphs of transient processes of oscillating element if $d < 1$, and $d \geq 1$?

LABORATORY RESEARCH 2

Analyzing stability of the automated control systems

2.1 **Objective** is to deepen students' knowledge while studying the Chapter "Stability of the automated control systems".

During the activities, students should be able to:

– identify characteristic polynom of the closed system of the automated control;
– make a matrix of Hurwitz coefficients;
– determine effect of the specified parameters of a linear automated system on its stability using Hurwitz stability criterion;
– construct stability boundary, and define areas of stable and unstable areas of the automated control system operation in terms of the specified coordinates;
– gain practical skills to analyze the automated systems using a computer.

2.2 Input data to perform the activities are the following:

– structural schemes and numerical parameters of the analyzed automated systems (Fig.2.1, Table 2.1.); and
– the MATLAB and MathCAD application package for computer-based simulation of regulating systems (RS) and mathematical calculations.

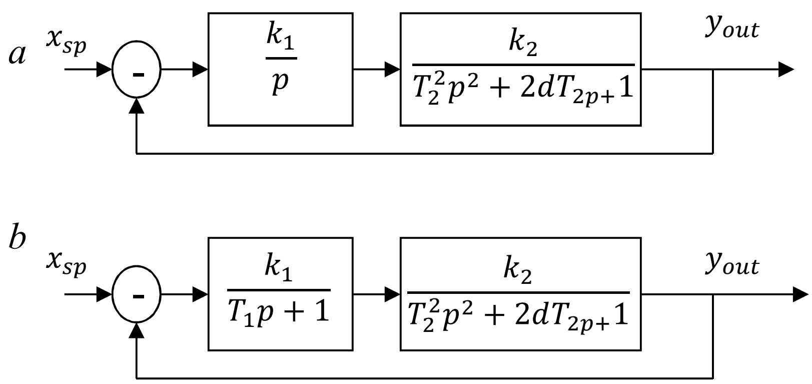


Fig. 2.1 Structural schemes of the analyzed automated systems:
a – astatic system of the automated control;
b – static system of the automated control.

Input data

Table 2.1

Variant	Scheme	Parameters of the elements					Coordinate system
		k_1	k_2	T_1	T_2	d	X, Y
1	a	2.0	1.0	–	2.0	0.6	k_1, T_2
2	a	2.0	2.0	–	3.0	0.7	k_2, T_2
3	a	1.0	1.0	–	4.0	0.8	k_1, d
4	a	1.0	2.0	–	1.0	0.9	k_2, d
5	a	1.0	3.0	–	2.0	1.0	T_2, d
6	a	4.0	0.5	–	1.0	1.1	k_1, k_2
7	a	0.5	0.5	–	4.0	1.2	k_2, k_1
8	a	0.5	2.0	–	3.0	1.3	T_2, k_1
9	a	4.0	3.0	–	2.0	1.4	d, k_2
10	a	3.5	2.0	–	2.0	0.5	d, T_2
11	b	1.0	1.0	0.5	1.0	1.3	k_1, T_2
12	b	5.0	1.0	0.6	3.0	1.2	k_2, T_2
13	b	2.0	0.2	0.7	2.0	1.1	k_1, d
14	b	3.0	2.0	0.8	1.0	1.0	k_2, d
15	b	4.0	1.0	0.9	4.0	0.9	T_2, d
16	b	1.0	1.0	1.0	3.0	0.8	k_1, k_2
17	b	1.0	2.0	0.9	2.0	0.7	T_1, T_2
18	b	1.0	3.0	0.8	1.0	0.6	k_1, T_1
19	b	3.0	4.0	0.7	1.5	0.5	T_1, d
20	b	2.0	0.5	0.3	1.0	1.0	k_2, T_1

2.3 Operating procedures

Following order is recommended:

- record a transfer function of open system for the structural scheme specified by a teacher;
- record characteristic polynom for the closed system;
- make a matrix of Hurwitz coefficients;
- determine stability boundary of a linear system of the automated control system as a function by the parameters of elements of the automated control system;
- construct stable and unstable areas of the automated control system;
- apply the MATLAB application package to identify transient process within the system in terms of a single step signal for stable and unstable operation of the automated system, and for the system operation in terms of the stable boundary;
- draw conclusions using the obtained results.

2.4 Methodological explanations.

Stability is among the most important characteristics of any automated system. Unstable system cannot perform its functions. Moreover, it may cause emergency of the controlled object. That is why, the problem to provide system stability is one of the central ones in the theory of the automated control.

The automated system stability is its characteristic to get back to a balance after the influence, caused the unbalance, is over. No unstable system can get back to a balance state distancing from it continuously.

Algebraic Hurwitz criterion is one of the most popular in the context of engineering practices determining stability. The criterion can be formulated as follows: to make the automated system stable, it is necessary and quite sufficient for each determinant of a matrix of Hurwitz coefficients to be positive. Even if one zero determinant is available in the absence of negative determinants, the system of the automated control will be within its stability boundary (i.e. output value will vary in terms of a harmonic law with stable amplitude and frequency).

The matrix of Hurwitz coefficients consists of coefficients of characteristics polynom of the closed automated control system being:

$$D_{cl}(p) = a_0 p^n + a_1 p^{n-1} + \dots + a_{n-1} p + a_n = 0, \qquad (2.1)$$

where n is the order of the automated control system.

The order of the matrix of Hurwitz coefficients is $n \times n$. Correspondingly, coefficients of a characteristic polynom a_i are diagonal elements of such a matrix c_{ii} $(i = \overline{1, n})$. Rows to the right of the diagonal elements are added by coefficients with successively increasing even indices if the diagonal element is in the even row; if the diagonal element is in the odd row, then coefficients with successively increased odd indices add it. Rows to the left of the diagonal element are added by coefficients with successively decreasing even indices, and coefficients with successively decreasing odd indices if the diagonal element is in the odd row. The matrix elements are equal to zero if coefficients of characteristic polynom, which indices are more than n or less than zero, should be instead of them.

To obtain characteristic polynom of the closed automated control system, it is required to record a transition function of an open automated system, and then sum up numerator and denominator polynoms.

Example 2.1. Identify stability of the automated system which structural scheme is in Fig. 2.2. Also, determine an equation of a stability boundary as $k_1 = f(T_2)$ function and construct the areas of stable and unstable operation of the automated control system in terms of (T_2, k_1) coordinates. The system parameters are $k_1 = 1$; $k_2 = 1$; $k_3 = 2$; $T_1 = 1$; and $T_2 = 0.5$.

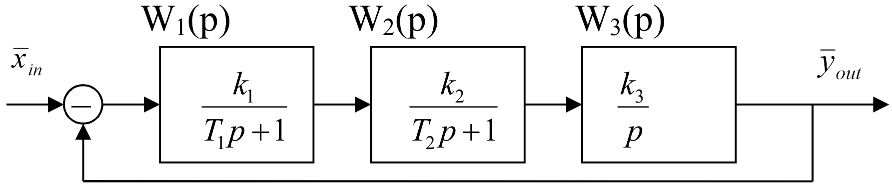


Fig. 2.2 Structural scheme for the example

Define transition function of the open automated control system:

$$W_{op}(p) = W_1(p)W_2(p)W_3(p) = \frac{k_1}{T_1 + 1} \times \frac{k_2}{T_2 + 1} \times \frac{k_3}{p} =$$

$$= \frac{k_1 k_2 k_3}{T_1 T_2 p^3 + T_1 p^2 + T_2 p^2 + p} = \frac{k_1 k_2 k_3}{T_1 T_2 p^3 + (T_1 + T_2)p^2 + p}.$$

Identify characteristic polynom of the closed automated control system:

$$D_{cl}(p) = T_1 T_2 p^3 + (T_1 + T_2)p^2 + p + k_1 k_2 k_3. \quad (2.2)$$

We get 3rd order characteristic polynom where $a_0 = T_1 T_2$, $a_1 = T_1 + T_2$, $a_2 = 1$, and $a_3 = k_1 k_2 k_3$. Make a matrix of Hurwitz coefficients:

$$\begin{pmatrix} (T_1 + T_2) & k_1 k_2 k_3 & 0 \\ T_1 T_2 & 1 & 0 \\ 0 & (T_1 + T_2) & k_1 k_2 k_3 \end{pmatrix}.$$

Substitute the parameter values and calculate the matrix elements. Hence:

$$\begin{pmatrix} 1.5 & 2 & 0 \\ 0,5 & 1 & 0 \\ 0 & 1.5 & 2 \end{pmatrix}.$$

Define determinants of the matrix of Hurwitz coefficients:

$$\Delta_1 = c_{11} = 2 > 0;$$

$$\Delta_2 = c_{11} \times c_{22} - c_{12} \times c_{21} = 0.5 > 0.$$

The latter (i.e. the third) determinant of the matrix is calculated using the formula:

$$\Delta_3 = \Delta_2 \times c_{33} = 1 > 0.$$

All the determinants are positive. Thus, the system is stable.

Identify the areas of stable and unstable operation of the automated control system in terms of (T_2, k_1) coordinates. If the system is within its stability boundary, then 2nd order diagonal minor should be equal to zero:

$$(T_1 + T_2) - k_1 k_2 k_3 T_1 T_2 = 0. \tag{2.3}$$

Using the abovementioned, identify an equation of stability boundary of the RS in terms of the specified coordinates:

$$k_1 = \frac{T_1 + T_2}{k_2 k_3 T_1 T_2} = \frac{1 + T_2}{2T_2}. \tag{2.4}$$

Varying T_2 parameter, construct stability boundary of the automated system operation in terms of $(T_2,\ k_1)$ coordinates (Table 2.2).

Data to calculate the stability boundary

Table 2.2

T_2	0.1	0.5	1	1.5	2	2.5	3	3.5	1
k_1	5.5	1.5	1	0.83	0.75	0.7	0.67	0.64	0.63

It is obvious that stability of the automated control system will be in correspondence with the graph area in terms of which following inequation is fulfilled:

$$\Delta_2 = (T_1 + T_2) - k_1 k_2 k_3 T_1 T_2 > 0.$$

Thus:

$$k_1 < \frac{T_1 + T_2}{k_2 k_3 T_1 T_2} = \frac{1 + T_2}{2T_2}.$$

The area of unstable operation of the RS will be determined using the inequation:

$$\Delta_2 = (T_1 + T_2) - k_1 k_2 k_3 T_1 T_2 < 0.$$

Thus :

$$k_1 > \frac{T_1 + T_2}{k_2 k_3 T_1 T_2} = \frac{1 + T_2}{2T_2}.$$

Fig. 2.3 demonstrates the areas of stable and unstable operation of the automated control system.

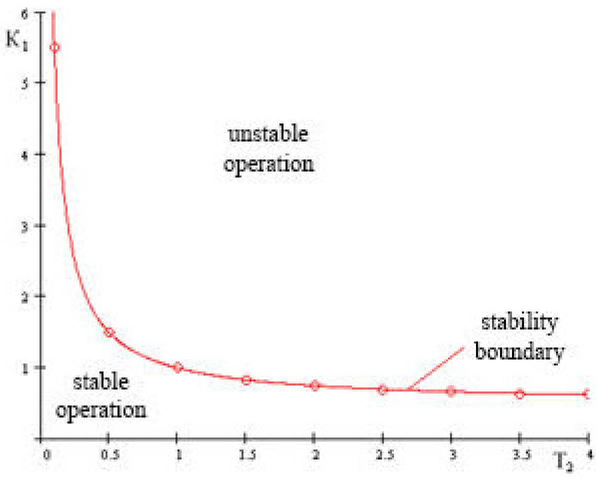


Fig. 2.3 Areas of stable and unstable operation of the automated system

Using the constructed graph, simulate operation of the automated system in terms of a stable mode (0.5; 1), unstable mode (2; 3), and stability boundary (1; 1). Fig. 2.4 demonstrates graphs of transient processes.

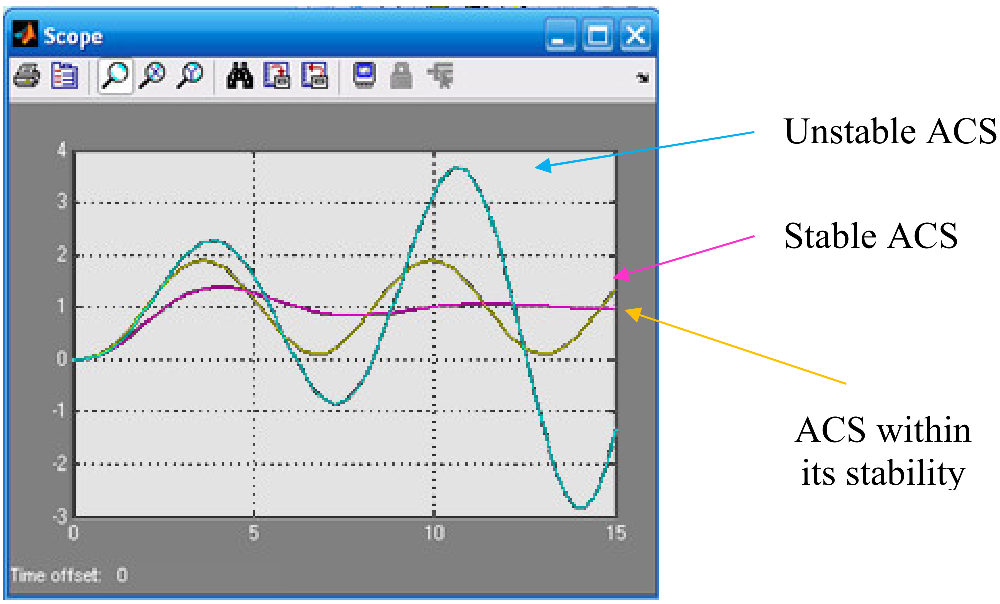


Fig. 2.4 Transient processes of a stable ACS; unstable ACS; and ACS within its stability boundary

2.5 Report contents.

Structural scheme of the analyzed system.

Characteristic equation of the closed system.

Stability calculation of the specified automated system.

Equations of the system stability boundary.

Irregularities of stable and unstable operation of the automated system.

Graphs of areas of stable and unstable operation of the automated system.

Graphs of transient processes of stable and unstable system as well as a system within its stability boundary represented in one coordinate system.

2.6 Control questions

What is understood as the automated system stability?

What is the system being considered as a stable one?

What is the system being considered as an unstable one?

What is the system being considered in terms of its stability boundary?

How can be determined a 3rd order critical intensification coefficient?

LABORATORY RESEARCH 3

Analyzing accuracy of static and astatic ACSs

3.1 Objective is to deepen students' knowledge while studying the Chapter "Accuracy of regulating systems".

In the process of the activities, students should be able to:

– identify accuracy of both static and astatic regulating systems;

– determine operation areas of the regulating systems which provide the specified accuracy of the regulating systems;

– develop the areas providing the specified accuracy of the regulating system in terms of parameters of its elements as well as parameters of input effect;

– gain practical skills to analyze the automated systems using a computer.

3.2 Input data to perform the activities are the following:

– structural scheme and numerical parameters of dynamic elements of the analyzed systems (Fig.3.1, Table 3.1.);

– the MATLAB and MathCAD application package to simulate the automated control systems and perform computer-based calculations.

3.3 Operating procedures

Following order is recommended:

– calculate static errors for both static and astatic systems of the automated control when a step signal is set to their inputs;

– construct a dependence graph of a static error of the static system upon the intensification coefficient of an open part;

– determine dependence of the intensification coefficient upon the input effect to provide the specified value of the static error;

– construct graphs of transient processes in terms of static and astatic systems when the effect, varying according to a linear law, is provided to their effect inputs;

– test a hypothesis on the independence of a velocity error in terms of a stable mode from a free coefficient of a law of input value variation;

– calculate parameters of astatic system of the automated control providing the velocity value as that not exceeding the specified one.

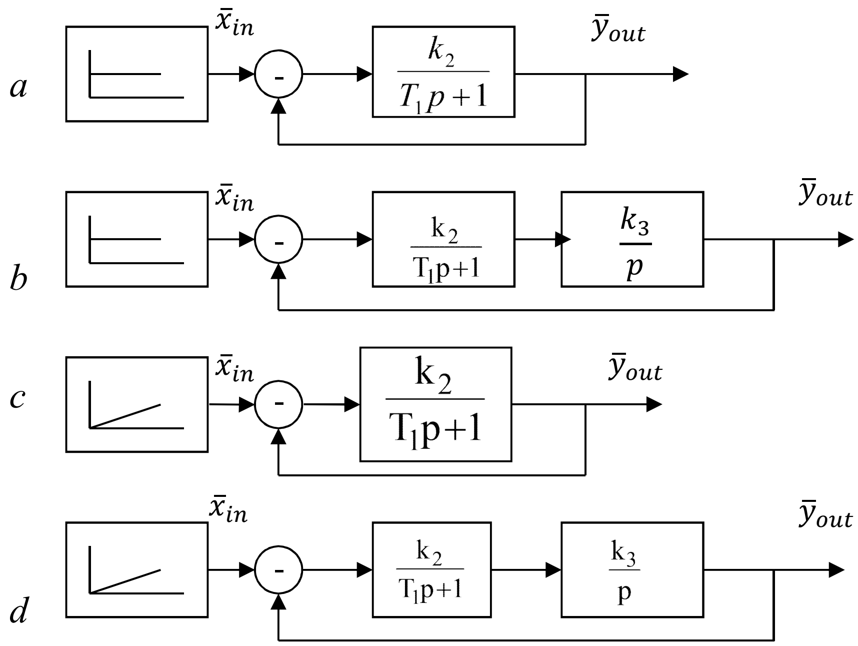


Fig. 3.1 Structural scheme to analyze accuracy of regulating systems:

a – a static system with step input effect;
b – an astatic system with step input effect;
c – a static system with linear input effect;
d – an astatic system with linear input effect.

3.4 Methodological explanations.

Provision of the required accuracy in terms of the stable mode is one of the requirements as for the regulating systems. Error value in various standard modes is applied to evaluate accuracy of a control system. Stable error is a difference between the specified value of the controlled variable and actual one.

The stable error value within a static regulating system in terms of a step input value is determined using the formula:

$$\varepsilon_{st} = \frac{x_{in}}{1 + K_{op}}, \tag{3.1}$$

where ε_{st} is the static error; and $K_{op}$is the overall intensification coefficient of an open part of the system.

When the transient process is over, the controlled variable value will be as follows

$$y_{out} = x_{in} - \varepsilon_{st} = \frac{x_{in} K_{op}}{1 + K_{op}}. \tag{3.2}$$

If such linear signal as $x_{in}(t) = a + k_1 t$ is set to the static system input, then a static error ε_{st} will vary with constant velocity. If the effect is quite long-term then the systems stops to be efficient.

Table 3.1

Input data

Variant	Parameters of elements				
	x_{in}	k_1	k_2	k_3	T, c
1	1.0	0.5	1.5	0.5	0.70
2	1.5	0.8	1.0	1.0	0.50
3	2.0	1.1	0.5	0.7	0.60
4	2.5	1.4	2.0	1.2	0.40
5	3.0	1.7	2.5	0.9	0.35
6	3.5	2.0	3.0	1.3	0.30
7	4.0	2.3	3.5	0.7	0.25
8	4.5	2.6	4.0	0.4	0.20
9	4.0	2.9	4.5	0.4	0.20
10	3.5	2.6	4.0	0.7	0.25
11	3.0	2.3	3.5	1.3	0.30
12	2.5	2.0	3.0	0.9	0.35
13	2.0	1.7	2.5	1.2	0.40
14	1.5	1.4	2.0	0.7	0.60
15	1.0	1.1	0.5	1.0	0.50
16	0.5	0.8	1.0	0.5	0.70
17	1.0	0.5	1.5	1.0	0.50
18	1.5	0.8	1.0	0.7	0.60
19	3.5	2.6	1.0	0.7	0.35
20	2.0	1.7	3.5	1.3	0.60

As for the astatic regulating system, the static error value in terms of a step input value $x_{in} = \text{const}$ is $\varepsilon_{st} = 0$; hence, $y_{out} = x_{in}$. If a linear signal is set to the system input, then the stable error will not increase temporally; when the transient process is over, value of the error will have a value determined as a velocity value:

$$\varepsilon_{vl} = \frac{k_1}{K_{op}}, \tag{3.3}$$

In terms of the stable mode, the controlled variable will experience its changes according to the law:

$$y_{out}(t) = a + k_1 t - \varepsilon_{vl} = \left(a - \frac{k_1}{K_{op}}\right) + k_1 t. \qquad (3.4)$$

Example 3.1. Calculate the accuracy of both static and astatic systems of the automated control (Fig. 3.1, *a* and 3.1, *b*) if step effect is set to their inputs. Construct a dependence graph of statistic error of the systems upon the intensification coefficient of their open parts $\varepsilon(k_2)$. Identify the dependence between reference-input signal and intensification coefficient of the open part $k_2(x_{in})$ to provide the specified error value ε_{sp}=0.1 input value is $x_{\text{вх}} = 5$; and values of dynamic coefficients of the elements are $k_2 = 3$; $T_1 = 2$; and $k_3 = 5$.

The automated control systems, represented in Figures 3.1, *a* and 3.1, *b* are 1st order and 2nd order systems respectively having one-sign positive coefficients. Such systems are of stable nature. Determine accuracy of the systems when step effect is set to their inputs.

The automated control system, represented in Fig. 3.1, *a* is the static system. Determine static error within the system using the formula (3.1):

$$\varepsilon_{st} = \frac{x_{in}}{1 + k_2} = \frac{5}{1 + 3} = 1.25. \qquad (3.5)$$

Verify the obtained results while simulating under the SIMULINK MATLAB environment in terms of a scheme represented in Fig. 3.2.

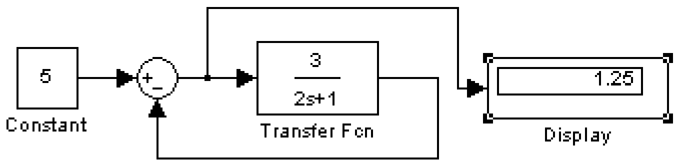


Fig. 3.2 Structural scheme to analyze the system accuracy

The simulation results have verified the calculations.

Calculate static error ε_{st} while varying intensification coefficient k_2 of an open ACS part. Construct dependence graph $\varepsilon(k_2)$ using the MathCAD environment
(Fig. 3.3).

It is understood from the graph in Fig. 3.3 that the increase in intensification coefficient of an open ACS part results in the decrease of static error.

To define dependence between the reference-input signal and intensification coefficient of an open $k_2(x_{in})$ part in terms of the specified ε_{sp}= 0.1, reduce 3.5 expression to the following:

$$k_2 = \frac{x_{in} - \varepsilon_{st}}{\varepsilon_{st}} = \frac{x_{in} - 0{,}1}{0{,}1}. \qquad (3.6)$$

Fig. 3.4 demonstrates the dependence graph $k_2(x_{in})$ constructed in terms of the MathCAD environment.

Verify the calculation results. Select any point within the graph, and determine its coordinates within the help of a function *Trace* in the MathCAD application package (Fig. 3.5).

$$k2 := 0, 0.01 .. 20$$

$$\varepsilon(k2) := \frac{5}{(1 + k2)}$$

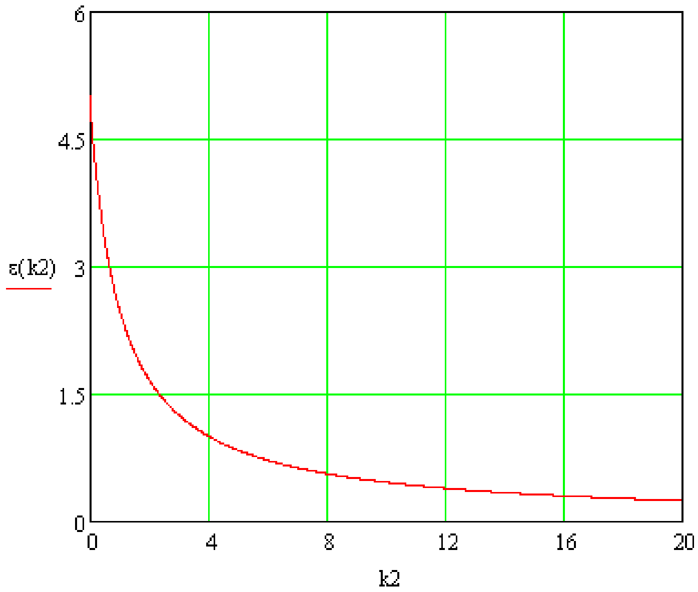


Fig. 3.3 Dependence graph $\varepsilon(K_2)$

$$x := 0.1, 0.11 .. 10$$

$$k2(x) := \frac{x - 0.1}{0.1}$$

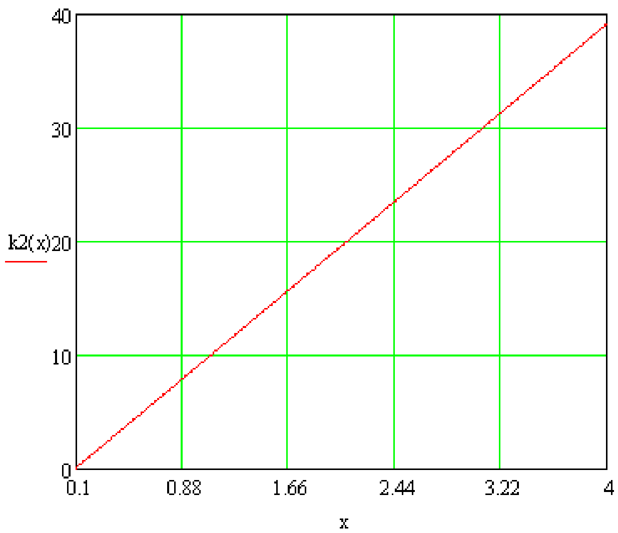


Fig. 3.4 Dependence graph $K_2(x_{in})$

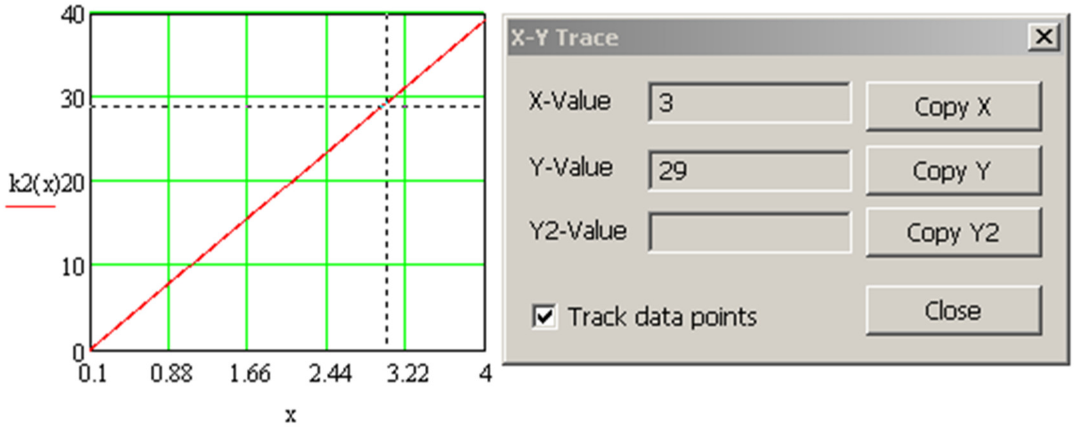


Fig. 3.5 Determination of the graph point coordinates

Hence, we have point coordinates $x_{in} = 3$ and $k_2 = 29$. Verify the obtained results using simulating in the SIMULINK MATLAB application package in terms of the abovementioned scheme (Fig. 3.2). Fig. 3.6 demonstrates the findings.

