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# INTEGRATED MODEL OF INVESTMENT PORTFOLIO OPTIMISATION

Investment portfolio optimization was an issue of scientific and practical interest since stock markets establishment.

However, since the global economy in general and Ukraine economy in particular are unstable, the challenge of financial resources efficient allocation becomes more urgent.

Traditionally, financial tools and mathematical-economical models that reduce risk and provide high-profitable investment are of particular interest in the periods of economical uncertainty.

Despite the existing variety of scientific and practical approaches to formation of investment strategy and risk management, classical Markowitz [1] and Sharpe models [2] are widely applied for direct distribution of funds among the assets.

The Pistunov-Sitnikov risk-revenue model was created as a solution to multicriteria optimization task ensuring risk minimum and profit maximum [3]. As a result convolution of two Markowitz criterion was formed.

$$\begin{cases} \dfrac{\sqrt{\sum\limits_{i} x_i^2 v_i^2} + \sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N} x_i x_j v_{ij}}{\sum\limits_{i}^{N} x_i d_i} \to \min \\ \sum\limits_{j=1}^{N} x_j = 1 \\ x_j \geq 0,\ j = 1,..,N \end{cases}$$

where $d_i$ – average rate of return of asset $i$, $v_i$– variation (standard deviation) rate of return on asset $i$, $v_{ij}$ – covariance of profitability of assets $j$ and $i$, $x_{i,j}$ – the share of capital spent on the purchase of securities $i$ and $j$.

We use the Pistunov-Sitnikov approach, that allows assembling two Markowitz models into one by putting the rate of return into numerator and the risk into the denominator.

Considering the Sharpe model we have $R_f$ – rate of return on the risk-free asset. Elimination of this parameter provides us with portfolio level of profitability i.e. maximum aiming function.

But there is a rate of return index in the denominator of Pistunov's optimal portfolio model.

Two options exist to combining two criteria. One way is to merge them by summing, but then their weighting have to develop. Another, more simple, option is to multiply two criterions, in this case the denominator will look like:

$$(\sum_{j=1}^{N} \alpha_j x_j + R_m \sum_{j=1}^{N} \beta_j x_j) \cdot \sum_{j=1}^{N} d_j x_j$$

Let us consider the main constrain of the Sharp's model:

$$\sqrt{p_m^2 (\sum_{j=1}^{N} \beta_j x_j)^2 + \sum_{j=1}^{N} p_j^2 x_j^2} \le p_{req}$$

This inequality ensures the implementation of the premise that the risk of the portfolio should not exceed pre-specified risk frontier. The counterpart of this constrain is also present in the Markowitz model, as well as set beforehand expected portfolio return. But the model Pistunov-Sitnikov allows to omit the definition of such values as predefined profit and risk. On the top of that, the index that characterizes risk ($\beta_i$) is present in the denominator. Therefore, the Sharp's model constrain is discarded completely. So a simplified version of the denominator is multiplied by the denominator of Pisunov's model and the numerator remains unchanged.

Thus, Integrated Pistunov-Sitnikov-Sharpe model is:

$$\begin{cases} \dfrac{\sqrt{\sum_i x_i^2 v_i^2} + \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j v_{ij}}{(\sum_{j=1}^{N} \alpha_j x_j + R_m \sum_{j=1}^{N} \beta_j x_j) \cdot \sum_{j=1}^{N} d