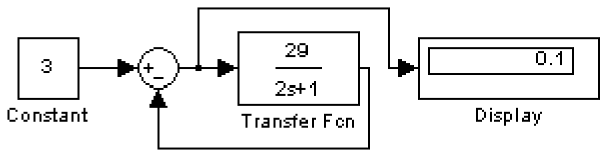


Fig. 3.6. Structural scheme to verify the calculation results $\varepsilon_{st} = 0.1$ has been obtained

Thus, the automated control system, represented in Fig. 3.1, b is of astatic type. In the context of the system, static error is equal to zero; moreover, it cannot depend upon intensification coefficient of an open part $K_{op} = k_2 k_3$ as well as upon a reference-input signal x_{in}. Verify the statement using simulation in the SIMULINK MATLAB application package with values of the reference-input signal and intensification coefficients, specified by the example conditions (Fig. 3.7), and the varied values (Fig. 3.8). As it is seen, the static error approaches zero in terms of both cases.

Example 3.2. Linear effect $x_{in}(t) = a + k_1$ has been set to the inputs of the automated control systems (Figures 3.1, *c*, and 3.1, *d*). Use simulation in the SIMULINK MATLAB application package to identify a velocity error ε_{vl} within the systems. Verify independence of a velocity error ε_{vl} from *a* coefficient for the astatic ACS (Fig. 3.1, *d*), and separate an area within $K_{op}(k_1)$ plane where $\varepsilon_{vl} \leq \varepsilon_{sp}$. Values of coefficients of input effect, parameters of the automated control system, and the specified error values are as follows: $a = 3$, $k_1 = 2$, $k_2 = 3$, $k_3 = 5$, $T_1 = 2$, $\varepsilon_{sp} = 0.1$.

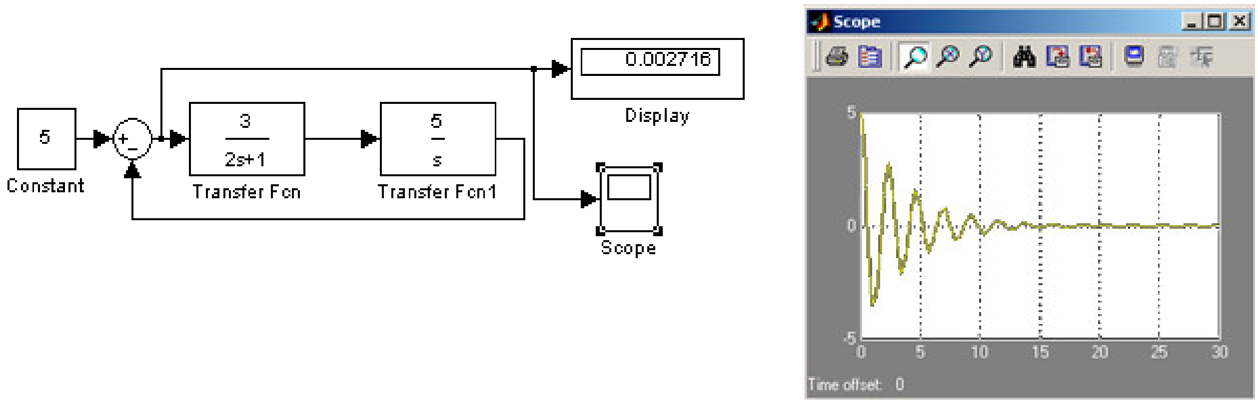


Fig. 3.7 Structural scheme of ACS and a transient process graph for input values of the parameters

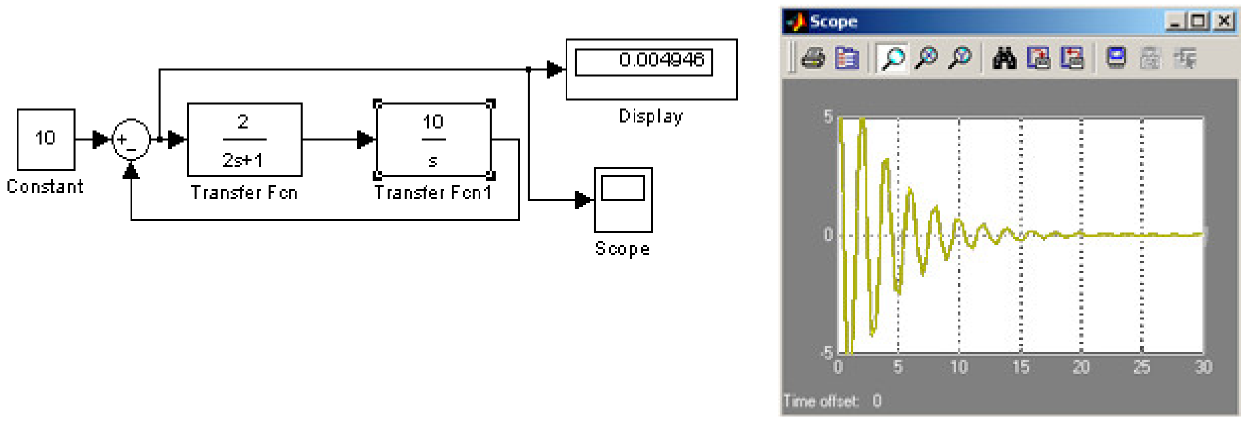


Fig. 3.8 Structural scheme of ACS and a transient process graph for the varied parameter values

Fig. 3.9 demonstrates structural scheme to analyze static and astatic ACSs (Figures 3.1, c, and 3.1, d) in the SIMULINK MATLAB application package. Such manual switches as *Manual Switch 2 and Manual Switch 3* help throw inputs of *Scope* and *Display* to the inputs of elements of static and astatic systems.

Both static and astatic systems of the automated control have been simulated according to the structural scheme. Fig. 3.10 represents graphs of the transient process as well as graphs of changes in the input effect and time errors of the static ACS. As it follows from the graphs, in terms of linear changes in the input effect, ε_{vl} error value increases infinitely.

Fig. 3.11 demonstrates graphs of the transient process as well as graphs of changes in the input effect and time errors of the astatic ACS. As it follows from the graphs, in terms of linear changes in the input effect, ε_{vl} error value approaches zero.

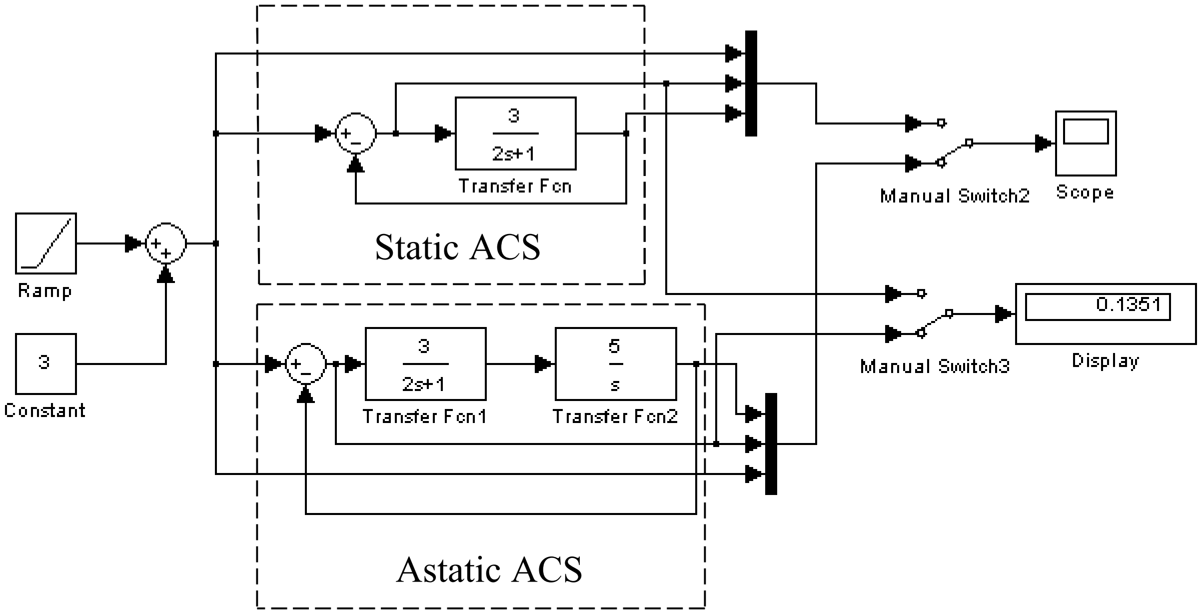


Fig. 3.9 Structural scheme to analyze ACSs

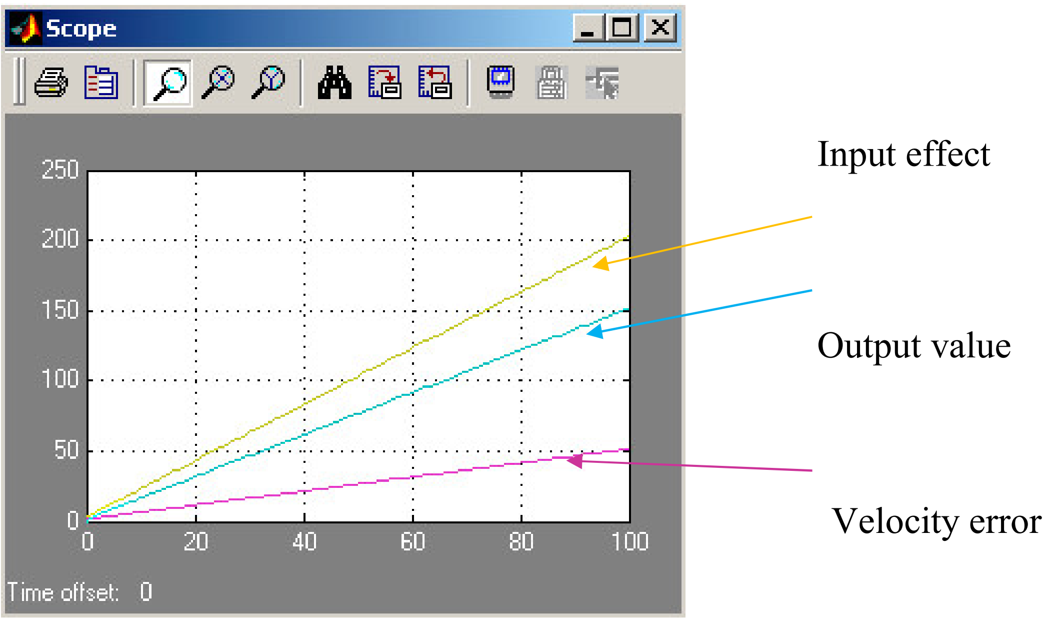


Fig. 3.10 Simulation results of the static ACS

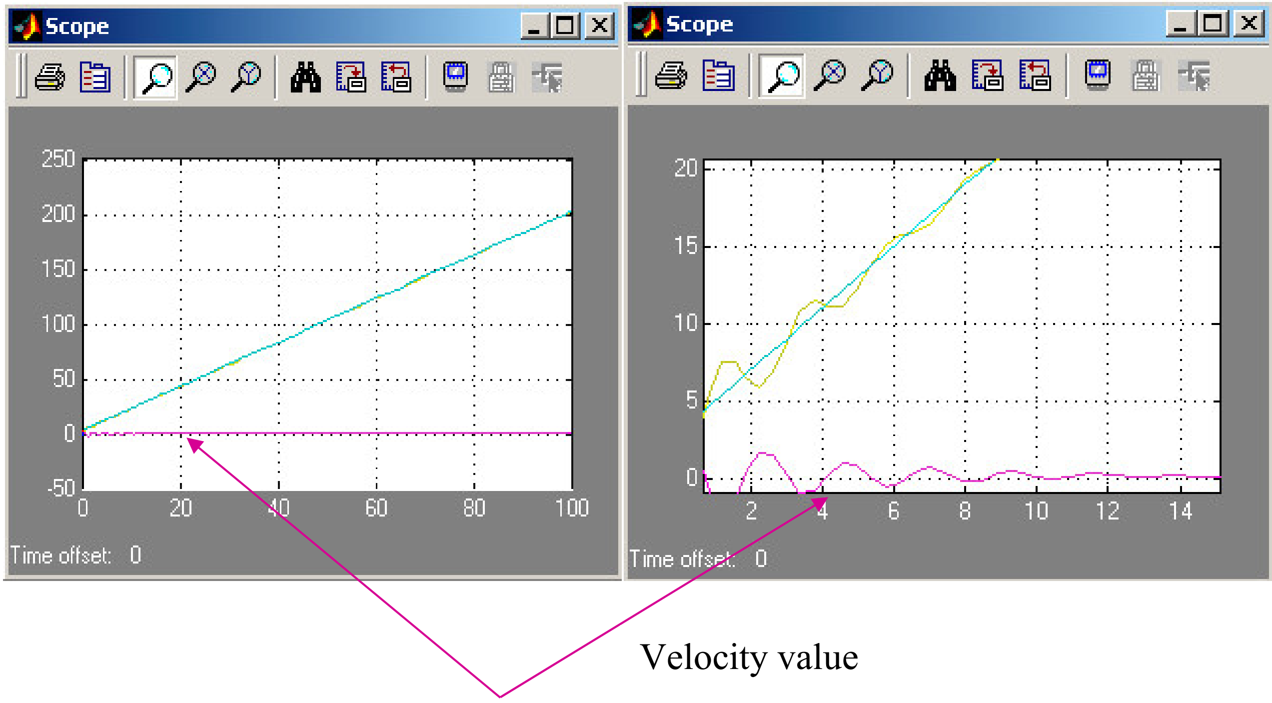


Fig. 3.11 Simulation results of the astatic ACS

Prove independence of the velocity error ε_{vl} from a parameter of the input effect in terms of astatic systems of the automated control. For the purpose, apply Spearman's criterion. While varying values of a parameter, use the SIMULINK MATLAB application package to identify corresponding values of the velocity error ε_{vl}; calculate Spearman's rank correlation coefficient. Table 3.2 represents the simulation results.

Table 3.2

Output data

a parameter	5	15	25	35	45	55	65	75	85	95
Velocity error ε_{vl}	0.13	0.16	0.08	0.06	0.03	-0.15	0.04	0.8	0	-0.16

Use the data from Table 3.2 to form ranks of a parameter (Table3.3) as well as velocity error ε_{vl} (Table 3.4).

Table 3.3

Value ranks of a parameter

a parameter	95	85	75	65	55	45	35	25	15	5
a_i rank	1	2	3	4	5	6	7	8	9	10

Table 3.4

Value ranks of ε_{vl} parameter

Velocity error ε_{vl}	0.8	0.16	0.13	0.08	0.06	0.04	0.03	0	-0.15	-0.16
ε_{vl}^{i} rank	1	2	3	4	5	6	7	8	9	10

Make sequences of a and ε_{vl} ranks with the help of the data represented by Tables 3.3 and 3.4. Table 3.5 demonstrates the rank sequences in addition to the differences in the ranks as well as the squared differences of the ranks.

Table 3.5

Sequences of ranks a and ε_{vl}

a_i rank	1	2	3	4	5	6	7	8	9	10
ε_{vl}^{i} rank	10	8	1	6	9	7	5	4	2	3
Differences in the ranks $d_i = a_i - \varepsilon_{vl}^{i}$	– 9	– 6	2	– 2	– 4	– 1	2	4	7	7
The squared d_i^2 difference	81	36	4	4	16	1	4	16	49	49

Determine Spearman's rank correlation coefficient:

$$\rho_s = 1 - \frac{6\sum_i^n d_i^2}{n^3 - n} = 1 - \frac{6 \times 260}{10^3 - 10} = -0.73, \tag{3.7}$$

where $n = 10$ is the sample size.

Test a zero hypothesis H_0 as for zero equality of the general coefficient of Spearman's rank correlation ρ_{Γ}. Use a Table of Student's t-distribution to find a value of a critical point of two-sided critical region $t_{cr}(\alpha, k)$. In this context, α is a significance level; $k = n - 2$ being degrees of freedom. Determine $t_{cr}(\alpha, k) = 3.36$ if $\alpha = 0.01$ and $k = 8$. Calculate the critical point:

$$T_{cr} = t_{cr}(\alpha, k)\sqrt{\frac{1 - {\rho_s}^2}{n - 2}} = 3.36\sqrt{\frac{1 - (-0.73)^2}{10 - 2}} = 0.81. \tag{3.8}$$

Since $|\rho_s| < T_{cr}$, there is no need to reject zero hypothesis concerning zero equality of the general coefficient of Spearman's rank correlation. Hence, there is no rank correlation between a, and ε_{vl} parameters. a parameter has no effect on the ε_{vl} velociity error.

Using coordinate plane $K_{op}(k_1)$ to determine the area within which $\varepsilon_{vl} \leq 0.1$. According to (3.3), we have:

$$\varepsilon_{vl} = \frac{k_1}{K_{op}} \leq 0.1. \tag{3.9}$$

Using (3.9), we obtain:

$$K_{op} \geq \frac{k_1}{0.1}. \tag{3.10}$$

In accordance with (3.10), $K_{op} = k_1 0.1^{-1}$, the MathCAD software has been applied to obtain $K_{op} = k_1 0.1^{-1}$ dependence (Fig.3.12). Also, Fig. 3.12 shows the area where $\varepsilon_{vl} \leq 0.1$ as well as the area where $\varepsilon_{vl} > 0.1$. Test accuracy of the calculations while simulating using the SIMULINK MATLAB application package according to 3.9 structural scheme. For the purpose, it is required to separate three points within the coordinate plane where $\varepsilon_{vl} < 0.1$, $\varepsilon_{vl} = 0.1$, and $\varepsilon_{vl} > 0.1$. Table 3.6 demonstrates coordinates of the points, and relevant values of velocity errors. The simulation results coincide with the calculations.

3.5 Report contents

Structural scheme to analyze accuracy of both static and astatic systems of the automated control.

Calculations of static error for static and astatic automated control systems when a step signal is set to their inputs.

Graph of dependence of a static error of the static automated control system upon intensification coefficient of an open part.

Calculation of the intensification coefficient dependence upon the input effect to provide the specified error value.

Graphs of transient processes within the static and astatic systems if the effect, varying according to a linear law, is set to their inputs.

Calculations to test the hypothesis concerning the velocity error independence in terms of the stable mode from a free coefficient of input value variation.

Calculations and graphs as for the separation of areas within $K_{op}(k_1)$ plane where velocity error is not more than the specified value.

3.6 Control questions

What is a system accuracy?

What is static error?

What is velocity error?

How is it possible to decrease the specified error within a static control system?

Is it possible to decrease the specified error within an astatic control system? Why?

What is the nature of the specified error variation within a static control system if time constant decreases (increases) in its dynamic elements?

How is it possible to decrease a velocity error within an astatic system?

k1 := 0,0.01 .. 10

$$Kpoз(k1) := \frac{k1}{0.1}$$

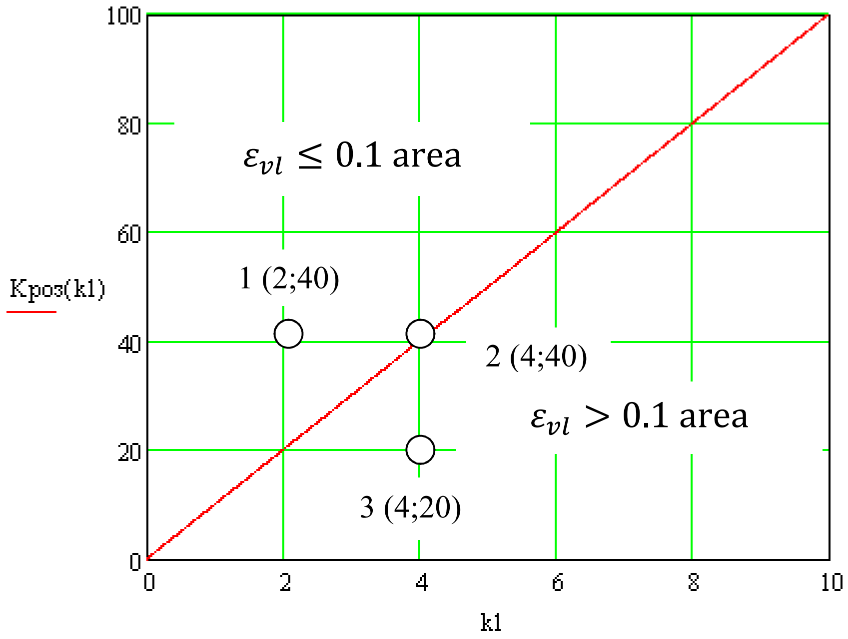


Fig. 3.12 $K_{op}(k_1)$ dependence graph

Table 3.6

Test results concerning velocity error of astatic ACS

Point	1 (2;40)	2 (4;40)	3 (4;20)
Velocity error ε_{vl}	0.03	0.08	0.20

LABORATORY RESEARCH 4

Analyzing series correctors

4.1 **Objective** is to deepen students' knowledge while studying the Chapter "Correctors and their synthesis method".

In the process of the activities, students should be able to:

– master effect of series correctors on the operation of the regulating system;
– identify parameters of series correctors to provide the required quality of the regulating system;
– determine a transient process within the corrected system using a computer.

4.2 Input data th perform the activities are the following:
– structural schemes and numerical parameters of the analyzed systems (Fig. 4.1 and 4.2; Table 4.1);
– the MATLAB and MathCAD application package to simulate the regulating systems, and perform computer-based mathematical calculations.

4.3 Operating procedures
Following order is recommended:
– calculate overall intensification coefficient of an open part of the regulating system to provide the specified static control error and velocity control error;
– connect additional intensification element in series with the elements of an open part of regulating system to achieve the required overall intensification coefficient;
– evaluate stability of the corrected regulating system;
– if the regulating system losses its stability, introduce a series corrector, and calculate its parameters to make the system stable;
– use the SIMULINK MATLAB application package to determine a transient process within the corrected regulating system;
– draw conclusions.

4.4 Methodological explanations

The structural changes in a regulating system, resulting from introduction of additional elements, is one of the methods to achieve the required control quality. There are the four types of such correctors: series; parallel; a corrector in terms of external effect; and multiply feedback.

Series correctors are introduced to a regulating system in series with dynamic elements of the open part. Several types of series correctors are available.

Introduction of derivative of an error is the simplest way to improve efficiency of a transient process. In practice, it can be implemented with the help of inertial differential element which transfer function is as follows $W(p) = k_c p(T_c p + 1)^{-1}$. Time constant T_c should be quite less than the transient time of the regulating system.

Introduction of integral of an error is another way to improve ACS efficiency. Implementation of the series corrector involves integral element which transfer function is as follows $W(p) = k_c p^{-1}$. Introduction of integral of an error makes it possible to increase the ACS astatism as well as accuracy control. However, negative $-\pi/2$ phase is introduced in the regulating system worsening its stability.

Such a series corrector as an *isodromic element,* which transfer function is $W(p) = k_{c1} + k_{c2} p^{-1}$ can help improve the system efficiency without any depreciation in its stability reserve.

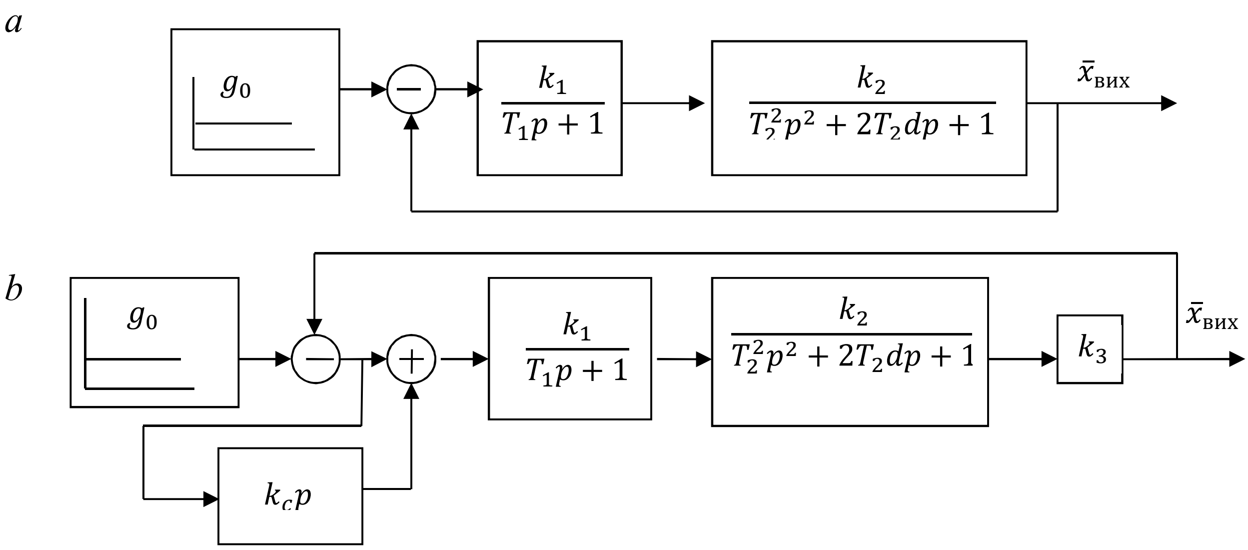


Fig. 4.1 Structural schemes of the analyzed systems:

a – output scheme of the automated control;
b – corrected scheme of the automated control.

Example 4.1. Identify the overall intensification coefficient *K* of the open ACS part (Fig. 4.1, *a*) to provide statistic error E_{st}=0.05. Achieve the system stability while introducing a series corrector of *derivative of error* type to its structure (Fig. 4.1, *b*). The system parameters are as follows: $g_0 = 0.5$; $k_1 = 2$; $k_2 = 1$; $T_1 = 2\ s$; $T_2 = 3\ s$; and $d = 0.4$.

The automated control system in Fig. 4.1 is of static type. If step effect $g_0 = 0.5$ is set to the system input, then error in terms of the stable mode will be:

$$E_{st} = \frac{g_0}{1+K}. \tag{4.1}$$

To provide E_{st} = 0.05, calculate the overall *K* coefficient of the regulating system open part using the formula:

$$K = \frac{g_0 - E_{st}}{E_{st}} = \frac{0.5 - 0.05}{0.05} = 9. \tag{4.2}$$

The intensification coefficient may be achieved when additional intensifying element is introduced in the open part of the ACS. The intensification element is:

$$k_3 = \frac{K}{k_1 k_2} = \frac{9}{1 \times 2} = 4.5. \tag{4.3}$$

Test the stability of the system with the additional intensifying element by means of Hurwitz criterion. Transfer function of the open system will be as follows:

$$W(p) = \frac{k_1}{T_1 p + 1} \times \frac{k_2}{T_2^2 p^2 + 2T_2 dp + 1} \times k_3 =$$

$$= \frac{K}{T_1 T_2^2 p^3 + \left(2T_1 T_2 d + T_2^2\right) p^2 + (T_1 + 2T_2 d)p + 1}. \tag{4.4}$$

Table 4.1

Input data

Variant	The system parameters					
	g_0	k_1	k_2	T, c	d	E_{st}
1	2	1.5	2	0.2	0.4	0.2
2	3	0.5	2	0.2	0.5	0
3	4	4	5	0.4	1.3	0.1
4	6	1	2	0.5	0.8	0
5	9	2.5	3.2	0.7	0.85	0.5
6	5	1	1	0.4	0.7	0
7	4	0.5	6	0.8	0.35	0.5
8	7	4	0.5	0.8	1	0
9	8	5	3	1.2	1.4	0.2
10	2	2	0.5	0.2	0.5	0
11	1	3	1	0.1	0.4	0.1
12	3	0.5	4	0.5	0.8	0
13	8	0.3	50	0.3	1.4	0.2
14	5	4	0.25	0.4	0.7	0
15	2	4	0.75	0.9	0.4	0.2
16	7	0.4	5	0.8	1	0
17	9	80	0.1	0.9	0.85	0.5
18	8	1	1	0.4	0.7	0
19	5	1	1	0.4	0.7	0
20	9	2.5	3.2	0.7	0.85	0.5

Identify the characteristic polynoms:

$$D(p) = T_1 T_2^2 p^3 + (2T_1 T_2 d + T_2^2)p^2 + (T_1 + 2T_2 d)p + (1 + K) =$$

$$= 18p^3 + 13.8p^2 + 4.4\, p + 10. \tag{4.5}$$

Make a matrix of Hurwitz coefficients:

$$\begin{pmatrix} 13.8 & 10 & 0 \\ 18 & 4.4 & 0 \\ 0 & 13.8 & 10 \end{pmatrix}.$$

Define determinants for the matrix of Hurwitz coefficients:

$$\Delta_1 = 13.8 > 0,$$

$$\Delta_2 = 13.8 \times 4.4 - 18 \times 10 = 60.72 - 180 < 0,$$

2nd order determinant is negative. Hence, the system is unstable.

Test the calculation results with the help of simulation. Fig. 4.3 demonstrates structural scheme to simulate the analyzed system using the SIMULINK MATLAB application package. Fig. 4.4 demonstrates the simulation results explaining that output value experiences its continuous time increase oscillating. Such changes correspond to unstable system.

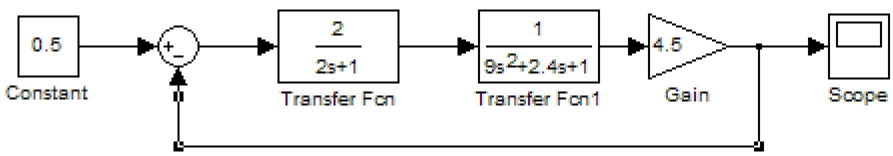


Fig. 4.3 Structural scheme of the analyzed system

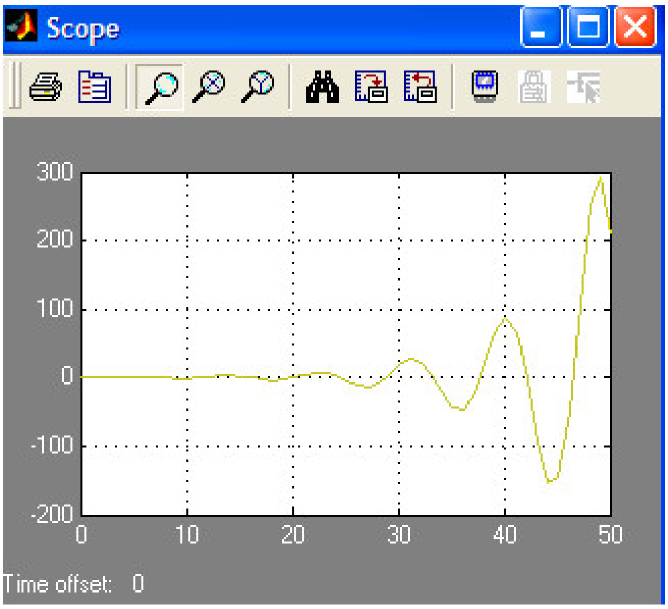


Fig. 4.4 Graph of the transient process

To provide stability of the regulating system, introduce corrector *of error* in series with elements of the open part (see Fig. 4.1, *b*). Then, transfer function of the open part of the system will look like:

$$W(p) = \frac{K \times (1 + k_c p)}{T_1 T_2^2 p^3 + (2T_1 T_2 d + T_2^2)p^2 + (T_1 + 2T_2 d)p + 1}. \quad (4.6)$$

Identify the characteristic polynoms:

$$D(p) = T_1 T_2^2 p^3 + (2T_1 T_2 d + T_2^2)p^2 + (T_1 + 2T_2 d + Kk_c)p + (1 + K) =$$

$$= 18p^3 + 13.8p^2 + (4.4 + 9k_c p) + 10. \quad (4.7)$$

Make a matrix of Hurwitz coefficients:

$$\begin{pmatrix} 13.8 & 10 & 0 \\ 18 & (4.4 + 9k_c) & 0 \\ 0 & 13.8 & 10 \end{pmatrix}.$$

1st order determinant is $\Delta_1 = 13.8 > 0$. To make the regulating system stable, $\Delta_2 > 0, \Delta_3 > 0$ conditions should also be fulfilled. Record a mathematical expression for 2nd order determinant:

$$\Delta_2 = 13.8 \times (4.4 + 9k_c) - 10 \times 18 > 0. \quad (4.8)$$

Define k_c using inequality (4.8)

$$k_c > \frac{10 \times 18 - 13.8 \times 4.4}{13.8 \times 9} \approx 0.96. \quad (4.9)$$

Thus, if coefficient is $k_c > 0.96$, then $\Delta_2 > 0$. The last Δ_3 determinator of the matrix of coefficients is calculated according to the formula:

$$\Delta_3 = \Delta_2 \times (1 + K). \quad (4.10)$$

If $k_c = 3$, then $\Delta_3 = 2533.2 > 0$.

Test the calculation results using simulation. Fig. 4.5 shows structural scheme to simulate the analyzed area in the environment of the SIMULINK MATLAB application package. Taking into consideration the fact that a correcting element (i.e. differential element) with $W(p) = 3p$ is such which cannot be implemented physically, it has been replaced by an actual differential element where time constant is more than two orders of magnitude less than time of a transient process within $W(p) = 3p(0.01p + 1)^{-1}$ system. Such a replacement has no effect on time characteristics of the automated control system.

Figures 4.6 and 4.7 represent the simulation results. It follows from the analysis of a graph concerning error variation (Fig. 4.6) that it experiences its time decrease oscillating and approaching the stable value. Fig. 4.7 shows the stable error value within a segment of the graph of error variation. In terms of the stable mode, the error corresponds to the specified value E_{st}= 0.05.

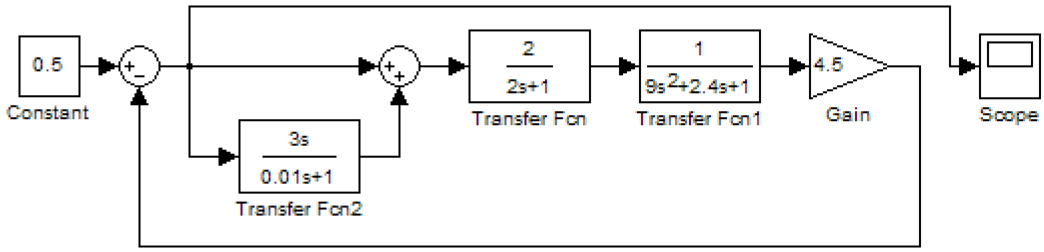


Fig. 4.5 Structural scheme of the analyzed system

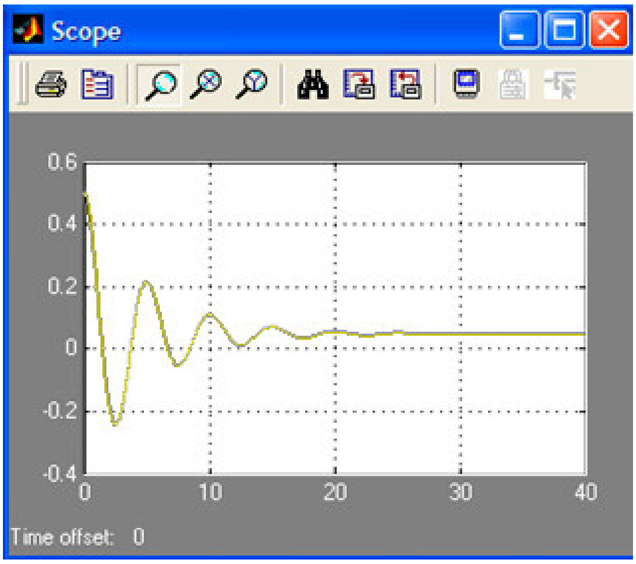


Fig. 4.6 Graph of the error time variation

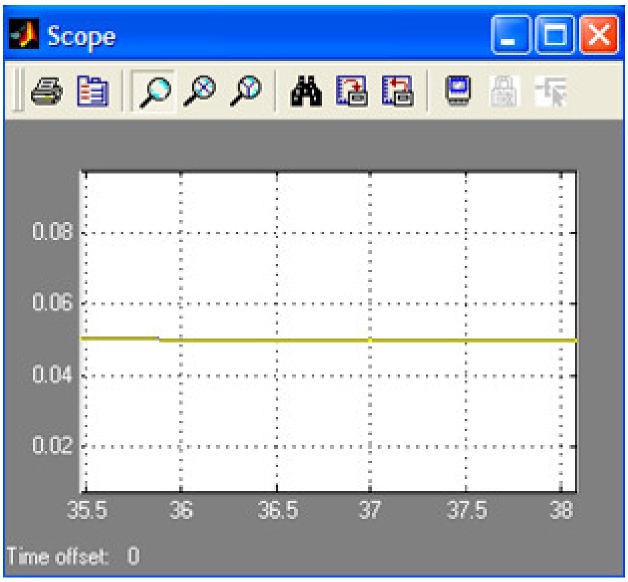


Fig. 4.7 Fragment of the graph of the error time variation

4.5 Report contents.

Structural schemes of the analyzed regulating systems.

Calculation of the parameters of additional elements of the system to provide the specified static error E_{st}.

Graphs of transient processes of both uncorrected and corrected automated systems.

4.6 Control questions

How is it possible to improve RS quality?

What are the additional elements introduced to RS to provide the specified quality indices?

What is series corrector?

What types of series correctors do you know?

What is the effect of a derivative of an error on the operation of the closed automated control system?

What is the result of integrating element introduction to the system?

Why isodromic element is introduced to the regulating system?

LABORATORY RESEARCH 5

Analyzing flexible feedback

5.1 Objective is to deepen students' knowledge while studying the Chapter "Parallel Correctors".

5.2 In the process of the activities, students should be able to:

– master the effect of such parallel corrector as *flexible feedback* on the characteristics of an oscillating element;

– calculate parameters of the parallel corrector as *flexible feedback* to meet the requirements for the quality indices of a transient process within the specified ACS area;

– identify the transient process within the corrected automated control system using a computer.

5.3 Input data th perform the activities are the following:

– structural schemes and numerical parameters of the analyzed systems (Fig. 5.1, Table 5.1);

– application packages MathCAD for mathematical calculations and MATLAB for computer-based simulation of the automated systems.

5.4 Operating procedures

Following order is recommended:

– identify a transfer function from ACS area with flexible feedback;

- identify intensification coefficient set of K_{fb} feedback to provide values of a damping coefficient d^* of a transient process within ACS area in terms of the specified range;
- construct a graph of dependence of intensification coefficient of flexible feedback K_{fb} upon the damping coefficient $d^*$of the transient process;
- test the calculation results using computer-based simulation of the specified ACS area.

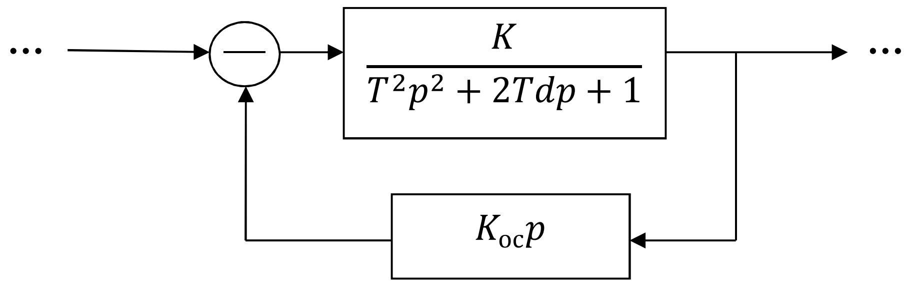


Fig. 5.1 ACS part with the flexible feedback

Input data

Table 5.1

#	K	T	d	d_1	d_2
1	2.2	2.2	0.1	0.2	1.3
2	4.6	2.4	0.2	0.3	1.2
3	1.8	2.6	0.3	0.4	1.4
4	3.4	2.8	0.4	0.5	1.1
5	2.8	3.2	0.1	0.2	1.3
6	4.2	3.4	0.3	0.4	1.1
7	5.6	3.6	0.1	0.2	1.2
8	3.6	3.8	0.2	0.3	1.4
9	1.4	4.2	0.3	0.4	1.1
10	3.2	4.4	0.4	0.5	1.2
11	2.6	4.6	0.5	0.6	1.1
12	5.4	4.8	0.2	0.3	1.2
13	1.4	5.2	0.1	0.3	1.4
14	4.4	5.4	0.2	0.5	1.1
15	3.8	5.6	0.3	0.4	1.2
16	5.2	5.8	0.4	0.6	1.3
17	4.8	6.2	0.5	0.6	1.2
18	1.2	6.4	0.3	0.5	1.4
19	2.4	6.6	0.1	0.5	1.1
20	2.2	6.8	0.2	0.4	1.3

5.4. Methodological explanations. Parallel correctors are implemented in the form of additional local feedbacks. The following belongs to the parallel correctors:

– proportional feedback

$$W_{fb}(p) = k_{fb}; \tag{5.1}$$

– inertial proportional feedback

$$W_{fb}(p) = \frac{k_{fb}}{T_{fb}p + 1}; \tag{5.2}$$

– flexible feedback

$$W_{fb}(p) = k_{fb}p; \tag{5.3}$$

– inertial flexible feedback

$$W_{fb}(p) = \frac{k_{fb}p}{T_{fb}p + 1}\cdot \tag{5.4}$$

where $W_{fb}(p)$ is the transfer function of the parallel corrector; and k_{fb}, T_{fb} are the intensification coefficient and time constant of the corrector respectively.

Generally, feedback is applied to increase oscillating element damping within the corresponding ACS area. In this context, the transfer function of ACS area with oscillating element and flexible feedback will look like:

$$W(p) = \frac{K/(T^2p^2 + 2TTp + 1)}{1 + (k_{fb}Kp/(T^2p^2 + 2Tdp + 1))} = \frac{K}{T^2p^2 + \left(2Td + k_{fb}K\right)p + 1}, \tag{5.5}$$

where K, T, and d are the intensification coefficient, time constant, and damping coefficient of a transient process.

It follows from (5.5) that flexible feedback can vary neither structure of ACS area, nor intensification coefficient K, nor time constant $T$of the oscillating element. However, damping coefficient varies and depends upon the intensification coefficient of feedback k_{fb}. Moreover

$$2Td + k_{fb}K = 2Td^*, \tag{5.6}$$

where d^* is the new damping coefficient.

Use (5.6) to identify k_{fb} for the provision of the specified d^* value:

$$k_{fb} = \frac{2Td^* - 2Td}{K} = \frac{2T(d^* - d)}{K}\cdot \tag{5.7}$$

Example 5.1. Such a parallel corrector as flexible feedback is connected to an area of the automated control system with $W(p) = 5(4p^2 + 0.8p + 1)^{-1}$ transfer function. Determine a set of intensification coefficient of flexible feedback k_{fb} to provide a value of the new damping coefficient $d^*$of a transient process within the specified ACS area in terms of $[0.3; 1.1]$. Construct $k_{fb} = f(d^*)$ dependence graph. Verify the calculation results by means of computer-based simulation.

$W(p) = 5(4p^2 + 0.8p + 1)^{-1}$ transfer function is a transfer function of oscillating elements with $K = 5$, $T = 2$, and $d = 0.2$ parameters.

Use (5.6) to identify a set of intensification coefficient of flexible feedback k_{fb} to provide a value of the new damping coefficient $d^*$of a transient process within the specified ACS area in terms of $[0.3; 1.1]$. Thus,

$$d^* = \frac{2Td + k_{fb}K}{2T} = d + \frac{k_{fb}K}{2T}, \tag{5.8}$$

then

$$0.3 \leq d + \frac{k_{fb}K}{2T} \leq 1.1, \tag{5.8}$$

or

$$0.3 \leq 0.2 + \frac{k_{fb}5}{2 \times 2} \leq 1.1. \tag{5.9}$$

Identical transformations will result in

$$0.08 \leq k_{fb} \leq 0.72. \tag{5.10}$$

Hence, if $k_{fb} = 0.08$ we obtain $d^* = 0.3$; if $k_{fb} = 0.72$ then $d^* = 1.1$.

Construct a dependence graph $k_{fb} = f(d^*)$. For the purpose, solve (5.6) relative to k_{fb}

$$k_{fb} = \frac{2T(d^* - d)}{K} \cdot \tag{5.11}$$

Substitute values of K, T, and d parameters in (5.11)

$$k_{fb} = \frac{2 \times 2(d^* - 0.2)}{5} = \frac{2 \times 2(d^* - 0.2)}{5} = 0.8\, d^* - 0.16. \tag{5.12}$$

Fig. 5.2 represents a dependence graph $k_{fb} = 0.8\, d^* - 0.16$, constructed in the MathCAD environment.

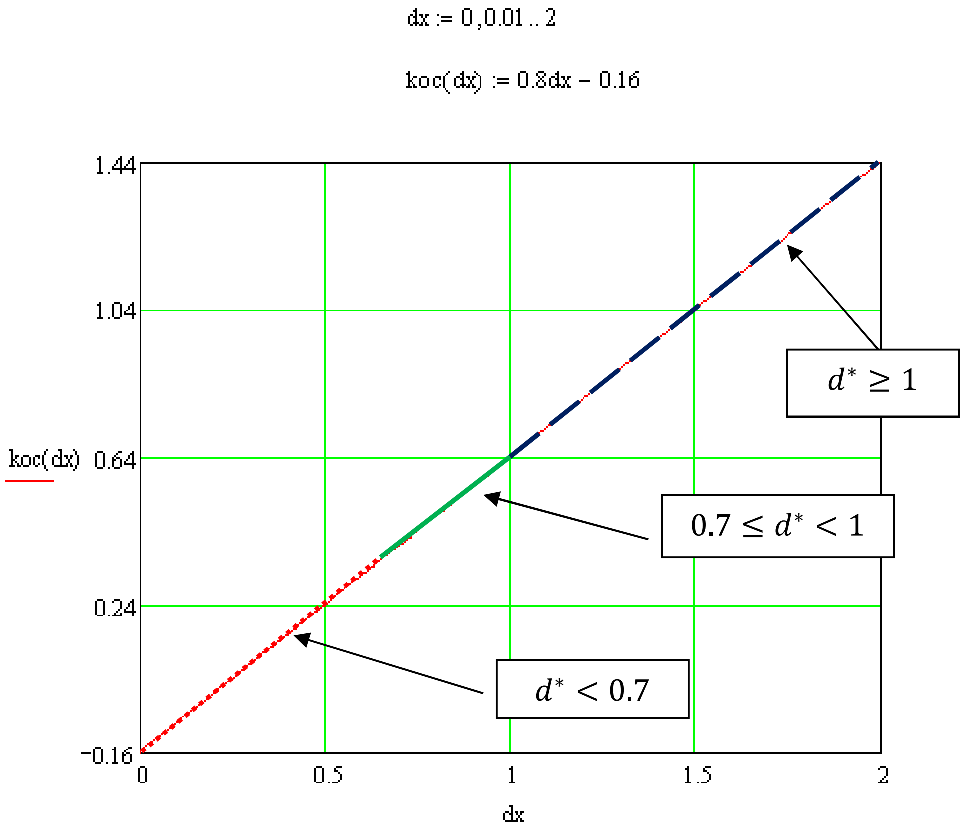


Fig. 5.2 Dependence graph $k_{fb} = f(d^*)$

Three line sections are singled out within the graph:

– $d^* < 07$ being the oscillating transient process;

– $0.7 \leq d^* < 1$ being the readjusted aperiodic transient process;

– $d^* < 0.7$ being the aperiodic transient without readjustment.

Test the calculations using simulation by the SIMULINK MATLAB application package. Fig. 5.3 represents a structural scheme of the analyzed ACS area; Table 5.2 shows parameters of the ACS area.

Fig. 5.4–5.6 demonstrate the simulation results. Graphs of the transient processes correspond to the system parameters obtained as the calculation results.

Data to simulate ACS area

Table 5.2

#	k_{fb}	d^*	Type of a transient process
1	0.08	0.3	Oscillating transient process
2	0.40	0.7	Readjusted transient process
3	0.64	1.0	Transient process without readjustment

Fig. 5.4–5.6 demonstrate the simulation results. Graphs of the transient processes correspond to the system parameters obtained as the calculation results.

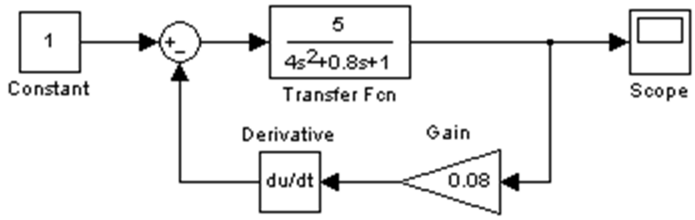


Fig. 5.3 Structural scheme of the analyzed ACS area

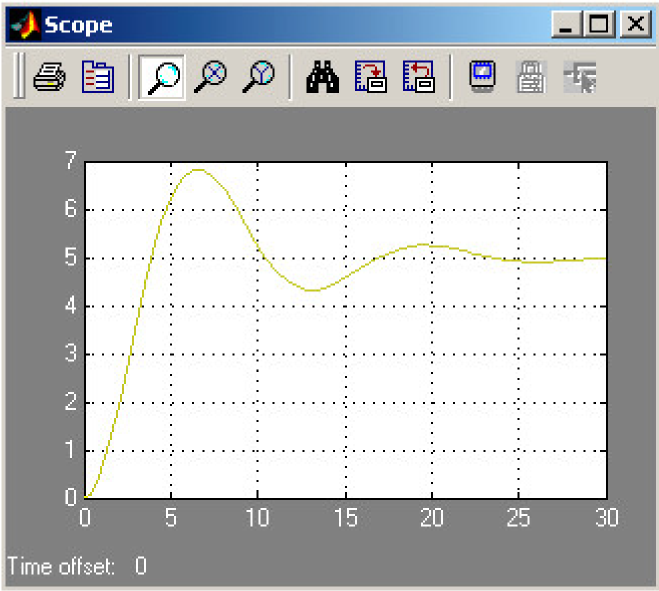


Fig. 5.4 Oscillating transient process

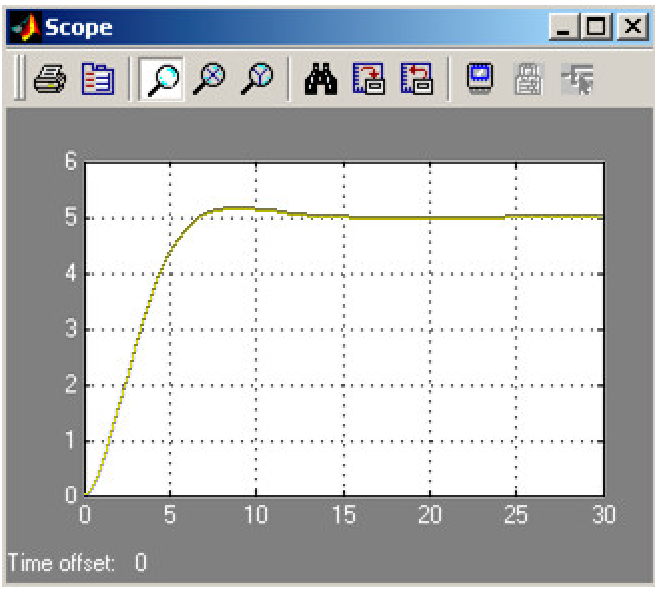


Fig. 5.5 The readjusted transient process

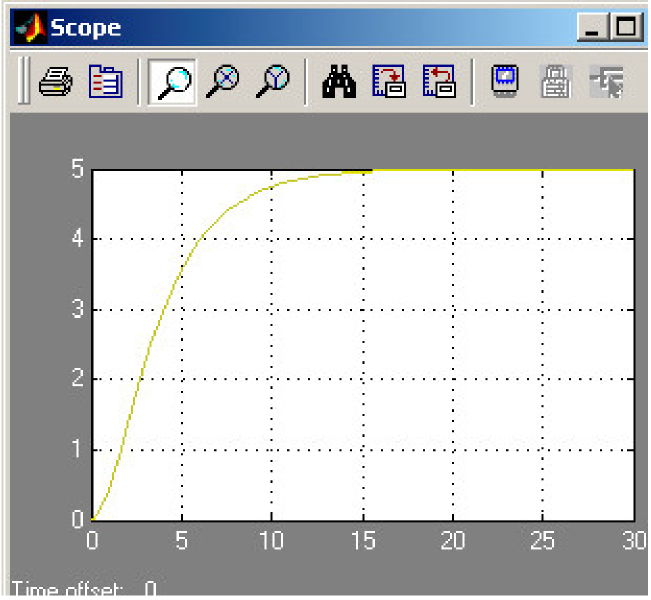


Fig. 5.6 Transient process without readjustment

5.5 Report contents.
Structural scheme and output data of the ACS area.
Calculation parameters of flexible feedback.
Dependence graph $k_{fb} = f(d^*)$.
Structural schemes and simulation results of the ACS area with corrector.

5.6 Control questions.
What types of correctors do you know?
What should be done for a parallel corrector introduction to the ACS?
What types of parallel correctors do you know?
What changes are experienced by the ACS area when flexible feedback surrounds it?
What are the parameters of oscillating element effected by the flexible feedback?
What are the parameters of aperiodic element effected by the flexible feedback?

LABORATORY RESEARCH 6

Analyzing stability of a linear regulating system with delay

6.1 Objective is to deepen students' knowledge while studying the Chapter "Regulating systems with delay".
In the process of the activities, students should be able to:
– calculate critical delay time;
– calculate stability margin of the regulating system;
– acquire practical skills to study the regulating systems using a computer.
6.2 Input data the perform the activities are the following:

– structural scheme, and numerical parameters of dynamic elements of the analyzed system (Fig. 6.1; Table 6.1); and
– the MATLAB application package for computer-based simulation of the regulating systems.

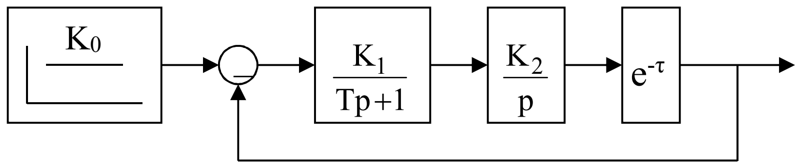


Fig. 6.1 Structural scheme of the analyzed system

6.3 Operatingp rocedures.
Following order is recommended:
– calculate critical delay time;
– calculate stability margin of the regulating system;
– construct a graph for stability margin and unstable operation of the regulating system;
– use the MATLAB application package to evaluate transient processes within the regulating system when the delay time is less and more than critical one;
– use the MATLAB application package to evaluate transient processes within the regulating system for the areas of its stable and unstable operations.

6.4 Methodological explanations
Mathematical models of the majority of production facilities in the mining and processing industry involve a net delay element. Regulating systems of such objects have transcendent characteristic equation; that is why algebraic criteria to determine their stability are unacceptable. At the same time, Mykhailov and Nyquist stability criteria maintain their values.

Mykhailov criterion is convenient to be applied for determination of stability margins and areas of delay systems. Within a stability margin, Mykhailov curve passes through a reference point in such a way that the whole following curve run corresponds to the stability. Hence, in terms of some ω value, it will be $D(j\omega) = 0$:

$$\begin{cases} X(\omega, \cos\tau\omega, \sin\tau\omega) = 0 \\ Y(\omega, \cos\tau\omega, \sin\tau\omega) = 0. \end{cases} \quad (6.1)$$

Equations of (6.1) system determine the stability margin.

Input data

Table 6.1

Variant	K_0	K_1	K_2	T, c	τ, c
1	1	0.5	2.0	0.5	0.4
2	1	0.7	2.2	0.6	0.3
3	1	1.0	1.8	0.7	0.2
4	1	1.2	2.4	0.4	0.3
5	1	1.4	1.6	0.7	0.4
6	1	1.5	2.5	0.6	0.2
7	1	1.3	1.7	0.5	0.3
8	1	1.6	2.1	0.8	0.4
9	1	1.0	1.3	0.9	0.2
10	1	0.9	1.9	0.6	0.3
11	1	0.8	2.2	1.0	0.4
12	1	0.6	2.3	1.2	0.4
13	1	0.9	2.0	1.3	0.3
14	1	1.2	1.3	1.1	0.2
15	1	1.4	1.6	1.0	0.3
16	1	1.5	1.5	0.9	0.2
17	1	1.3	1.4	0.8	0.4
18	1	1.2	1.0	1.2	0.4

Example 6.1. Identify stability margin of the closed regulating system with delay in terms of $[T; K]$ coordinates where transfer function of the open part is $W(p) =$
$= Ke^{-\tau p}(p(Tp+1))^{-1}$. Assume that $\tau = 3$.

First, solve the problem in general terms. Make characteristic polynoms of the closed regulating system

$$D(p) = Tp^2 + p + Ke^{-\tau p}. \qquad (6.2)$$

Record a characteristic vector

$$D(j\omega) = -T\omega^2 + j\omega + Ke^{-\tau j\omega}, \qquad (6.3)$$

or

$$D(j\omega) = -T\omega^2 + j\omega + K(\cos\tau\omega - j\sin\tau\omega). \qquad (6.4)$$

Single out actual part and imaginary part in (6.4) and record an equation of the regulating system stability

$$\begin{cases} X(\omega) = -T\omega^2 + K\cos\tau\omega = 0 \\ Y(\omega) = \omega - K\sin\tau\omega = 0. \end{cases} \qquad (6.5)$$

We have from level two of the system (6.5)

$$K = \frac{\omega}{\sin\tau\omega}. \qquad (6.6)$$

Identify T using equation one of the system

$$T\omega^2 = K\cos\tau\omega, \tag{6.7}$$

$$T = \frac{K\cos\tau\omega}{\omega^2}\cdot \tag{6.8}$$

After substituting (6.6) into (6.8), we will obtain

$$T = \frac{\cos\tau\omega}{\omega^2}\cdot\frac{\omega}{\sin\tau\omega}\cdot \tag{6.9}$$

$$T = \frac{\cos\tau\omega}{\omega\sin\tau\omega} = \frac{1}{\omega \mathrm{tg}\tau\omega}\cdot \tag{6.10}$$

To construct a graph, determine ω variation range using $K > 0$, and $T > 0$ conditions. We have

$$\begin{cases} \omega(\sin^{-1}\tau\omega) > 0 \\ (\omega \mathrm{tg}\tau\omega)^{-1} > 0. \end{cases} \tag{6.11}$$

Since $\omega > 0$, then

$$\begin{cases} \sin\tau\omega > 0 \\ \mathrm{tg}\tau\omega > 0. \end{cases} \tag{6.12}$$

Simultaneously, expressions within left sides of the inequalities (6.12) are more than zero if $0 < \tau\omega < \pi/2$. Thus, $0 < \omega < \pi/(2\,\tau)$.

Use (6.6) and (6.10) equations to construct stability margin for the specified $\tau =$ $= 3$ value within the coordinate plane $[T; K]$ varying ω from 0 to $\pi/6$ (Fig. 6.2).

Apply the graph data to identify coordinates of unstable operation of the system [2; 0.5], stable operation of the system [2; 0.35], and the system operation within its stability margin [2; 0.41]. Transient processes, represented in Fig. 6.3–6.5 correspond to the coordinates.

Nyquist criterion is applicable to identify critical delay time τ_{cr} of a control system. Delay time, in terms of which the control system is within its stability margin, is critical one.

According to a Nyquist stability criterion, the closed automated control system will be within its stability margin if amplitude and phase frequency response of its open part passes through a point within the complex plane with $[-1; j0]$ coordinates in terms of some ω_0 frequency. Then, if $\omega = \omega_0$, a modular unit of the complex intensification coefficient of the open ACS part will be equal to:

$$A(\omega_0) = 1. \tag{6.13}$$

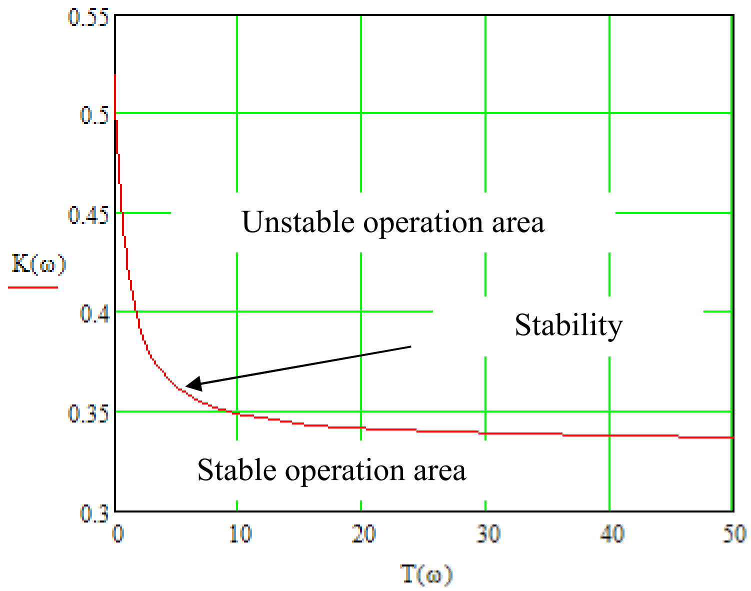


Fig. 6.2 Graph of stability margin

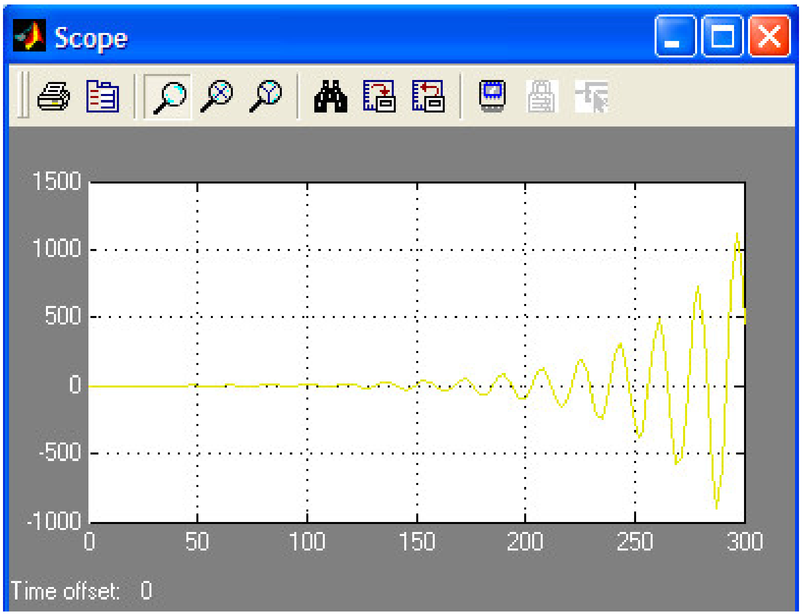


Fig. 6.3 Graph of a transient process of an unstable system

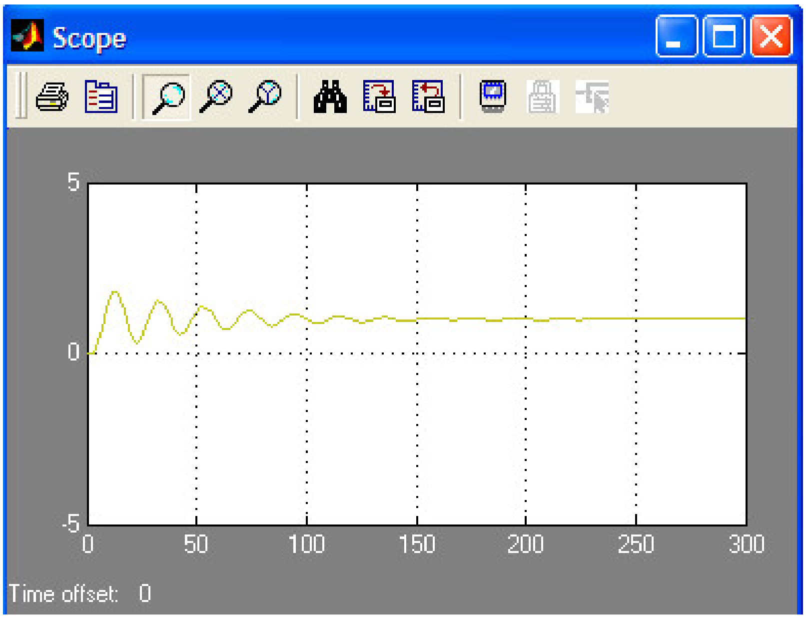


Fig. 6.4 Graph of a transient process of a stable system

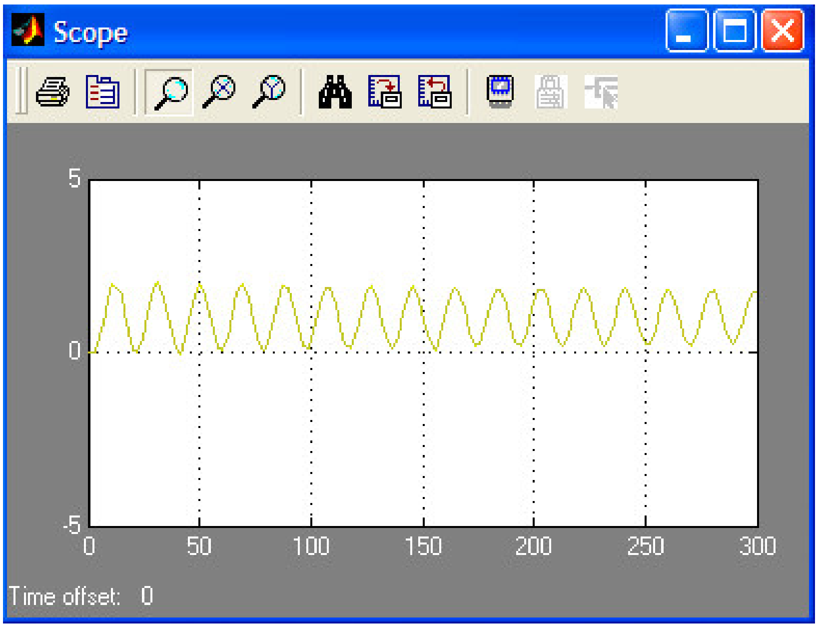


Fig. 6.5 Graph of a transient process of a system within its stability margin

In terms of the similar frequency, the phase will be:

$$\varphi(\omega_0) = -\pi. \tag{6.14}$$

Net delay element cannot effect on a modular unit of the complex intensification coefficient of the open ACS part. However, it varies its phase according to

$$\varphi(\omega) = \varphi_0(\omega) - \tau\omega. \tag{6.15}$$

Then if $\omega = \omega_0$, we have

$$\begin{cases} A_0(\omega_0) = 1 \\ \varphi_0(\omega_0) - \tau_{cr}\omega_0 = -\pi, \end{cases} \tag{6.16}$$

where $A_0(\omega_0)$ and $\varphi_0(\omega_0)$ are modular unit and a phase of the complex intensification coefficient of the open part of the automated control system respectively in terms of $\omega = \omega_0$ frequency exclusive of the net delay element.

Equation two of (6.16) system helps obtain

$$\tau_{cr} = \frac{\pi + \varphi_0(\omega_0)}{\omega_0}. \tag{6.17}$$

Example 6.2. Determine critical delay time τ_{cr} of the closed ACS which open part has $W(p) = K/((Tp+1)\, e^{-\tau p})$ transfer function. Values of the parameters are $K = 2$ and $T = 3$.

First, solve the problem in general terms. Open ACS system consists of aperiodic element and net delay element connected in series. Dependence of modular unit of the complex intensification coefficient of aperiodic element upon frequency is as follows

$$A_0(\omega) = \frac{K}{\sqrt{T^2\omega^2 + 1}}. \tag{6.18}$$

Determine ω_0 for which $A_0(\omega_0) = 1$. Thus, we have

$$\frac{K}{\sqrt{T^2\omega_0^2 + 1}} = 1. \tag{6.19}$$

After identical transformation, we will obtain

$$\omega_0 = \frac{\sqrt{K^2 - 1}}{T}. \tag{6.20}$$

Dependence of the complex intensification coefficient phase of aperiodic element upon frequency is

$$\varphi_0(\omega) = -\mathrm{arctg}(T\omega). \tag{6.21}$$

Use formula (8.17) to identify critical delay time

$$\tau_{cr} = \frac{\pi + \varphi_0(\omega_0)}{\omega_0} = \frac{T(\pi - \text{arctg}(\sqrt{K^2 - 1}))}{\sqrt{K^2 - 1}}. \qquad (6.22)$$

Hence,

$$\tau_{cr} = \frac{3(\pi - \text{arctg}(\sqrt{2^2 - 1}))}{\sqrt{2^2 - 1}} = 4.08. \qquad (6.23)$$

Fig. 6.6 represents simulation scheme of the unknown ACS; Fig. 6.7 represents the transient process.

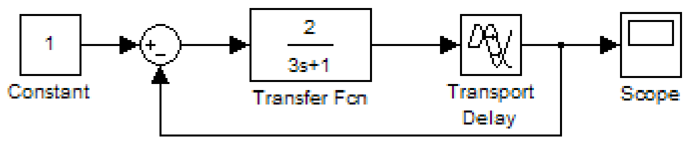


Fig. 6.6 Block diagram of the closed ACS with delay

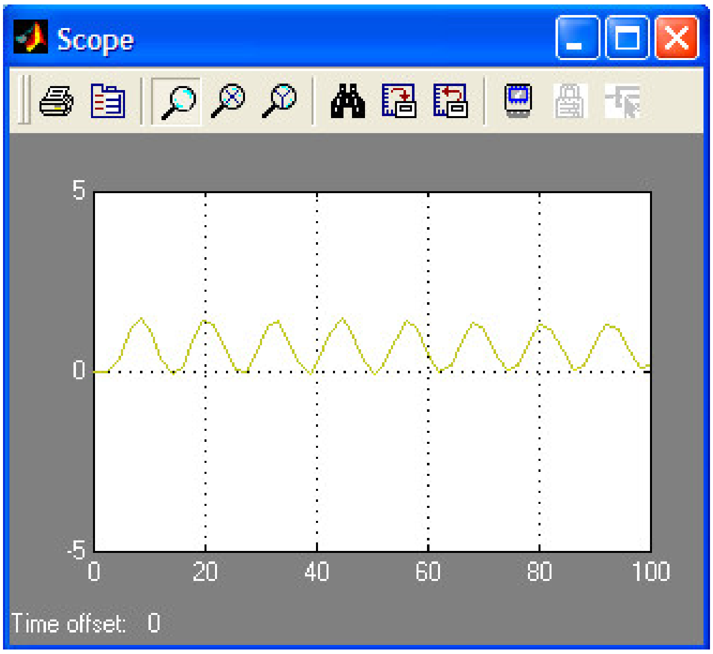


Fig. 6.7 Transient process within the ACS

6.5 Report contents

Structural scheme of the analyzed automated control system and output data.

Calculations of the critical delay time and boundary areas of stable and unstable operations of the regulating system.

Graph of the areas of stable nd unstable operations of the regulating system.

Graph of transient processes for delay time being less than critical and more than it.

Graphs of transient processes for the parameters of the regulating system concerning stable area and unstable one.

6.6 Control questions

What are the regulating systems with delay?

What is net delay element?

List the ways to connect net delay element to the regulating system.

Record differential equation of a positional delay element.

What are the methods applied to evaluate stability of the regulating system with delay?

What is the critical delay time?

How is it possible to identify stable and unstable operation areas of the regulating system?

LABORATORY RESEARCH 7

Analyzing FCS correctors with a linear delay object

7.1 Objective is to deepen students' knowledge while studying the Chapter "Systems of the automated control with delay".

In the process of the activities, students should be able to:

- calculate critical intensification coefficient, and oscillation period of output value of FCS with PID-controller;
- calculate parameters of PID-controller using Ziegler–Nichols method;
- calculate critical intensification coefficient nd oscillation period of output value of FCS with predicative PID-controller;
- calculate parameters of predicative PID-controller using Ziegler–Nichols method;
- compare the controlling FCS quality with PID-controller and predicative PID-controller;
- acquire practical skills to analyze the automated control systems using a computer.

7.2 Input data th perform the activities are the following:

- structural schemes and numerical parameters of dynamic elements of the analyzed automated control systems (Figures 7.1-7.2; Table 7.1); and
- the MathCAD and MATLAB application package for computer-based calculation and simulation of the automated systems.

7.3 Operating procedures

Following order is recommended:

- expand a net delay element into 2nd order Pade series;
- ealculate a critical intensification coefficient of the open FCS part with PID-controller in terms of Hurwitz criterion;

– calculate the oscillation period of the FCS output value of the output value of FCS with PID-controller;
– calculate PID-controller parameters using Ziegler–Nichols method;
– specify PID-controller settings of FCS analogue within the SIMULINK MATLAB environment;
– use Nyquist criterion to formulate an equation to identify frequencies of stable oscillations of output value in FCS with predicative PID-controller;
– use the MathLAB application package to solve a transcendent equation as for the oscillation frequency;
– calculate the critical intensification coefficient of a predicative PID-controller;
– calculate predicative PID-controller parameters using Ziegler–Nichols method;
– specify settings of predicative PID-controller using FCS analogue within the SIMULINK MATLAB environment;
– use the SIMULINK MATLAB environment to develop transient processes within FCS with PID-controller and predicative PID-controller;
– compare controlling quality of FCS with PID-controller and predicative PID-controller.

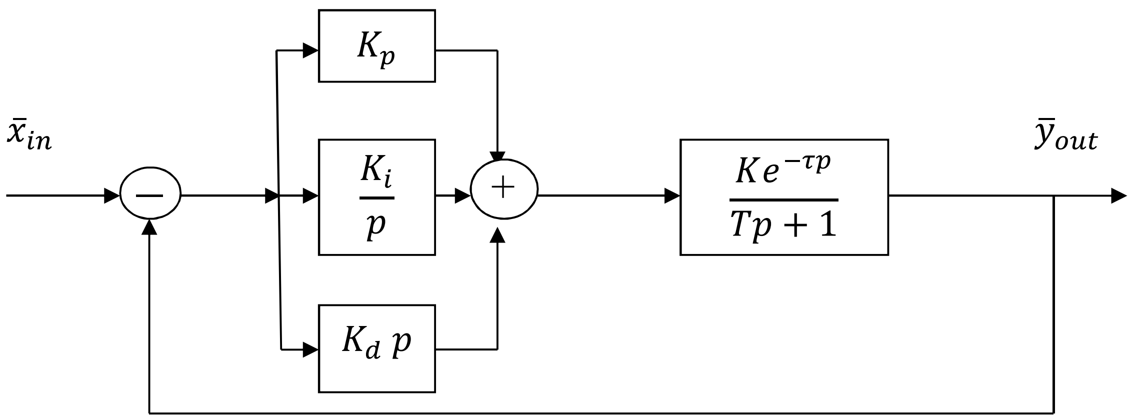


Fig. 7.1 Structural scheme of the analyzed FCS with PID-controller

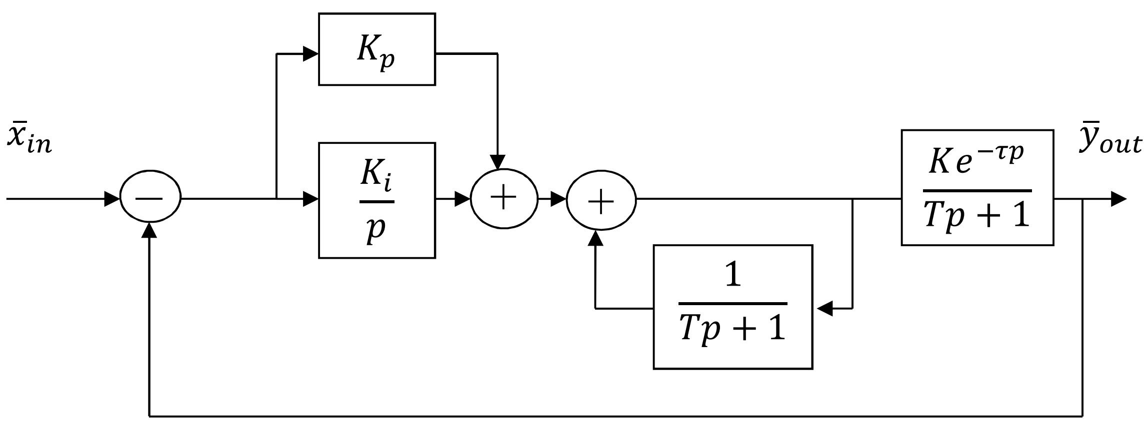


Fig. 7.2 Structural scheme of the analyzed FCS with predicative PID-controller

Input data

Table 7.1

Variant	K	T	τ
1	.	2.4	1,44
2	1.12	3.6	2.16
3	0.45	4.8	2.88
4	3.15	6.6	3.96
5	2.84	7.2	4.32
6	1.52	1.4	0.84
7	0.16	2.6	1.56
8	3.88	3.6	2.16
9	2.14	4.2	2.52
10	2.76	1.8	1.08
11	3.42	7.4	4.44
12	0.92	6.2	3.72
13	2.56	5.8	3.48
14	1.64	4.4	2.64
15	0.78	3.2	1.92
16	2.32	2.6	1.56
17	3.72	1.8	1.08
18	3.28	2.4	1.44
19	0.58	3.2	1.92
20	1.36	7.6	4.56

7.4 Methodological explanations

Numerous scientific papers concern the problem of delay object control. The increased interest in the control of such objects is quite justifiable since the availability of net delay within a control loop complicates heavily generation of the efficient ACSs. Among the current methods of delay object control, the following can be singled out:

1. relay control;
2. PID-control;
3. the predicted control;
4. the specific class of controllers using algorithms.

The specific controller is selected relying upon $\frac{\tau}{t_{tr}}$ ratio (Fig. 7.3) where τ is the net delay time; and t_{tr} is the transient process time.

Relay correctors are applied if objects with minor delay are controlled. Two position controller is preferable owing to its simple adjustment and operation. However, decrease in hysteresis of the corrector intended to improve its control efficiency results in the increased switching frequency of actuating device. The latter factors into origination of output value variation as well as into a short life of commutation components.

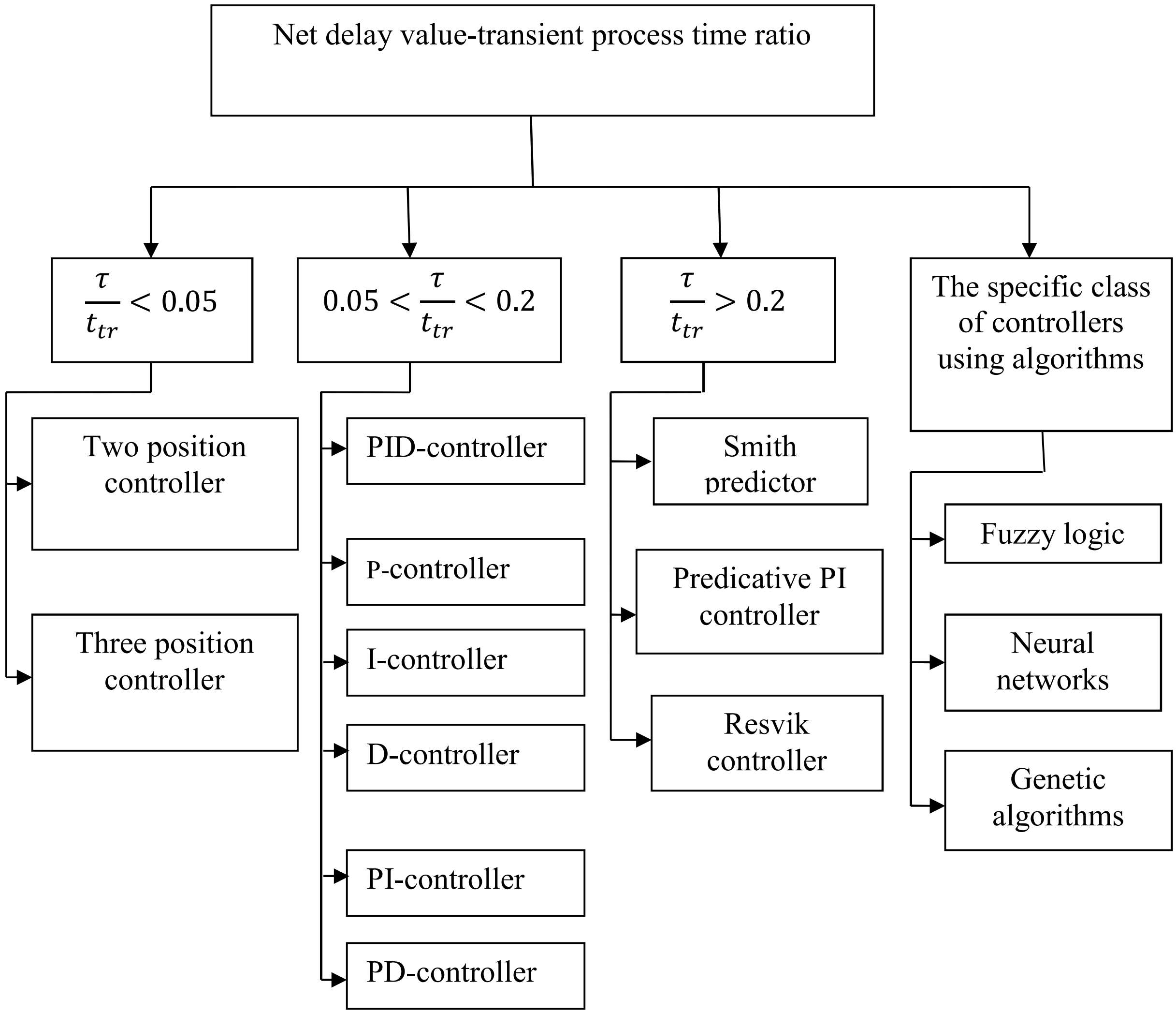


Fig. 7.3 Controller selection in terms of $\frac{\tau}{t_{tr}}$ ratio

As for the three positional correctors, their actuator may be either in a full open position or in a normal (central) position or totally closed. To compare with a two positional controller, the abovementioned one has quicker response to controlling effect; its accuracy and performance are higher.

PID-controller (i.e. proportional-integral-derivative controller) shapes a control signal $u(t)$ in terms of the law:

$$u(t) = K_p\varepsilon(t) + K_i \int_0^t \varepsilon(t)dt + K_d \frac{\varepsilon(t)}{dt}, \tag{7.1}$$

where $\varepsilon(t)$ is a control error.

Fig. 7.4 shows structural scheme of PID-controller.

PID controller involves three components – proportional P, integral I, and derivative D.

P-control is one of the simplest and most popular control laws. Output signal is proportional to a control error. Adjustment simplicity, lack of inertia, and high response are the P-controller advantages. Availability of static error, due to which output value cannot stabilize within the specified value, is its disadvantage.

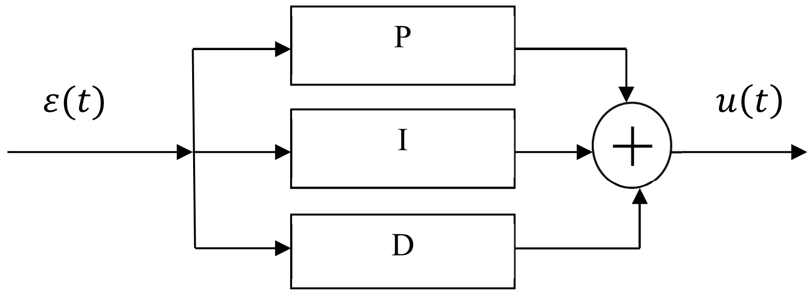


Fig. 7.4 Structural scheme of PID controller

To eliminate the static error, P-controller is added by integral component (i.e. I controller) being proportional temporally to an integral before the output value deviates from its specified value. If no external and external disturbance is available, the output value stabilizes at its specified value.

Nonavailability of control error in terms of the stable state is the I-controller advantage; slow response and potential self-oscillations if K_i parameter has been selected incorrectly.

Derivative element (i.e. D-controller) is intended to forecast future deviations and counteract them. D-controller accelerates response of the automated control system; however, that results in the significant readjustment, and stability conditions experience their worsening.

Depending upon combinations of P-, I-, and D-components, following variations of the controllers are possible – PI, PD, and PID. PID-controller and its modifications are connected in series with the ACS open part (Fig.7.1).

There are several methods to adjust PID-controller parameters; Ziegler-Nichols method is the most popular among them. It belongs to empiric techniques and relies upon the experimental data obtained in terms of a real object.

The adjustment procedure begins with the experimental analysis of a system involving a proportional controller (i.e. P-controller) and the specified control object. Starting from zero, intensification coefficient K_p of the P-controller increases until constant amplitude oscillations are set within a system output; i.e. until the system turns out to be at its stability margin. The controller coefficient, in terms of which a system achieves its stability margin, is recorder and specified through K_p^*. Then, T^* period of oscillations, being stable within the system, is measured.

Parameter values of the selected controller type are calculated using formulas listed in Table 7.2.

Parameters of generic controllers

Table 7.2

	K_p	K_i	K_d
P-controller	$0.50K_p^*$	–	–
PI-controller	$0.45K_p^*$	$0.54K_p^*/T^*$	–
PID-controller	$0.60K_p^*$	$1.20K_p^*/T^*$	$0.075K_p^*T^*$

To control the objects with significant transport delay ($\tau/t_{tr} \geq 0.2 \ldots 0.5$), the specific structures of PID-controllers are used since they involve blocks forecasting the object behaviour after τ time. The controller structure was proposed by Smith in 1957; it is called Smith predictor.

Purpose of Smith predictor is to forecast what signal should occur at the object output before its actual springing. The forecasting may involve a model of a control object consisting of a fine-rational part of the transfer function W_o, transport delay $e^{\tau p}$. Owing to the fact that the delay may be excluded from the model, a principal possibility emerges to forecast the object behaviour before the output signal occurs.

Fig. 7.4 demonstrates one of the potential implementations of such a system. Its operating schedule is as follows.

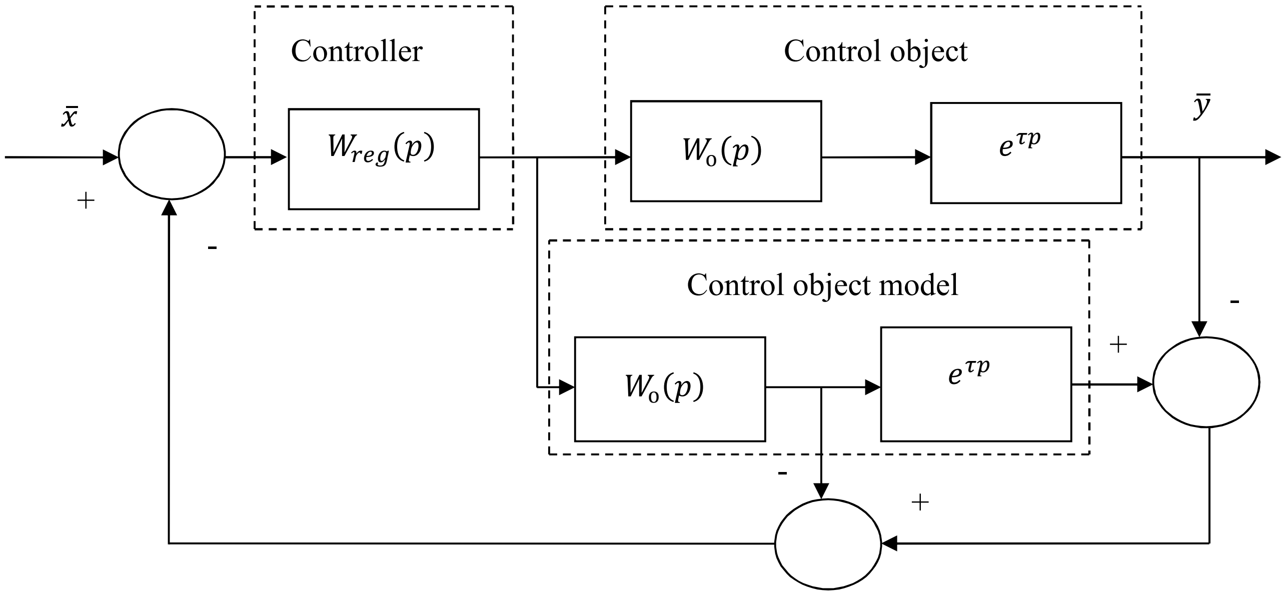


Fig. 7.4 Control system with Smith predictor

If the control object model (i.e. Smith predictor) is not available, the transfer ACS function with PID-controller and transport delay element within a forward loop closed by means of single feedback will look like (Fig. 7.4)

$$W_{cl}(p) = \frac{W_{reg}(p)W_o(p)e^{\tau p}}{1 + W_{reg}(p)W_o(p)e^{\tau p}}, \tag{7.2}$$

where $W_{рег}(p)$ is the transfer controller function.

If Smith predictor is connected to a system as an internal loop, then difference in signals within the object output and model is equal to zero. Hence, transfer function of the closed system will be as follows

$$W_{cl}(p) = \frac{W_{reg}(p)W_o(p)e^{\tau p}}{1 + W_{reg}(p)W_o(p)}. \tag{7.3}$$

To compare with the characteristic polynom (7.2), (7.3) one does not depend upon a transport delay. It means that the transport delay element cannot effect the system velocity and response.

Topological transformations of structural schemes may help obtain numerous mutually equivalent systems with Smith predictor.

Predictive PI-controller (PPI-controller), and Resvik controller can be considered as Smith predictor varieties. Fig. 7.2 represents structural scheme of the PPI-controller.

Fuzzy controlling algorithms are not advantageous to compare with the classic control methods. Moreover, a problem of stability in the context of a system with nonlinear control algorithms complicates heavily.

Example 7.1. Use Ziegler-Nichols to identify adjustments of PID-controller of the automated control system (Fig. 7.1) while controlling an object with $W(p) =$
$= 0.015e^{-2p}(7.3p + 1)^{-1}$ transfer function. Specify the calculated adjustments and determine experimentally quality indices of a transient process using FCS in terms of the SIMULINK MATLAB environment.

Represent the transfer function of an open share of the automated control system with in-series connected control object and P-controller (ignore temporarily both integral and differential parts of a control law) as follows

$$W(p) = \frac{K_p K}{Tp + 1} e^{-p\tau}. \tag{7.4}$$

Expand a net delay element into 2nd order Padè series

$$e^{-p\tau} \approx \frac{\tau^2 p^2 - 6\tau p + 12}{\tau^2 p^2 + 6\tau p + 12} = \frac{4p^2 - 12p + 12}{4p^2 + 12p + 12}. \tag{7.5}$$

Then, the transfer function of the open AUC part will look like

$$W_{op} = \frac{K_p 0.015(4p^2 - 12p + 12)}{(7.3p + 1)(4p^2 + 12p + 12)}, \tag{7.6}$$

where K_p is the proportionality constant of P-controller.

We have 3rd order ACS. Identify $K_p = K_p^*$ proportionality constant, in terms of which the automated control system achieves its stability margin. Make a characteristic polynom for the closed system:

$$D_{cl}(p) = (7.3p + 1)(4p^2 + 12p + 12) + K_p^* 0.015(4p^2 - 12p + 12). \tag{7.7}$$

Perform identical transformations:

$$\begin{aligned} D_{cl}(p) &= 29.2p^3 + 91.6p^2 + 99.6p + 12 + 0.06K_p^* p^2 - \\ &- 0.18K_p^* p + 0.18K_p^* = 29.2p^3 + (91.6 + 0.06K_p^*)p^2 + \\ &+ (99.6 - 0.18K_p^*)p + (12 + 0.18K_p^*). \end{aligned} \tag{7.8}$$

The 3rd order automated control system will be within its stability margin if 1st order determinant of Hurwitz coefficient matrix is more than zero and 2nd order determinant is equal to zero. Make Hurwitz coefficient matrix

$$\begin{pmatrix} (91.6+0.06K_p^*) & (12+0.18K_p^*) & 0 \\ 29.2 & (99.6-0.18K_p^*) & 0 \\ 0 & (91.6+0.06K_p^*) & (12+0.18K_p^*) \end{pmatrix}.$$

1st order determinant will be

$$\Delta_1 = 91.6 + 0.06K_p^* > 0. \quad (7.9)$$

Obviously, $\Delta_1 > 0$ if

$$K_p^* > -1527. \quad (7.10)$$

2nd order determinant will be equal to

$$\Delta_2 = (91.6+0.06K_p^*) \times (99.6-0.18K_p^*) - 29.2 \times (12+0.18K_p^*) = 0. \quad (7.11)$$

Expand the brackets and solve an equation relative to K_p^*. We obtain:

$$9123.36 - 16.49K_p^* + 5.98K_p^* - 0.01{K_p^*}^2 - 350.4 - 5.26K_p^* = 0 \quad (7.12)$$

$$-0.01{K_p^*}^2 - 15.77K_p^* - 8772.96 = 0,$$

$${K_п^*}^2 + 15.77K_п^* + 8772.96 = 0. \quad (7.13)$$

Identify the discriminant, and determine K_p^*:

$$D = 1577^2 - 4 \times 1 \times (-877296) = 5996113.$$

$$\sqrt{D} = 24448.7.$$

$$(K_п^*)_{1.2} = \frac{-1577 \pm 2448{,}7}{2} = [-2027.5; 435.85].$$

Taking into consideration (7.10), we have $K_p^* = 435.85$.

Define oscillation period T^* within the stability margin. For the purpose, calculate oscillation frequency ω^* applying $K_p^* = 435.85$ to (7.8) expression and substitute $j\omega$ for p. Hence, we derive a characteristic complex of the closed system:

$$D_{cl}(j\omega) = 29.2(j\omega)^3 + (91.6 + 0.06 \times 435.85) \times (j\omega)^2 +$$
$$+(99.6 - 0.18 \times 435.84) \times j\omega + (12 + 0.18 \times 435.85) = 0. \quad (7.14)$$

A characteristic complex will be equal to zero if its actual and imaginary parts are equal to zero. Identify the imaginary part

$$-29.2{\omega^*}^3 + (99.6 - 0.18 \times 435.85) \times \omega^* = 0. \quad (7.15)$$

Solve (2.13) equation relative to ω^*:

$$-29.2\omega^{*3} + (99.6 - 0.18 \times 435.85) \times \omega^{*} = 0,$$
$$-29.2\omega^{*3} + 21.5 \times \omega^{*} = 0,$$
$$-29.2\omega^{*2} + 21.5 = 0,$$
$$-29.2\omega^{*2} = -21.5,$$
$$\omega^{*2} = 0.72,$$
$$\omega^{*} = 0.85,$$

Determine T^{*}:

$$T^{*} = \frac{2\pi}{\omega^{*}} = \frac{2 \times 3.14}{0.85}.$$

Test the validity of the calculations using simulation in terms of the SIMULINK MAMLAB environment by means of a structural scheme in Fig. 7.5.

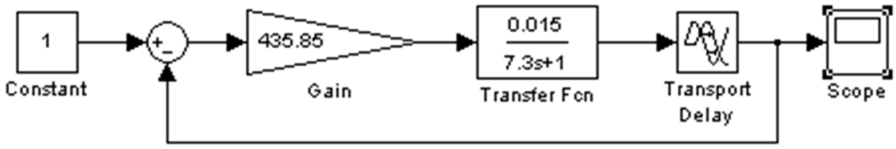


Fig. 7.5 ACS structural scheme for simulation

Fig. 7.6 demonstrates general form of a transient process within the ACS; Fig. 7.7 shows a part of the transient process to test oscillation period T^{*}.

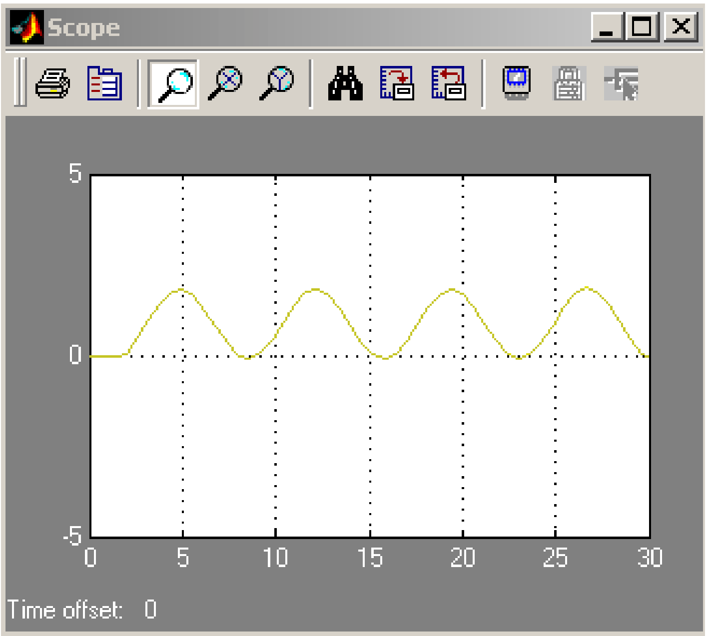


Fig. 7.6 General form of a transient process within the ACS

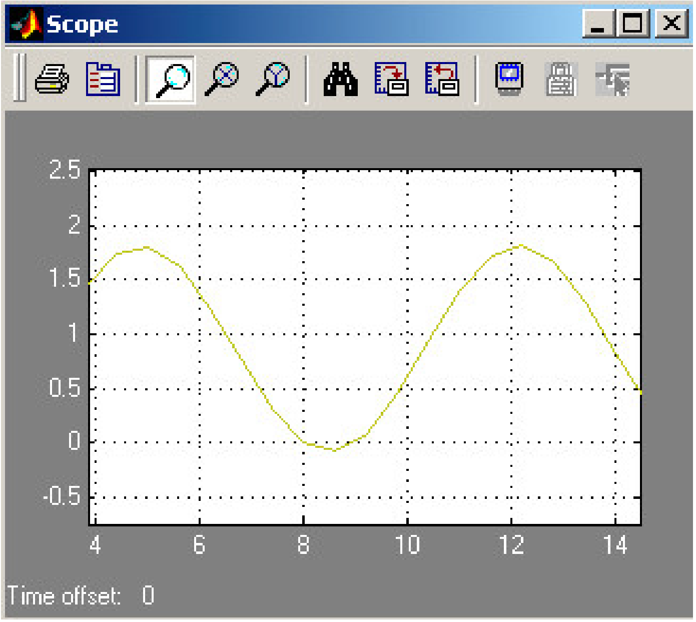


Fig. 7.7 A part of the transient process

The simulation results coincide completely with the calculations.

Table 7.3 represents th values of parameters of correctors calculated with the consideration of $T^* \approx 7.3\ s$ and $K_{p1}^* \approx 435.85$.

Parameters of generic correctors

Table 7.3

	K_p	K_i	K_d	Transfer function
P-controller	217.93	–	–	$W_p(p) = 217.93$
PI-controller	196.13	32.24	–	$W_{pi}(p) = 196.13 + \frac{32.24}{p}$
PID-controller	261.51	71.65	238.63	$W_{pid}(p) = 261.51 + + \frac{71.65}{p} + 238.63p$

To analyze ACS with PID-controllers in terms of the SIMULINK MATLAB environment, a model of the automated control system has been developed; Fig. 7.8 shows its structural scheme. Flotation process model is represented by the in-series connected *Transfer Fon* and *Transport Delay* blocks. PID-controller simulation is implemented using combination of its three parts: P-controller (i.e the *Slider Gain 1* blocks); PI-controller (i.e. the *Slider Gain 2*, and *Integrator* blocks connected in series); and PD-controller (i.e. the *Slider Gain 2* and *Derivative* blocks connected in series. Reference-input signal is introduced to the ACS by means of the *Step* block (i.e. step effect). In addition, FCS structural scheme also includes *Manual Switches* to change the structural scheme operatively according to the problem being solved during the analysis. The *Ground* blocks are used to set zero signals to the required blocks. The *Scope* block is applied to represent time variation of the output value.

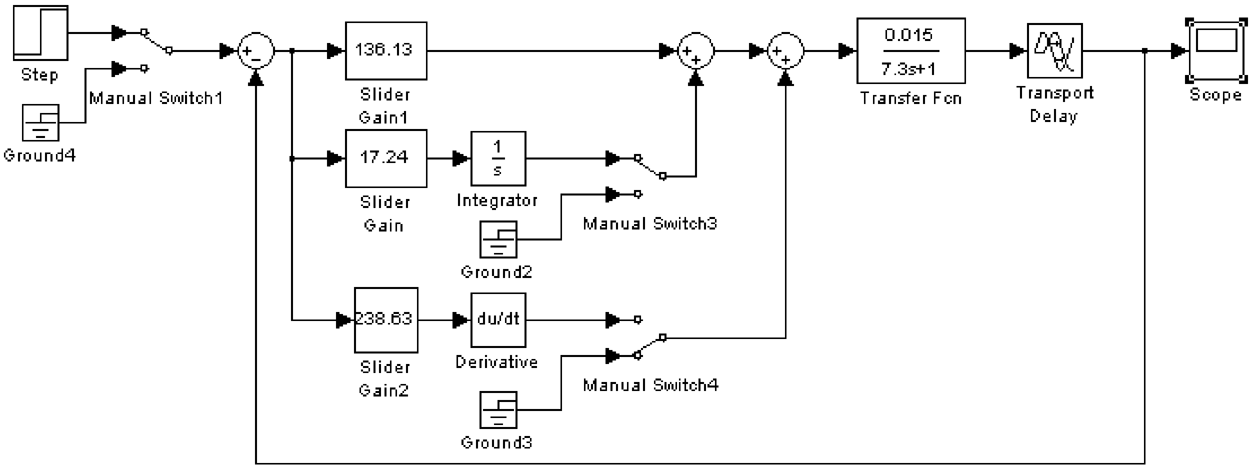


Fig. 7.8 FCS model with PID-controller

According to the simulation results, concerning FCS operation with set points, shown in Table 7.3, the transient processes, demonstrated in Figures 7.9-7.11 have been obtained. Fig. 7.12 illustrates a transient process where PID-controller was not used; Fig.7.13 illustrates a transient process with the best controller adjustment selected experimentally using a model

$$W_{reg}(p) = 136.13 + \frac{17.24}{p} \cdot \tag{7.16}$$

Controlling time was determined with the help of the time after which difference between the controlled value and the stable value is not more than 5%. Stability degree has been identified using the known formula

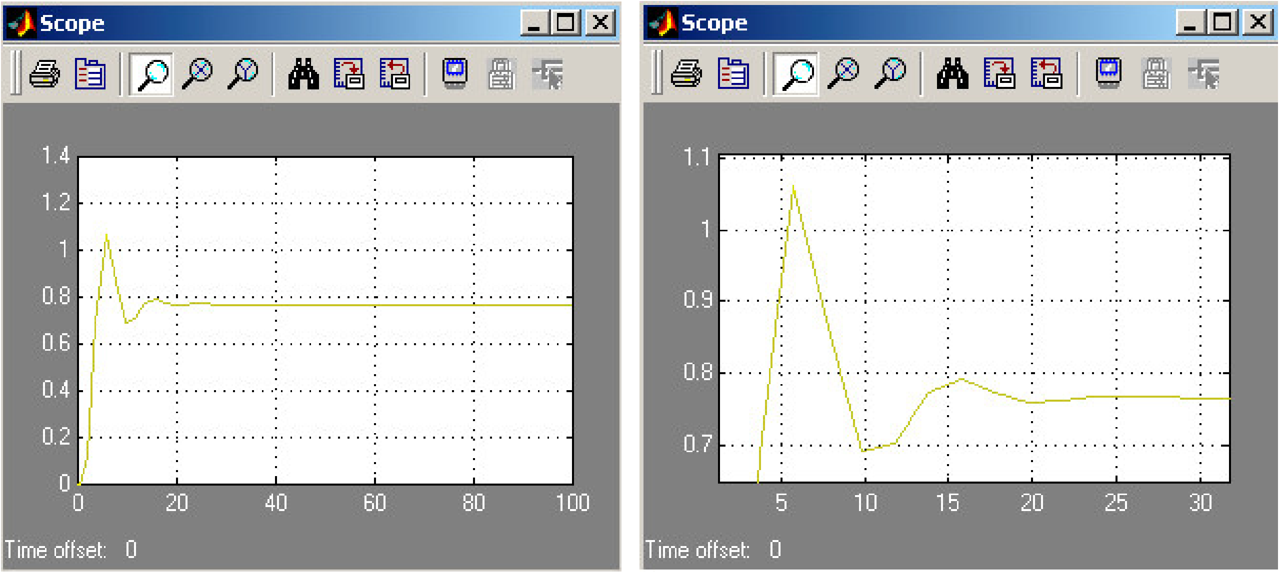
$$\eta \approx \frac{3}{t_{reg}}, \tag{7.17}$$

where t_{reg} is the controlling time.

Quality indices of a control system with PID-controller

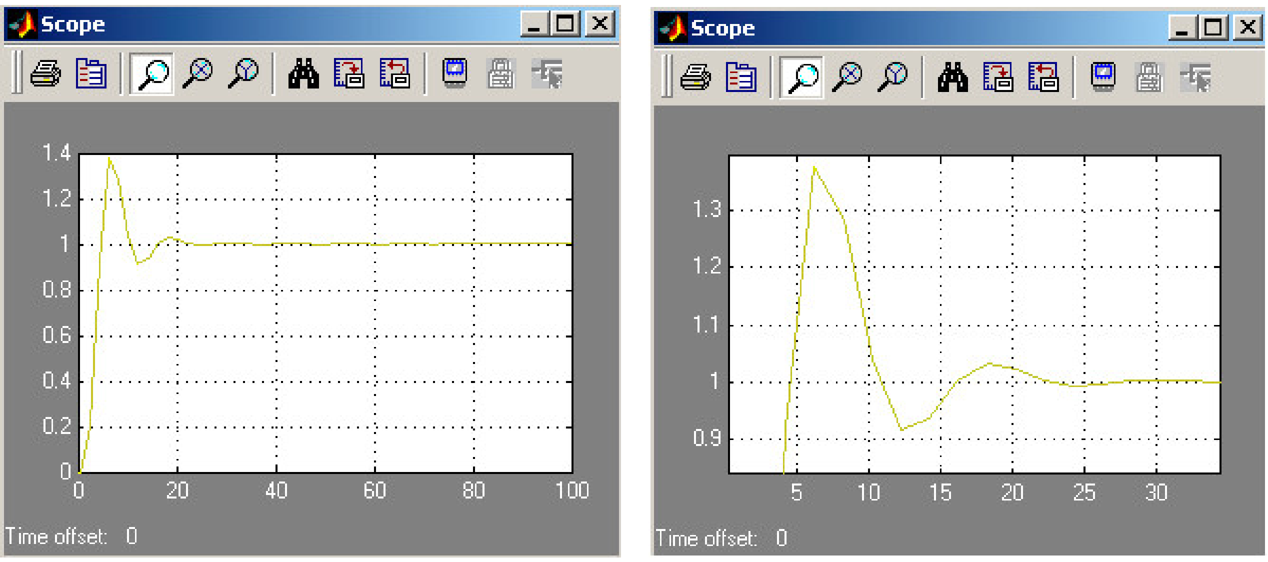
Table 7.4

Controller	Parameter				
	Stability degree	Static error	Controlling time, s	Readjustment, %	Oscillations
No controller	0.08	0.985	36	0	0
P-controller	0.21	0.22	14	5	1
PI-controller	0.20	0	15	40	1
PID-controller	0.20	0	15	52	0
PID-controller with the best adjustment	0.25	0	12	7	0



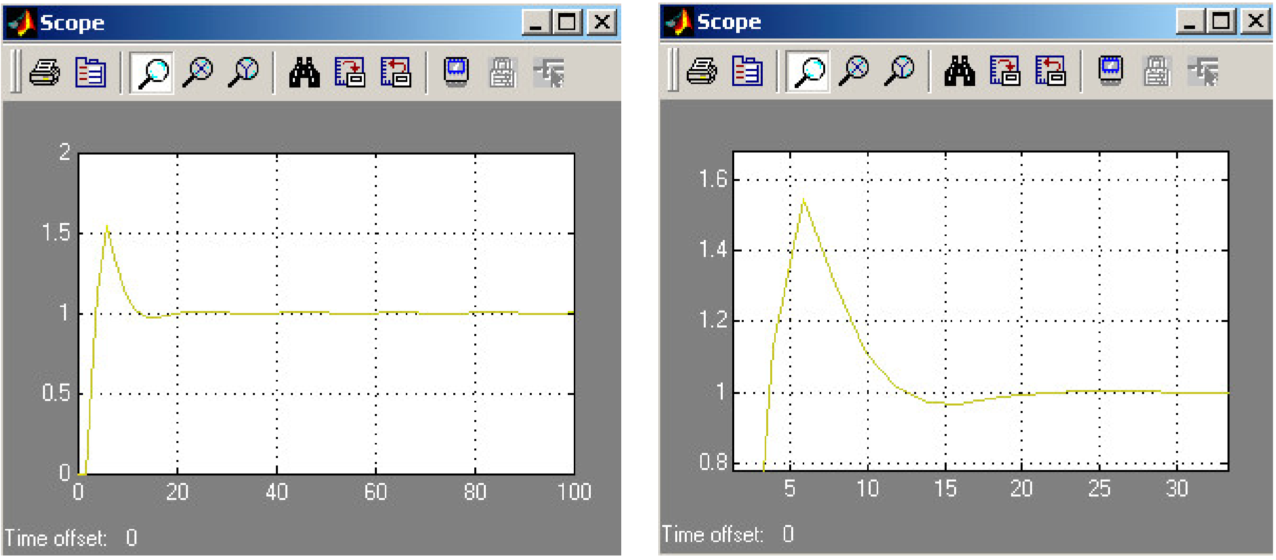
a *b*

Fig. 7.9 Transient process within FCS using P-controller:
a – overall view; *b* – view to evaluate quality indices



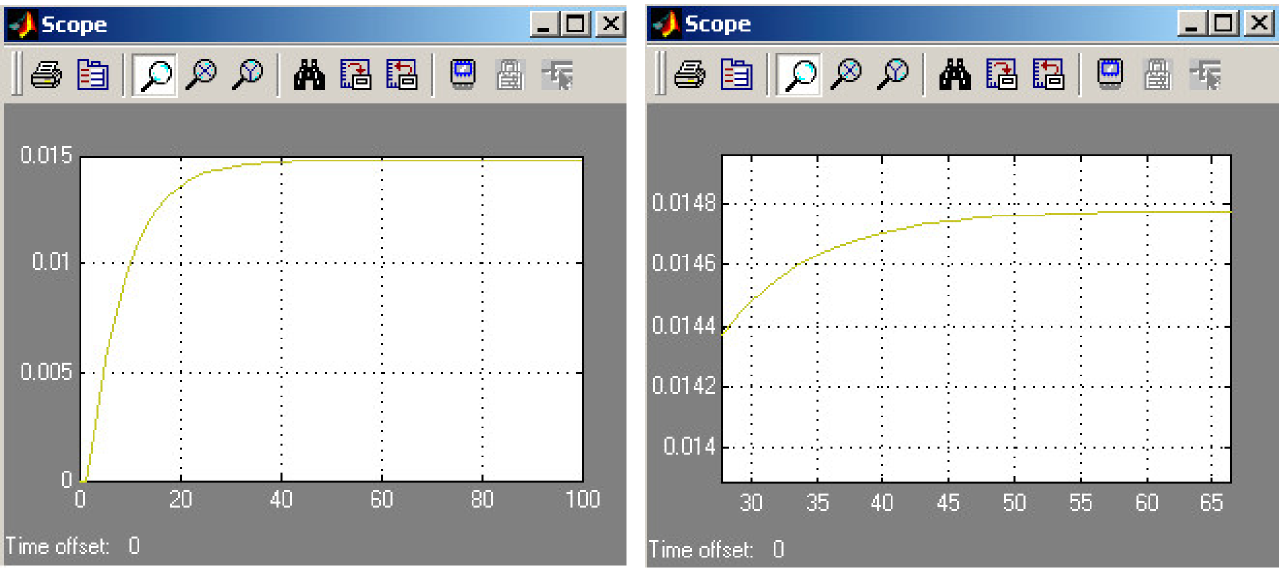
a *b*

Fig. 7.10 Transient process within FCS using PI-controller:
a – overall view; *b* – view to evaluate quality indices



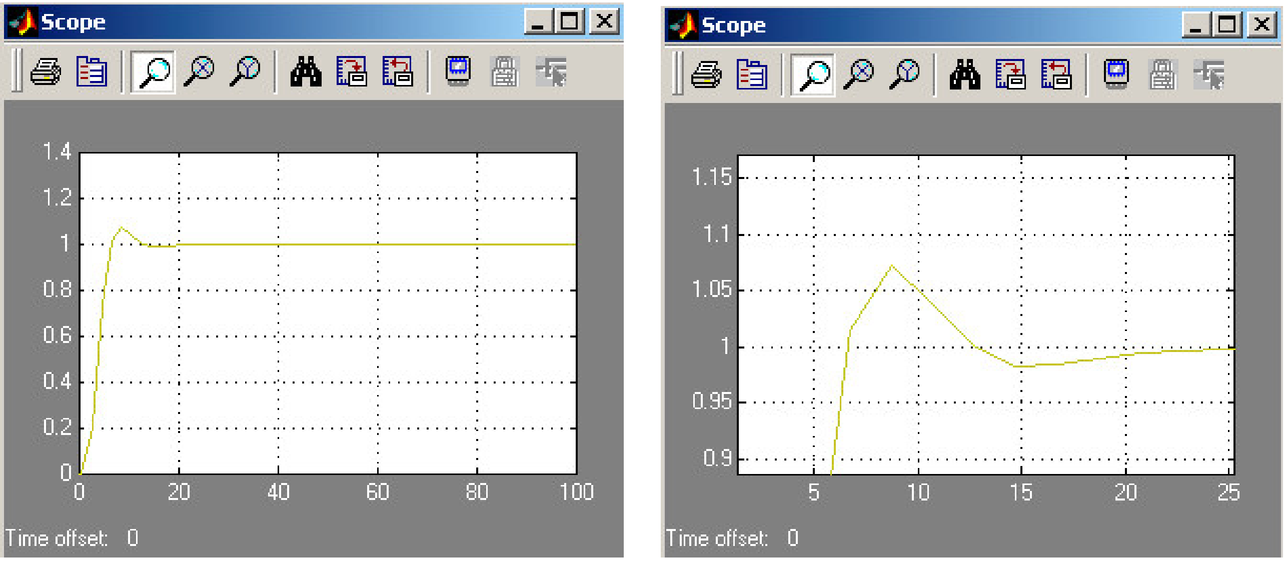
a *b*

Fig. 7.11 Transient process within FCS using PID-controller:
a – overall view; *b* – a view to evaluate quality indices



a *b*

Fig. 7.12 Transient process within FCS where no controller has been used:
a – overall view; *b* – view to evaluate quality indices



a *b*

Fig. 7.13 Transient process within FCS with the best controller adjustments: *a* – overall view; *b* – a view to evaluate quality indices

Quality indices of a control system with PID-controller

Table 7.4

Controller	Parameter				
	Stability degree	Static error	Controlling time, s	Readjustment, %	Oscillations
No controller	0.08	0.985	36	0	0
P-controller	0.21	0.22	14	5	1
PI-controller	0.20	0	15	40	1
PID-controller	0.20	0	15	52	0
PID-controller with the best adjustment	0.25	0	12	7	0

Example 7.2. Apply Ziegler-Nichols method to identify PID-controller adjustments of the automated control system in terms of the object in 7.1 example (Fig. 7.2). Specify the calculated adjustments and determine quality indices of the transient process experimentally using FCS model in terms of the SIMULINK MATLAB environment.

Apply Ziegler-Nichols method to identify PID-controller parameters. Define the critical intensification coefficient K_p^* for the system, shown in Fig. 2.7, ignoring the integral part of the control law. Transfer function of the open part is:

$$W_{op}(p) = K_p^* \frac{1}{1-(e^{-p\tau}/Tp+1)} \frac{Ke^{-p\tau}}{Tp+1}. \quad (7.18)$$

Identical transformations result in

$$W_{op}(p) = K_p^* \frac{K_p^* K e^{-p\tau}}{Tp+1-e^{-p\tau}}. \quad (7.19)$$

Make a characteristic polynom of the closed ACS

$$D(p) = Tp + 1 - e^{-p\tau} + K_p K e^{-p\tau}. \quad (7.20)$$

The automated control system will be within its stability margin (i.e. it will perform stable oscillations with constant amplitude and frequency) according to Mykhailov criterion if the characteristic polynom (7.20) is equal to zero (Mykhailov godohraph will pass through a point with [0; j0)] coordinates within the complex plane). Thus, we obtain

$$Tp + 1 - e^{-p\tau} + K_p K e^{-p\tau} = 0. \quad (7.21)$$

Replace $j\omega$ variable for p one; use Euler formula to record (7.21) expression as follows

$$Tj\omega + 1 - (\cos\tau\omega - j\sin\tau\omega) + K_p K(\cos\tau\omega - j\sin\tau\omega) = 0. \quad (7.22)$$

Derive the equation system recording (7.22) for imaginary and actual parts of the expression

$$\begin{cases} 1 - \cos\tau\omega + K_p K\cos\tau\omega = 0 \\ T\omega + \sin\tau\omega - K_p K\sin\tau\omega = 0. \end{cases} \quad (7.23)$$

Apply (7.23) equation system to determine frequency of stable oscillations ω and critical intensification coefficient K_n^*. Then, use formula $T = 2\pi\omega^{-1}$ to calculate stable oscillation period T^*. The following is obtained from the first equation of the system

$$K_p = -\frac{1-\cos\tau\omega}{K\cos\tau\omega}. \quad (7.24)$$

The following is obtained from the second equation of the system

$$K_p = \frac{T\omega + \sin\tau\omega}{K\sin\tau\omega}. \quad (7.25)$$

Equate left parts to right parts of (7.24) and (7.25) equations

$$-\frac{1-\cos\tau\omega}{K\cos\tau\omega} = \frac{T\omega + \sin\tau\omega}{K\sin\tau\omega}. \quad (7.26)$$

Perform identical transformations of (2.24) equation

$$-\frac{1-\cos\tau\omega}{\cos\tau\omega} = \frac{T\omega + \sin\tau\omega}{\sin\tau\omega},$$

$$\frac{\cos\tau\omega - 1}{\cos\tau\omega} = \frac{T\omega + \sin\tau\omega}{\sin\tau\omega},$$

$$\frac{\sin\tau\omega(\cos\tau\omega - 1)}{\cos\tau\omega} = T\omega + \sin\tau\omega,$$

$$\frac{\sin\tau\omega(\cos\tau\omega - 1)}{\cos\tau\omega} - \sin\tau\omega = T\omega,$$

$$\frac{\sin\tau\omega\cos\tau\omega - \sin\tau\omega - \sin\tau\omega\cos\tau\omega}{\cos\tau\omega} = T\omega,$$

$$\frac{-\sin\tau\omega}{\cos\tau\omega} = T\omega,$$

$$-\mathrm{tg}\tau\omega = T\omega. \qquad (7.27)$$

Equation (7.27) has not any analytical solution relative to the unknown value ω. The equation may be solved graphically; for instance, under the MATHCAD environment.

In the context of the control object, (7.27) equation will look like

$$-\mathrm{tg}2\omega = 7{,}3\omega. \qquad (7.28)$$

Since tangent is a periodic function, (7.28) equation has many solutions. Hence, starting from zero, increase in the intensification coefficient K_p of P-controller will have numerous values of stable oscillations with different ω^* frequencies for different K_p^*. Identify the first K_{p1}^* value corresponding to the minimum ω_1^* value. For the purpose, evaluate the interval within which ω_1^* will stay ω_1^*. Since right member of (7.28) equation is more than zero, then

$$-\mathrm{tg}2\omega > 0,$$

Hence

$$\mathrm{tg}2\omega < 0,$$

Thus

$$-\frac{\pi}{2} + \pi k \le 2\omega < \pi k, \;\; k \epsilon Z.$$

Finally

$$-\frac{\pi}{4} + \frac{\pi}{2}k \le \omega < \frac{\pi}{2}k, \;\; k \epsilon Z. \qquad (7.29)$$

Table 7.5 explains the intervals for ω^* values calculated in terms of (7.29) expression.

ω^* intervals

Table 7.5

k	0	1	2
Intervals	$-0.79 \div 0$	$0.79 \div 1.57$	$2.35 \div 3.14$

Since ω frequency cannot be negative, then $\omega_1^* \in [0{,}79; 1{,}57)$. Identify ω_1^* . Use the MATHCAD environment to construct individual graphs for left member and right member of (7.28) equation and define coordinates of their junction point. Fig. 7.14 and 7.15 demonstrate the solution results within the specified ω_1^* availability interval.

Fig. 7.14 shows the constructed graphs as well as their junction point; Fig. 7.15 demonstrates abscissa and ordinate of the joint point of the graphs determined using *X-Y Trac* function. Thus, $\omega_1^* = 0.86$ rad/s. Identify a period of stable oscillations $T^* = 2\pi {\omega_1^*}^{-1} = 2\pi\, 0.86^{-1} \approx 7.3$ s. Insert $\omega_1^* = 0.86$ rad/s into (7.24) and determine K_{n1}^*:

$$K_{p1}^* = -\frac{1-\cos\tau\omega_1^*}{K\cos\tau\omega_1^*} \approx -\frac{1-\cos(2\times 0.86)}{0.015\cos(2\times 0.86)} \approx 515.15. \qquad (7.30)$$

$\tau := 2 \qquad T := 7.3$

$\omega := 0.79, 0.80 .. 1.57$

$y(\omega) := -\tan(\tau\cdot\omega) \qquad g(\omega) := T\cdot\omega$

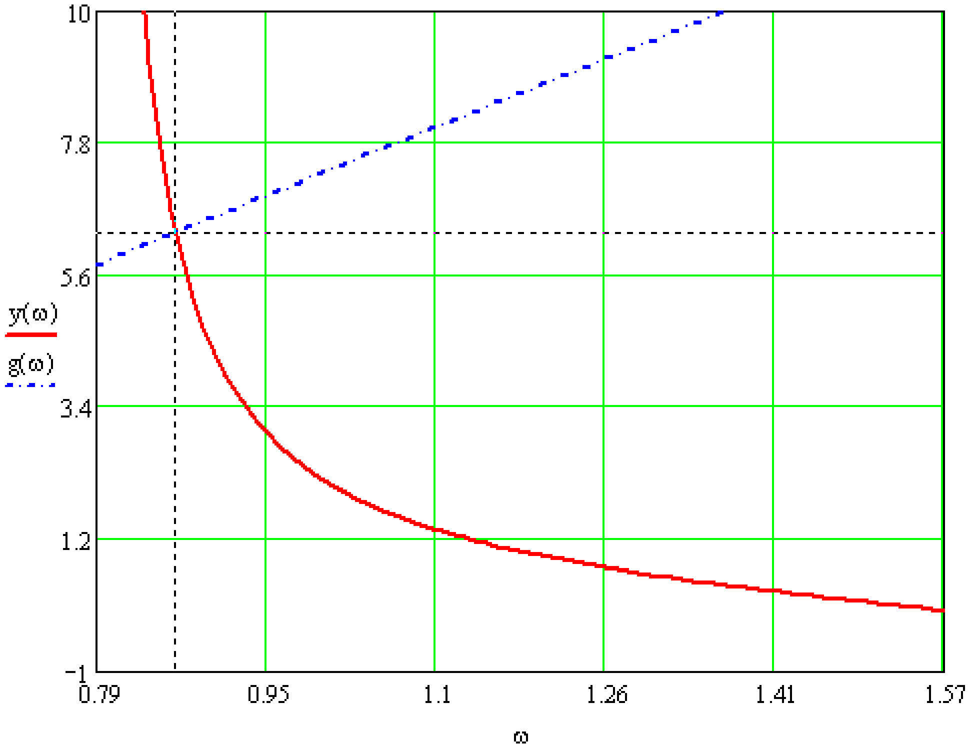


Fig. 7.14 Graphical solution of the equation

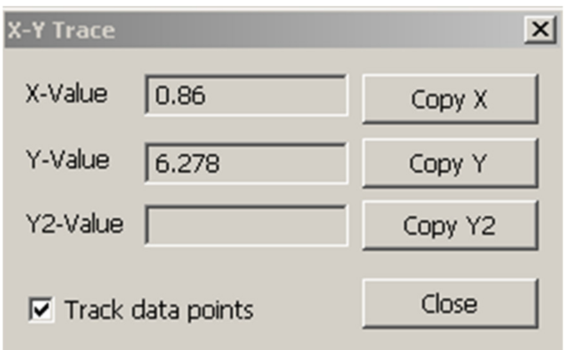


Fig. 7.15 ω_1^* value determination

Validate correctness of the performed calculations using simulations under the SIMULINK MATLAB environment in terms of a structural scheme in Fig. 7.16.

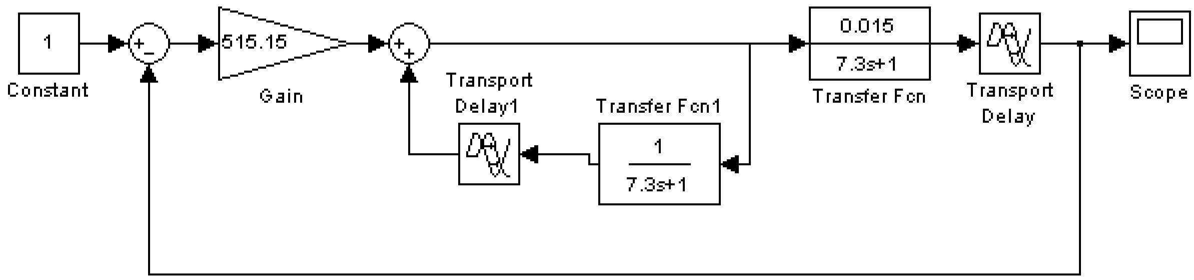


Fig. 7.16 ACS structural scheme for simulation

Fig. 7.17 demonstrates overall view of the transient point within the ACS; Fig. 7.18 shows the transient process part to test oscillation period T^*.

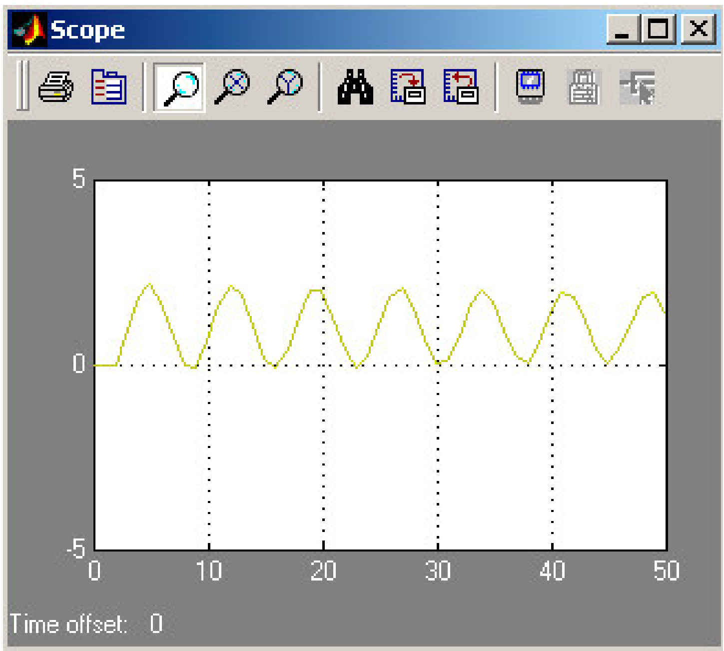


Fig. 7.17 Overall view of the transient process within the ACS

The simulation results coincide completely with the calculations.

Table 7.6 shows parameter values of PI-controller calculated with consideration that $T^* \approx 7.3\ s$ and $K_{p1}^* \approx 515.15$.

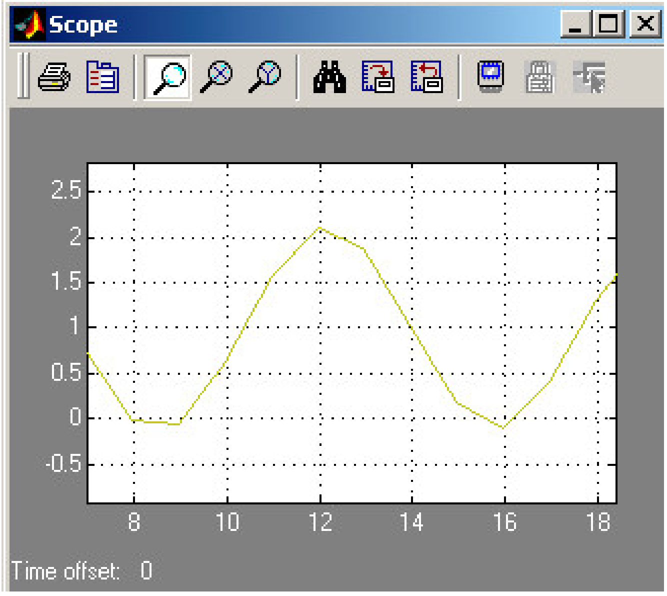


Fig. 7.18 The transient process part

Parameters of generic controllers

Table 7.6

	K_p	K_i	K_d	Transfer function
PI-controller	231.82	38.11	–	$W_{pi}(p) = 231.82 + \dfrac{38.11}{p}$

Fig. 7.19 demonstrates structural scheme of the controlled object model, developed under the SIMULINK MATLAB environment, to analyze FCS with PPI-controller. This scheme differs from the scheme, represented in Fig. 7.8, in the fact that it does not involve a differential part of PID-controller and its internal circuit contains a predictor represented by the in-series connected *Transfer Fon 1* and *Transport Delay 1* blocks.

Transient process, shown in Fig. 7.20, result from the simulation of FCS operation with set points of PI-controller (Table 7.6). Fig.7.21 demonstrates the transient process with the best controller adjustments selected experimentally with the use of the model (Table 7.7)

$$W_{reg}(p) = 140. \tag{7.31}$$

Formula (7.17) has helped determine the stability degree.

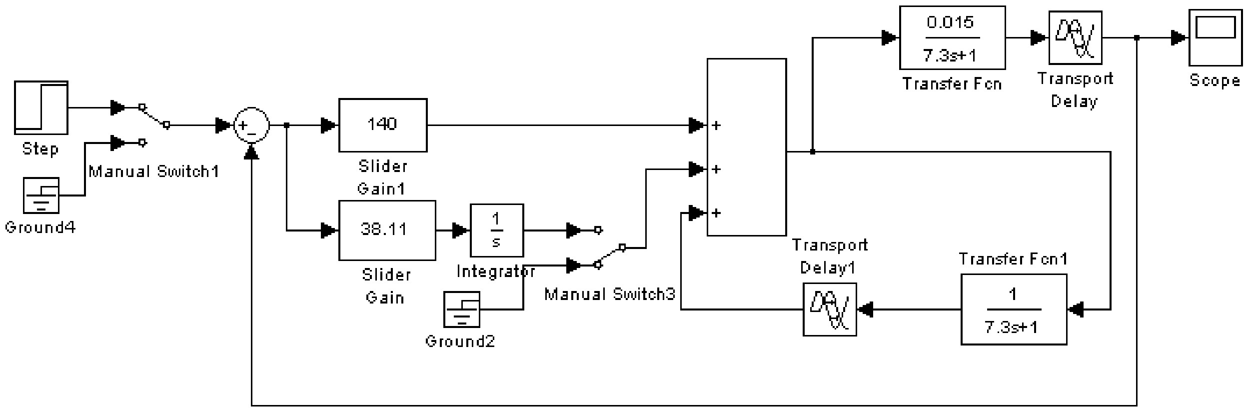
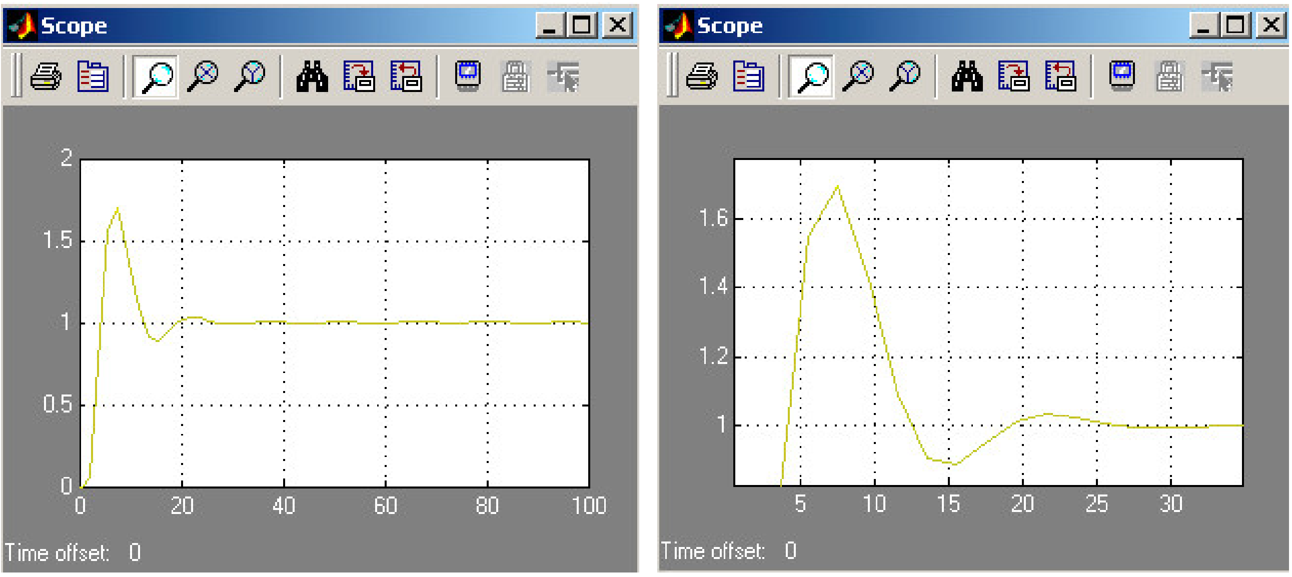


Fig. 7.19 FCS model with a predictive PI-controller



a *b*

Fig. 7.20 The transient process within FCS with PI-controller
a – overall view; *b* – view to evaluate quality indices

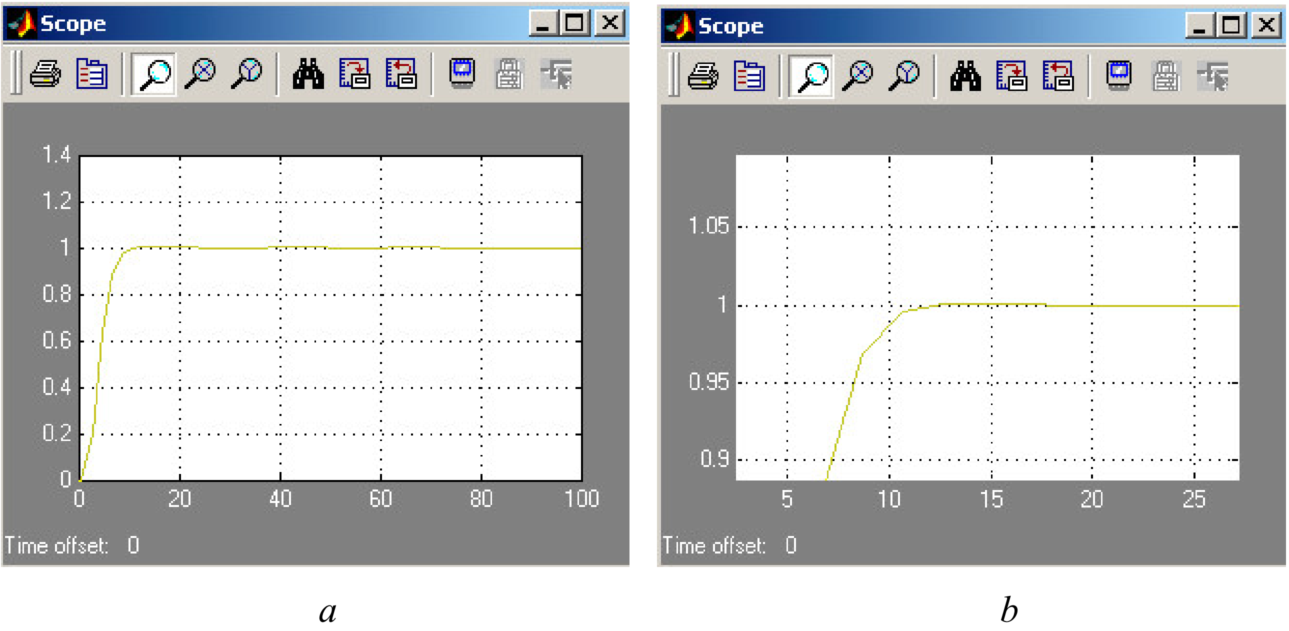


Fig. 7.21 The transient process within FCS with the best controller adjustments
a – overall view; *b* – view to evaluate quality indices

Quality indices of a control system with PPI-controller

Table 7.7

Controller	Parameter				
	Stability degree	Static error	Time to control, s	Readjustment, %	Oscillations
PPI-controller	0.17	0	18	70	1
PPI-controller with the best adjustments	0.38	0	8	0	0

According to the analysis results in examples 1 and 2, it is possible to conclude that in the context of the considered control object, PPI-controller is better to compare with the classic PID-controller since its stability degree is higher, controlling interval is shorter, and readjustment is not available.

7.6 Report contents.
Output data.
Structural schemes of the analyzed automated control systems.
Calculation of the parameters of PID-controller and predictive PI-controller using Ziegler-Nichols method.

Graphs of transient processes within FCS involving PID-controller and predictive PI-controller.
Quality evaluation of transient processes within FCS involving PID-controller and predictive PI-controller.
Comparison of the best transient processes within FCS involving PID-controller and predictive PI-controller.

7.7 Control questions
What control methods are applied for objects with delay?
What are the advantages and disadvantages of relay control?
What are the advantages and disadvantages of PID control law?
What are the advantages and disadvantages of a predictive control?
What are the advantages and disadvantages of controllers using algorithms?
What is Ziegler-Nichols method?
Why is it necessary to expand objects with delay into Padé series?

LABORATORY RESEARCH 8

Analyzing a sampling control system

8.1 Objective is to deepen students' knowledge while studying the Chapter "Sampling control systems"

In the process of the activities, students should be able to:
- make a transfer function of a pulse system with a zero-order extrapolator and fixation for the period;
- identify stability of a sampling control system;
- calculate a transient process within the automated control system;
- acquire practical skills while analyzing pulse systems using a computer.

8.2 Input data to perform the activities are the following:
- structural scheme and numerical parameters of the analyzed automated system (Fig. 8.1, Table 8.1); and
- the MATLAB application package for computer-based simulation of the automated system.

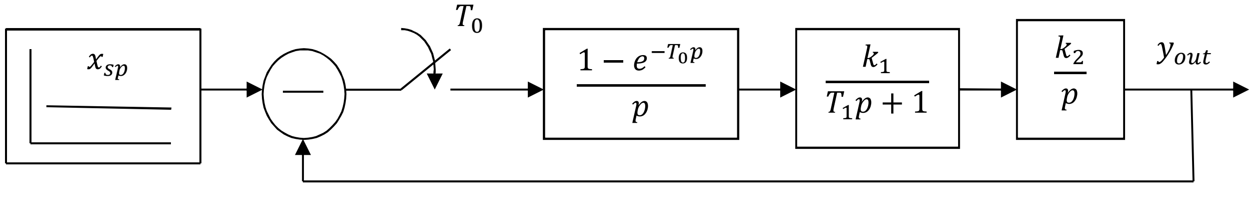


Fig. 8.1 Structural scheme of the pulse ACS

Input data

Table 8.1

Variant	x_{sp}	T_0, s	k_1	T_1, s	k_2
1	1	0.5	5.2	1.0	1.3
2	2	0.2	6.4	1.6	1.2
3	3	0.7	7.8	1.4	1.5
4	4	0.8	6.3	1.6	1.6
5	5	0.9	5.5	1.8	1.7
6	6	1.0	5.4	1.7	1.8
7	7	0.9	4.3	1.5	1.9
8	8	0.8	4.9	1.3	2.0
9	9	0.7	3.4	1.1	1.9
10	10	0.6	5.6	0.9	1.8
11	9	0.5	6.1	1.2	1.7
12	8	0.6	6.4	1.4	1.6
13	7	0.7	7.9	1.6	1.5
14	6	0.8	6.2	1.8	1.4
15	5	0.9	7.8	1.7	1.3
16	4	1.0	8.7	1.5	1.2
17	3	0.9	10.0	1.3	1.0
18	2	0.8	8.4	1.1	1.1
19	1	0.7	7.6	0.9	1.2
20	2	0.6	7.8	1.0	1.3

8.3 Operating procedures

Following order is recommended:

– make a transfer function of the continuous part of the pulse automated control system;
– identify the transfer function of the pulse automated control system with a zero-order extrapolator and fixation for the period;
– determine the automated control system stability;
– calculate a transient process within the automated system when a single step signal is set to its input;
– use the MATLAB application package to verify the calculation results by means of computer-based simulation of a sampling control system.

8.4 Methodological explanations

Sampling control systems combine a pulse element and a continuous part of the system. The pulse element transforms the continuous input effect into the equispaced pulses.

Transfer system of the open sampling control system with a zero-order extrapolator and fixation for the period is as follows:

$$W(z) = \frac{z-1}{z} Z\left\{\frac{W_{cp}(p)}{p}\right\} \cdot \quad (8.1)$$

,

where $z = e^{pT_0}$ is the complex variable; Z is the operation of z transformation; and $W_{cp}(p)$ is the transfer function of the continuous part of the system.

Transfer function of the closed system will be

$$\Phi(z) = \frac{W(z)}{1 + W(z)} \cdot \quad (8.2)$$

The closed system will be closed if roots of characteristic equation are inside a circle which radius is equal to a unit. If the characteristic equation is transformed in terms of $z = (1+\omega)(1-\omega)^{-1}$ (being bilinear transformation), then each root of a stable system will have negative real part. In such a case, it is possible to evaluate stability of the pulse ACS using the criteria applied for continuous linear systems.

Quality indices of sampling control systems are identified with the help of a transient process graph. In practice, several techniques to calculate transient process within a system are applied. However, Laurent transformation of z image of the output value is the most popular one.

Example 8.1. Determine transfer function, evaluate stability, and calculate a transient process within a sampling control system represented in Fig. 8.1. The system parameters are as follows: $x_{sp} = 1$; $k_1 = 1{,}4$; $k_2 = 6$; $T_0 = 0.6$; and T_1=1.2. Verify the calculation results using computer-based simulation.

Determine a transfer function of the sampling control system continuous part

$$W_{cp}(p) = \frac{k_1 k_2}{(T_1 p + 1)p} = \frac{K}{(T_1 p + 1)p}, \quad (8.3)$$

where $K = k_1 k_2$ is the overall intensification coefficient of the open ACS.

The transfer function of the open part a zero-order extrapolator and fixation for the period will be

$$W(z) = \frac{z-1}{z} Z\left\{\frac{W_{cp}(p)}{p}\right\} = \frac{z-1}{z} Z\left\{\frac{K}{(T_1 p + 1)p^2}\right\} \cdot \quad (8.4)$$

Factorize the expression denominator in curly brackets (8.4), taking into consideration the fact that it has multiple roots $p_1 = p_2 = 0$ and $p_3 = -T_1{}^{-1}$ root

$$W(z) = \frac{z-1}{z} \cdot \frac{K}{T_1} Z\left\{\frac{1}{p^2 (p + T_1{}^{-1})}\right\} \cdot \quad (8.5)$$

Expand the nonintegral in curly brackets (8.5) into the total of the simplest nonintegrals

$$\frac{1}{p^2(p+T_1{}^{-1})}=\frac{A}{p}+\frac{B}{p^2}+\frac{C}{p+T_1{}^{-1}}\cdot \tag{8.6}$$

Identical transformation of a right (8.6) member results in

$$\frac{1}{p^2(p+T_1{}^{-1})}=\frac{(A+C)p^2+\left(AT_1{}^{-1}+B\right)p+BT_1{}^{-1}}{p^2(p+T_1{}^{-1})}\cdot \tag{8.7}$$

Two like nonintegrals with equal denominators should have similar numerators too

$$1=(A+C)p^2+\left(\frac{A}{T_1}+B\right)p+\frac{B}{T_1}\cdot \tag{8.8}$$

Use (8.8) expression to derive the equation system by making equal coefficients in terms of variable *p* with similar degree indices:

$$\begin{cases} A+C=0 \\ \dfrac{A}{T_1}+B=0 \\ \quad\dfrac{B}{T_1}=1. \end{cases} \tag{8.9}$$

Solution for (8.9) is as follows: $B=T_1$; $A=-T_1^2$; and $C=T_1^2$.

Expression (8.5) will look like:

$$\begin{aligned} W(z)&=\frac{z-1}{z}\cdot\frac{K}{T_1}Z\left\{-\frac{T_1^2}{p}+\frac{T_1}{p^2}+\frac{T_1^2}{p+AT_1{}^{-1}}\right\}= \\ &=K\frac{z-1}{z}\cdot Z\left\{-\frac{T_1}{p}+\frac{1}{p^2}+\frac{T_1}{p+AT_1{}^{-1}}\right\}\cdot \end{aligned} \tag{8.10}$$

Perform Z transformation of the simplest nonintegrals is curly brackets (8.10) using Table 8.2:

$$\begin{aligned} W(z)&=K\frac{z-1}{z}\left(-\frac{T_1 z}{z-1}+\frac{T_0 z}{(z-1)^2}+\frac{T_1 z}{z-e^{-\frac{T_0}{T_1}}}\right)= \\ &=K\left(-T_1+\frac{T_0}{z-1}+\frac{T_1(z-1)}{z-e^{-\frac{T_0}{T_1}}}\right)\cdot \end{aligned} \tag{8.11}$$

Representation of certain lattice functions and time generation functions as well as their Laplace images

Table 8.2

Generating continuous function		Lattice function	Simple Z transformation
Original	Laplace transformation		
$1(t)-1(t-T_0)$	$\frac{1-e^{pT_0}}{p}$	$-\Delta[n]=\nabla[n-1]$	1
$1(t)$	$\frac{1}{p}$	$1[n]$	$\frac{z}{z-1}$
t	$\frac{1}{p^2}$	nT_0	$\frac{T_0 z}{(z-1)^2}$
$e^{-\alpha T_0}$	$\frac{1}{p+\alpha}$	$e^{-\alpha n T_0}=d^n$	$\frac{z}{z-d}, d=e^{-\alpha T_0}$

Insert values of T_0, K, and T_1 parameters into (8.11):

$$W(z)=K\left(-T_1+\frac{T_0}{z-1}+\frac{T_1(z-1)}{z-e^{-\frac{T_0}{T_1}}}\right)=8.4(-1.2+\frac{0,6}{z-1}+\frac{1,2(z-1)}{z-e^{-\frac{0.6}{1.2}}})\cdot \quad (8.11)$$

Identical transformations of the right (8.11) member result in

$$W(z)=\frac{1.09z+0.84}{(z-1)(z-0.61)}\cdot \quad (8.12)$$

Use (8.2) formula to identify a transfer function of the closed sampling control system

$$\Phi(z)=\frac{W(z)}{1+W(z)}=\frac{(1.09z+0.84)((z-1)(z-0.61))^{-1}}{1+(1.09z+0.84)((z-1)(z-0.61))^{-1}}=$$

$$=\frac{1.09z+0.84}{z^2-0.52z+1.45}\cdot \quad (8.13)$$

Test the closed sampling control system stability. For that purpose, determine roots of its characteristic polynom. The characteristic polynom is as follows

$$D=z^2-0.52z+1.45. \quad (8.14)$$

Roots of the characteristic polynom, derived in (8.14), are $z_{1,2}\approx 0,26\pm 1.18i$. Identify $|z_1|$ and $|z_2|$ moduli of such complex numbers as z_1 and z_2

$$|z_1| = |z_2| \approx \sqrt{0.26^2 + (\pm 1.18)^2} \approx 1.21. \quad (8.15)$$

Since $|z_1| > 1$ and $|z_2| > 1$, then the closed sampling system is unstable.

Identify a transition process within the system using Laurent transformation.$x_{sp} = 1$ effect, which representation is $x_{sp}(z) = z(z-1)^{-1}$ according to Table 8.2, has been set to an input of the closed sampling control system.

determine Z image of output $y_{out}(z)$ value

$$y_{out}(z) = x_{sp}(z)\Phi(z) = \frac{z}{z-1} \cdot \frac{1.09z + 0.84}{z^2 - 0.52z + 1.45} =$$

$$= \frac{1.09z^2 + 0.84z}{z^3 - 1.52z^2 + 1.97z - 1.45}. \quad (8.16)$$

Divide the numerator by a common denominator located in the right member of (8.16)

$$\begin{array}{r|l}
-\;\begin{array}{l} 1.09z^2 + 0.84z \\ 1.09z^2 - 1.66z + 2.15 - 1.58z^{-1} \end{array} & \dfrac{z^3 - 1.52z^2 + 1.97z - 1.45}{1.09z^{-1} + 2.5z^{-2} + 1.65z^{-3}} \\
\hline
-\;\begin{array}{l} 2.52z - 2.15 + 1.58z^{-1} \\ 2.52z - 3.8 + 4.93z^{-1} - 3.63z^{-2} \end{array} & \\
\hline
-\;\begin{array}{l} 1.65 - 3.35z^{-1} + 3.63z^{-2} \\ 1.65 - 2.15z^{-1} + 3.25z^{-2} - 2.39z^{-3} \end{array} & \\
\hline
-0.84z^{-1} + 0.38z^{-2} + 2.39z^{-3} & .
\end{array}$$

In terms of the complex variable z, the share coefficients, raised to the correspondent power, are the values of output $y(t)$ value during 0; T_0; $2T_0$; $3T_0$ ... time moments. Table 8.3 represents the output $y_{out}(t)$ value during corresponding time moments at the start of the transient process according to the performed calculations.

Values of the output variable

Table 8.3

	$0 \cdot T_0 = 0$	$1 \cdot T_0 = 0.6$	$2 \cdot T_0 = 1.2$	$3 \cdot T_0 = 1.8$
$y_{out}(t)$	0	1.09	2.5	1.67

Test the performed calculations using simulation under the SIMULINK MATLAB environment. Fig. 8.2 shows structural scheme to simulate a sampling control system. The scheme implements two simulation techniques for the sampling control system.

Simulation in terms of technique one uses such blocks as *Zero-Order Hold*, *Transfer Fon,* and *Transfer Fon 1*. The *Zero-Order Hold* block simulates operation of a zero-order extrapolator and fixation for the period; the *Transfer Fon* and *Transfer Fon 1* blocks simulate continuous part of the open sampling control system.

The *Discrete Transfer Fon* block is applied in the context of technique two. The block simulates operation of the closed sampling control system in terms of its transfer function (8.13).

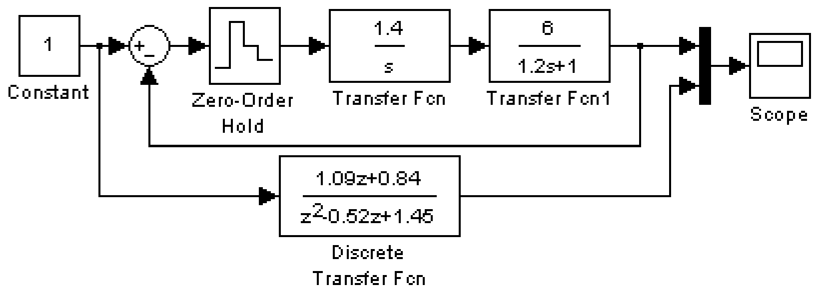


Fig. 8.2 Structural scheme of the analyzed sampling control system

Fig. 8.3 demonstrates transient process within the sampling control system. The transient process corresponds to unstable ACS coinciding with the calculation results. Fig. 8.4 shows values of the output variable during $t\{0; 0.6; 1.2; 1.8\}$ time moments. Values of the output variable, obtained using the two simulation techniques during the listed time periods, also coincide with the calculation results (Table 8.3).

8.5 Report contents

Output data and structural scheme of the analyzed system.

Transfer function of the open part of a sampling control system with a zero-order extrapolator and fixation for the period.

Transfer function of the closed sampling system with a zero-order extrapolator and fixation for the period.

Calculations of the automated control system stability.

Calculations of a transient process.

Structural scheme to simulate a sampling control system and graphs of transient processes within the system determined by the simulation results.

8.6 Control questions

What is a sampling control system?

What is the ideal pulse element?

What is an extrapolator function?

What is a real pulse element?

How is it possible to identify stability of a sampling control system ?

Why is bilinear transformation of a characteristic equation used?

How is it possible to calculate a transient process within a sampling control system?

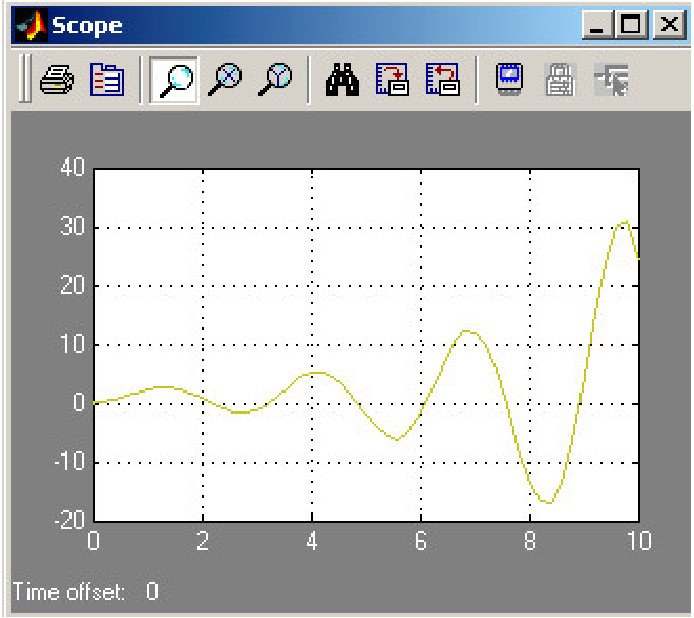


Fig. 8.3 Transient process within the sampling control system

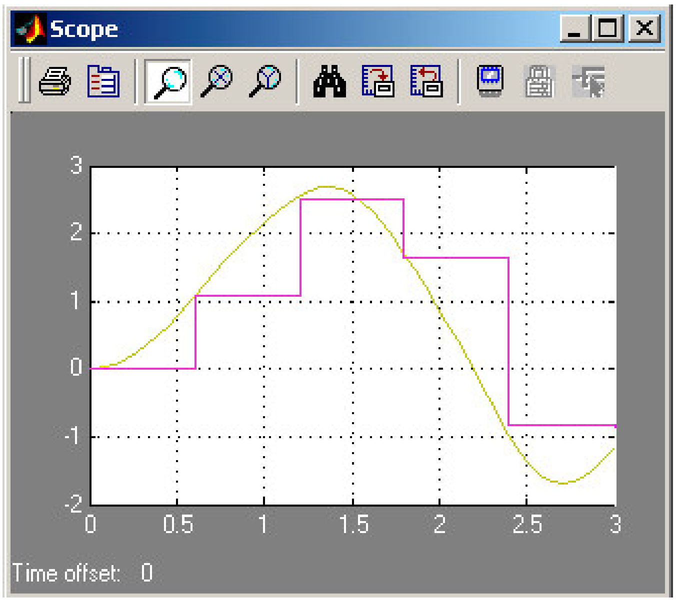


Fig. 8.4 Before testing the output variable values

LABORATORY RESEARCH 9

Analyzing a linear ACS[1]

9.1 Objective is to deepen students' knowledge while studying sections of theory of linear systems of the automated control

[1]In 2016, tasks of the laboratory research (developed by M.M. Tryputen) have been proposed to the participants of 2nd stage of the All-Ukrainian Student Competition in the field of "System Engineering" (specialism area "Computerized Control Systems and Automation").

In the process of the activities, students should be able to:
– determine the automated control system order in terms of a velocity error value, and properties of a characteristic vector of ACS;
– simulate structural scheme of the open part of a linear ACS in terms of characteristics of dynamic elements and law of changes in the input and output values;
– identify ACS stability conditions assuming that parameters of all its elements experience time changes;
– acquire practical skills to analyze linear ACSs using a computer.

9.2 Output data to perform the activities are the following:
– structural scheme and numerical parameters of the analyzed automated system (Figures 9.1-9.7, Table 9.1);
– the MATLAB application package for computer-based simulation of the automated control system.

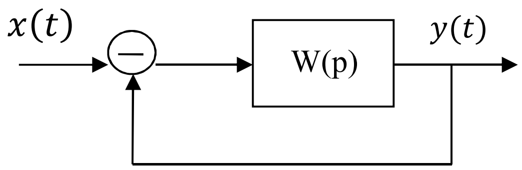


Fig. 9.1 Structural scheme of a FCS

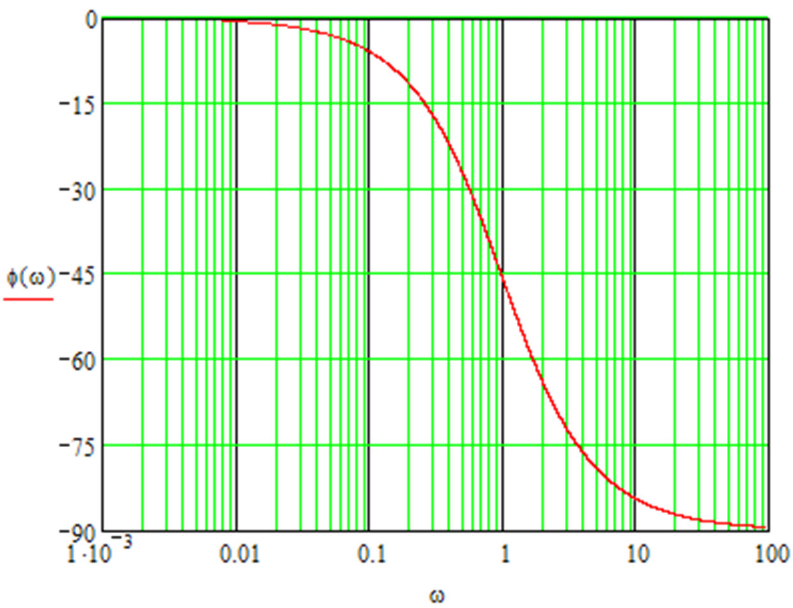
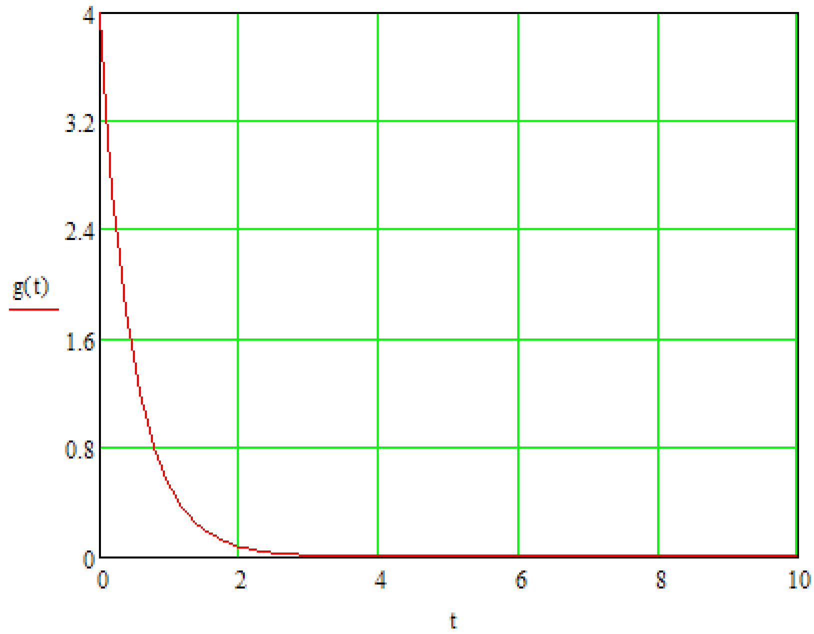


Fig. 9.2 Logarithmic phase frequency response of a dynamic element



$$g(t) = 4e^{-2t}$$

Fig. 9.3 Impulse transient function of a dynamic element

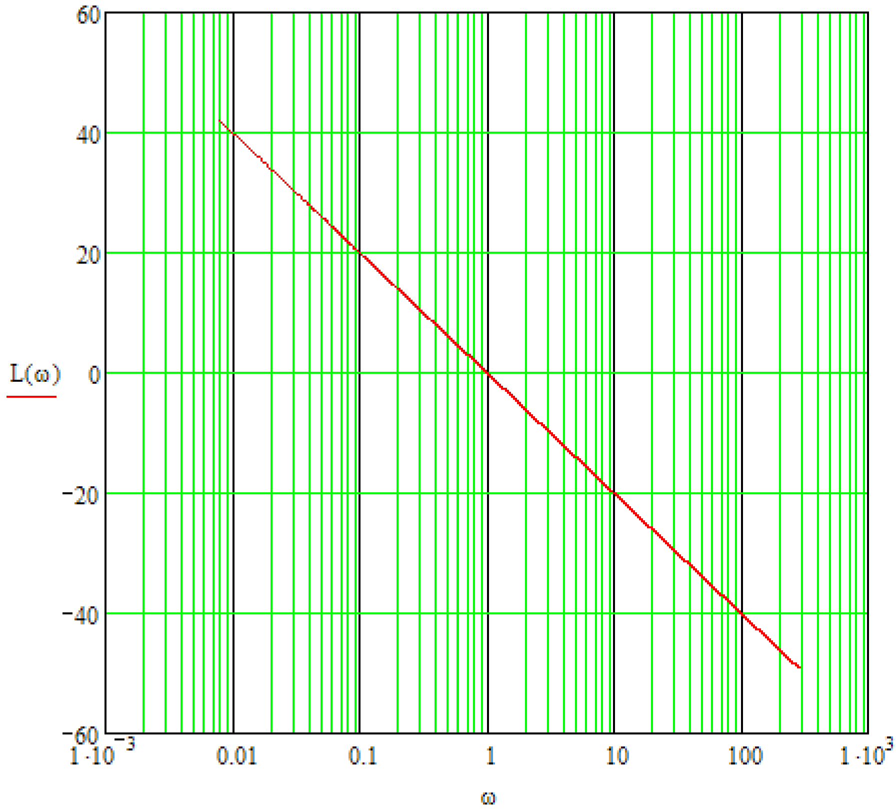


Fig. 9.4 Logarithmic amplitude frequency response of a dynamic element

Fig. 9.5 Transient function of a dynamic element

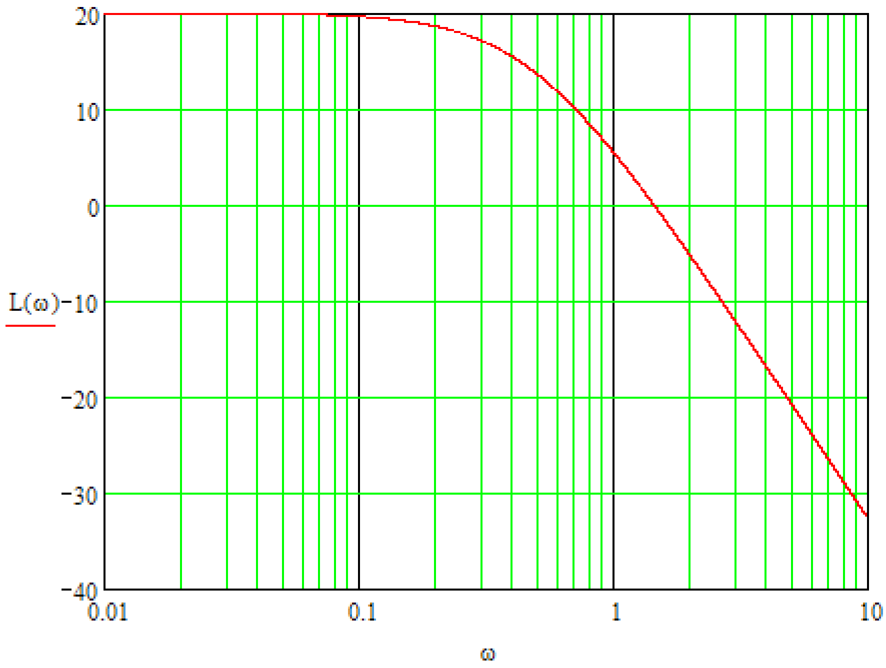


Fig. 9.6 Logarithmic amplitude frequency response of a dynamic element

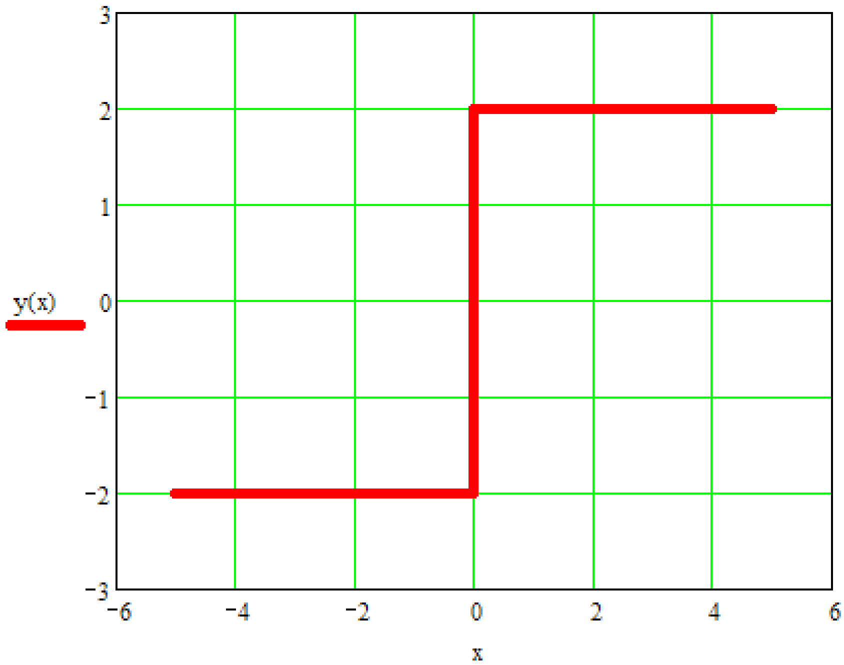


Fig. 9.7 Input-output characteristic of a dynamic element

Input data

Table 9.1

Variant	Rotation angle of a characteristic vector $D(j\omega)$	Law of input value variation x_{in}	Law of output value variation y_{out}
1	1.5π	$x_{in}(t) = 3t + 7$	$y_{out}(t) = 3t + 5.5$
2	1.5π	$x_{in}(t) = t + 5.5$	$y_{out}(t) = t + 5$
3	1.5π	$x_{in}(t) = 4t + 9$	$y_{out}(t) = 4t + 7$
4	1.5π	$x_{in}(t) = 2t + 8$	$y_{out}(t) = 2t + 7$
5	1.5π	$x_{in}(t) = 5t + 8.5$	$y_{out}(t) = 5t + 6$
6	1.5π	$x_{in}(t) = 3t + 6.5$	$y_{out}(t) = 3t + 5$
7	1.5π	$x_{in}(t) = t + 1$	$y_{out}(t) = t + 0.5$
8	1.5π	$x_{in}(t) = 2t + 2$	$y_{out}(t) = 2t + 1$
9	1.5π	$x_{in}(t) = 5t + 2.5$	$y_{out}(t) = 5t$
10	1.5π	$x_{in}(t) = 4t + 7.5$	$y_{out}(t) = 4t + 5.5$
11	1.5π	$x_{in}(t) = t + 3.5$	$y_{out}(t) = t + 3$
12	1.5π	$x_{in}(t) = 5t + 3.5$	$y_{out}(t) = 5t + 1$
13	1.5π	$x_{in}(t) = 3t + 1.5$	$y_{out}(t) = 3t$
14	1.5π	$x_{in}(t) = 2t$	$y_{out}(t) = 2t - 1$
15	1.5π	$x_{in}(t) = 4t$	$y_{out}(t) = 4t - 2$
16	1.5π	$x_{in}(t) = t$	$y_{out}(t) = t - 0.5$
17	1.5π	$x_{in}(t) = 2t + 1$	$y_{out}(t) = 2t$
18	1.5π	$x_{in}(t) = 5t + 5$	$y_{out}(t) = 5t + 2.5$
19	1.5π	$x_{in}(t) = 3t + 1.5$	$y_{out}(t) = 3t$
20	1.5π	$x_{in}(t) = 4t + 1$	$y_{out}(t) = 4t - 1$

9.3 Operating procedures

Following order is recommended:

- determine the dynamic elements which may be a part of the ACS;
- determine the ACS order;
- determine the overall intensification coefficient of the open ACS part;
- determine the dynamic elements involved by the open part;
- determine stability conditions of the closed ACS;
- verify the simulation calculation results of the simulated ACS using a computer under the SIMULINK MATLAB environment.

9.4 Methodological explanations

Analysis and synthesis of the automated control systems is rather complicated problem which solution needs combination and use of different sections of the automated control theory. The skill to identify the effect of the automated control system parameters as well as its components (elements) on the ACS characteristics is the foundation to determine correctly its advantages and disadvantages as well as implement the control processes in accordance with the formulated requirements.

Example 9.1. Characteristic $D(j\omega)$ vector of the closed automated control system, characterized by additivity and uniformity, turned to 1.5π angle after ω frequency changed from 0 to ∞. When linear $x_{in}(t) = 2t + 5$ effect is set to the system input (Fig. 9.1), output value varies in terms of $y_{out}(t) = 2t + 4$ law under the transient process termination. According to the characteristics of dynamic elements, shown in Figures 9.2–9.7, it is required to simulate structural scheme of the open ACS part; identify the ACS stability conditions assuming that the parameters of all their elements experience time changes; and support the calculation results by means of simulation.

1. Determine transfer $W(p)$ function of the open ACS part.

Characteristics of additivity and uniformity means that the ACS is not linear one. Hence, the nonlinear element, shown in Fig. 9.7, cannot be its part. Response of an output value to input effect, varying in terms of a linear law, means univalently that after termination of a transient process, ε_{vl} velocity error becomes a stable value. It is possible if only the FCS is both stable and astatic system with 1st order astatism. That is why the conservative element, which transient process is shown in Fig. 9.5, is not a component of the ACS elements as well.

In terms of the stable system and Mykhailov criterion, characteristic vector $D(j\omega)$ will turn to $n \cdot \pi/2$ angle after ω frequency varies from 0 to ∞ being n order of the system). Since the characteristic vector has turned to $1.5\pi = 3 \cdot \pi/2$ angle that the FCS order is three $n = 3$.

In the context of astatic stable system with 1st order astatism, the velocity error ε_{vl} is

$$\varepsilon_{vl} = x_{in}(t) - y_{out}(t) = \frac{a_1}{K}, \tag{9.1}$$

where a_1 is a proportionality coefficient in terms of independent variable t within a linear law of input effect; and K is the overall intensification coefficient of a transfer function of the open $W(p)$ part of the FCS. Hence,

$$2t + 5 - (2t + 4) = \frac{2}{K}\cdot \quad (9.2)$$

Thus $K = 2$.

Determine transfer functions of dynamic elements shown in Fig. 9.2, 9.3, 9.4, and 9.6.

Fig. 9.2 demonstrates the logarithmic phase frequency response $\varphi(\omega)$ (where phase is specified in degrees) being typical for 1st order aperiodic element. φ dependence upon ω is as follows

$$\varphi(\omega) = -\mathrm{arctg}(T\omega). \quad (9.3)$$

Graph is Fig. 2 passes through a point with [1; -45] coordinates. We obtain

$$-45 = -\mathrm{arctg}(T). \quad (9.4)$$

Thus $T = 1$. Transfer function of the element is

$$W_1(p) = \frac{k_1}{p + 1}, \quad (9.5)$$

where k_1 is the intensification coefficient.

Fig. 9.3 demonstrates graph of a pulse transfer function of an aperiodic element. Pulse transfer function of the element is as follows

$$g(t) = \frac{k_2}{T_2} e^{-\frac{t}{T_2}}. \quad (9.6)$$

Thus

$$\begin{cases} \dfrac{1}{T_2} = 2 \\ \dfrac{k_2}{T_2} = 4 \end{cases} \quad (9.7)$$

Hence, $T_2 = 0.5$; and $k_2 = 2$. We have the transfer function

$$W_2(p) = \frac{2}{0{,}5p + 1}\cdot \quad (9.8)$$

Slope of the logarithmic amplitude frequency response in Fig. 9.4 is $-20\, dB/dec$. Such a characteristic is typical for an integrating element; it passes through a point with $[1; 20\lg(k_3)]$ coordinates. In this context, k_3 is the intensification coefficient. Consequently, $20\lg(k_3) = 0$. It is obvious that $k_3 = 1$. Transfer function of the integrating element looks like

$$W_3(p) = \frac{1}{p}\cdot \quad (9.9)$$

Slopes of the logarithmic amplitude frequency response in Fig. 9.6 are $0\, dB/dec$ i $-40\, dB/dec$. Such a characteristic is typical for an oscillating element.

Ordinate of horizontal share of the characteristic is connected with the intensification coefficient k_4 as follows

$$20\lg(k_4) = L(\omega_c). \qquad (9.10)$$

where ω_c is the corner frequency between slope angles $0\,dB/dec$ and $-40\,dB/dec$. In this case

$$20\lg(k_4) = 20. \qquad (9.11)$$

Thus $k_4 = 10$, and $W_4(p) = 10\,(T^2p^2 + 2dTp + 1)^{-1}$. In this context, T is the time constant; and d is the damping coefficient.

Taking into consideration the abovementioned characteristics of the ACS, its open part may involve the elements with such transfer functions as $W_1(p)$, $W_2(p)$, and $W_3(p)$ or $W_3(p)$, and $W_4(p)$. However, the overall intensification coefficient of the open ACS part with $W_3(p)$ and $W_4(p)$ elements does not correspond to the early calculated ($K = 2$) one despite their connection. In-series connected elements may become the open part of the ACS if $k_1k_2k_3 = 2$. It becomes possible in terms of $k_3 =$
$= 1$.

Hence, transfer function of the open ACS part consists of $W_1(p) = (p+1)^{-1}$, $W_2(p) = 2(0{,}5p+1)^{-1}$, and $W_3(p) = p^{-1}$ elements connected in series.

Simulate the FCS under the SIMULINK MATLAB environment.

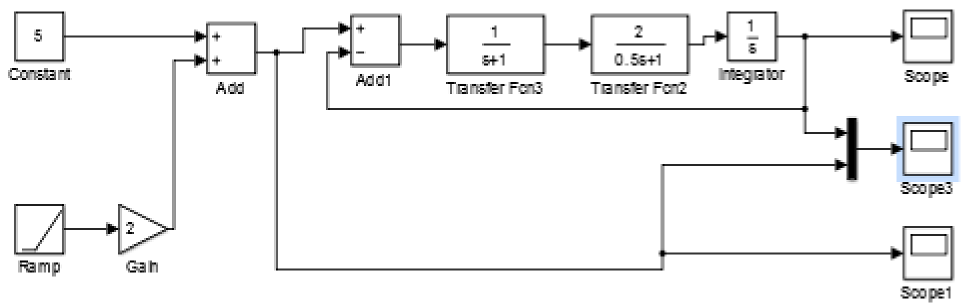


Fig. 9.8 The analyzed ACS

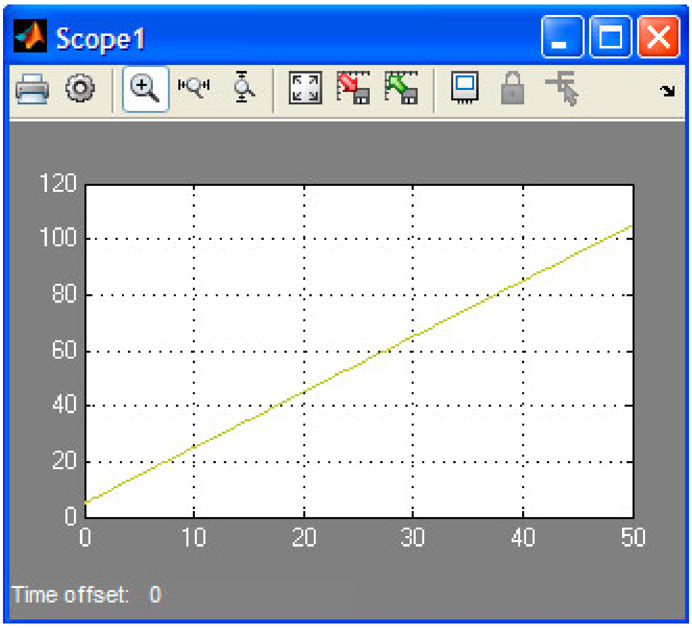


Fig. 9.9 Time changes in the input effect

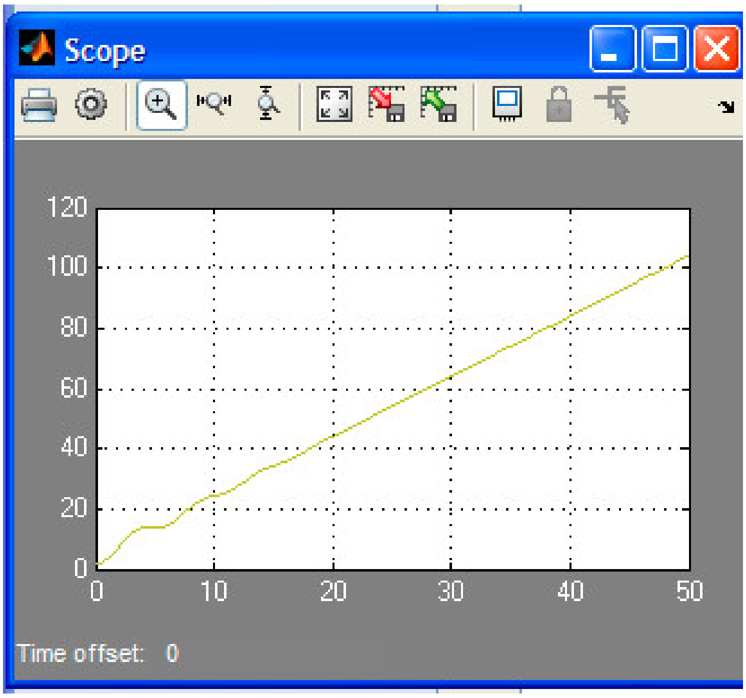


Fig. 9.10 Time changes in the output effect

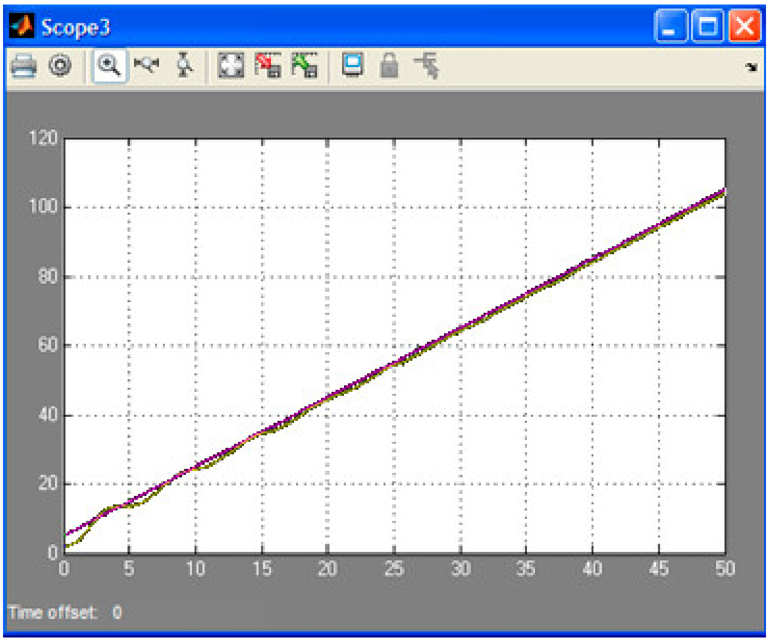


Fig. 9.11 Time changes in the input and output effects

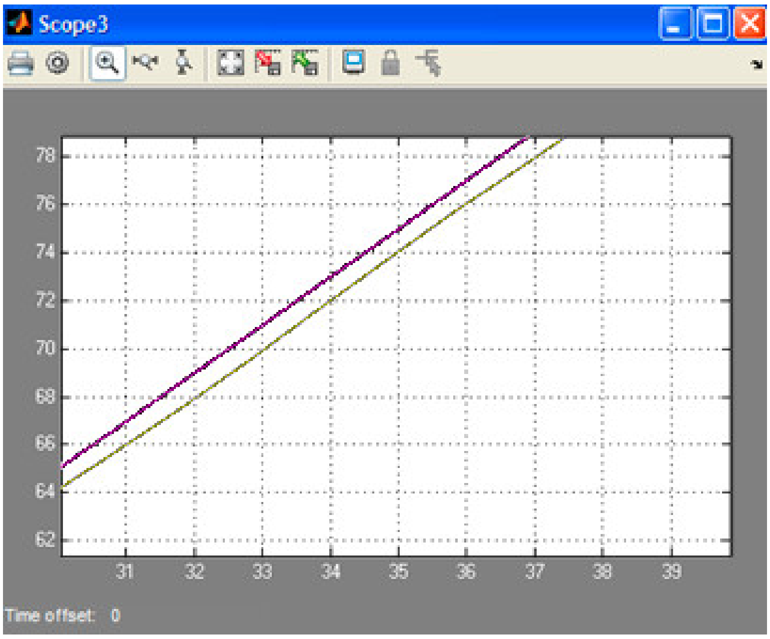


Fig. 9.12 Changes in the input and output effects in terms of the stable mode

Analysis of the graphs, concerning the changes in input and output values, helps conclude that the proposed ACS system corresponds to the problem statement.

2. Identify stability condition of the ACS.

Specify the overall intensification coefficient of the system open part as well as time constants of the dynamic elements as K, T_1, and T_2 respectively. Then, a transfer function of the ACS open part will look like:

$$W(p) = \frac{K}{p(T_1 p + 1)(T_2 p + 1)} = \frac{K}{T_1 T_2 p^3 + (T_1 + T_2)p^2 + p}. \quad (9.12)$$

Record characteristic polynom of the ACS:

$$D(p) = T_1 T_2 p^3 + (T_1 + T_2)p^2 + p + K. \tag{9.13}$$

Make a matrix of Hurwitz coefficients using the characteristic polynoms:

$$\begin{vmatrix} (T_1 + T_2) & K & 0 \\ T_1 T_2 & 1 & 0 \\ 0 & (T_1 + T_2) & K \end{vmatrix}. \tag{9.14}$$

The ACS will be stable if all determinants of the Hurwitz coefficient matrix are positive.

1st order determinant is:

$$\Delta_1 = T_1 + T_2 > 0. \tag{9.15}$$

It is obvious that $\Delta_1 > 0$ since the time constants T_1 and T_2 cannot be negative.

2nd order determinant is:

$$\Delta_2 = T_1 + T_2 - T_1 T_2 K. \tag{9.16}$$

The determinant will be positive if:

$$T_1 + T_2 > T_1 T_2 K, \tag{9.17}$$

or

$$\frac{1}{T_2} + \frac{1}{T_1} > K. \tag{9.18}$$

The last inequality is the ACS stability condition since the last determinant of the matrix of Hurwitz confidents is:

$$\Delta_3 = \Delta_2 \times K. \tag{9.19}$$

Since $K > 0$, then if $\Delta_2 > 0$ is fulfilled, the Δ_3 determinant will be more than 0. Test the determined condition of the RS using simulation.

Stable system.

The system will be stable if ${T_2}^{-1} + {T_1}^{-1} > K$; for instance, $K = 1$, $T_1 = 2$, and $T_2 = 0.1$.

Fig. 9.13 demonstrates the transient process graph.

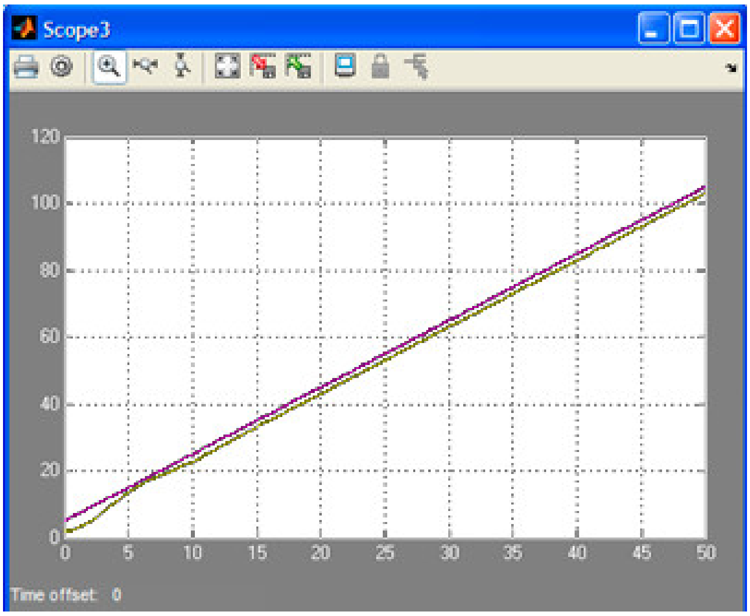


Fig. 9.13 Graph of a transient process within the stable RS

Unstable system.

The system will not be stable if $T_2{}^{-1} + T_1{}^{-1} < K$; for instance, $K = 5$, $T_1 = 4$, and $T_2 = 6$.

Fig. 9.14 demonstrates the transient process graph.

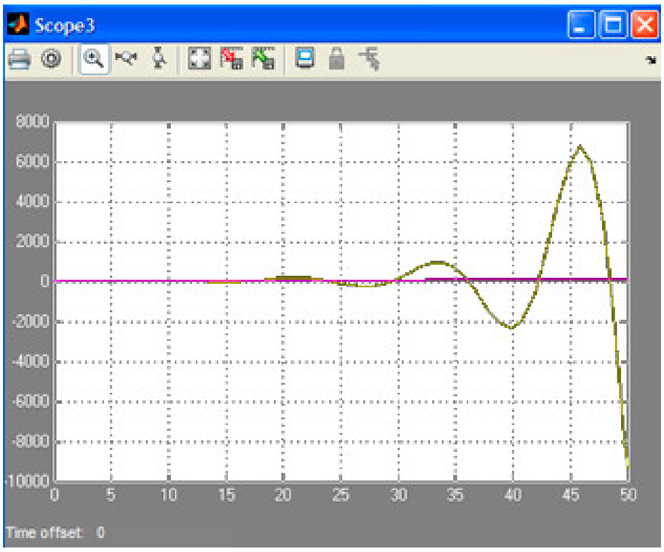


Fig 9.14 Graph of a transient process within the unstable ACS

System within its stability margin.

A system will be within its stability margin if $T_2{}^{-1} + T_1{}^{-1} = K$; for instance, $K = 3$, $T_1 = 1$, and $T_2 = 0.5$.

Fig. 9.15 demonstrates the transient process graph.

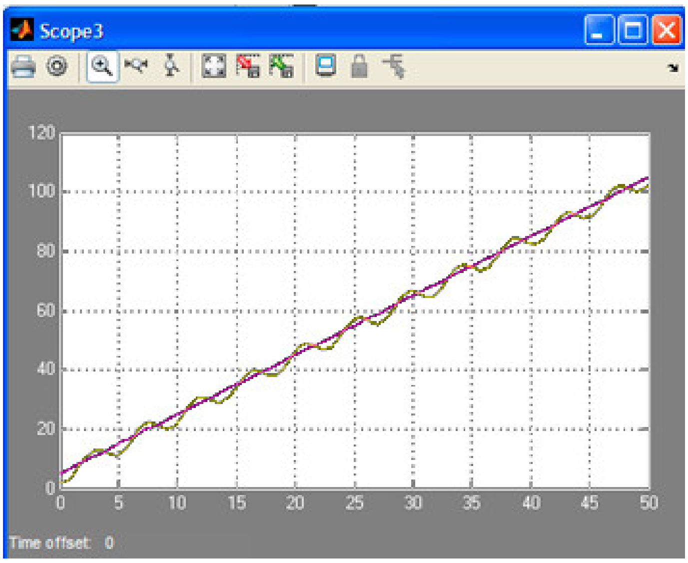


Fig. 9.15 Graph of a transient process within the ACS being within its stability margin

The simulation results have supported correctness of the FCS stability determination.

9.5 Report contents

Output data, and structural scheme of the analyzed system.

Calculations to identify the closed ACS order.

Calculations to determine the overall intensification coefficient of the open FCS part.

Calculations to identify dynamic elements of the open FCS part.

Calculations concerning the automated control system stability.

Structural scheme and simulation results as for the closed FCS as well as graphs of transient processes within the system determined relying upon the simulation results.

9.6 Control questions

What is the order of the automated control system?

What is the angle, the characteristic vector of a stable system will turn to?

How is it possible to identify a velocity error within the astatic system?

What is a characteristic point through which a logarithmic frequency response of integral element will pass?

What is the critical intensification coefficient?

Which of the automated control systems has characteristics of additivity and uniformity?

What is a slope of a logarithmic frequency response of an oscillation element?

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ТЕОРІЯ АВТОМАТИЧНОГО КЕРУВАННЯ
(ЛІНІЙНІ ТА ОСОБЛИВІ СИСТЕМИ)

Методичні рекомендації до виконання лабораторних робіт студентами спеціальності 151 «Автоматизація та комп'ютерно-інтегровані технології»

Methodical recommendations for laboratory works for students for specialty 151 «Automation and computer-integrated technologies»

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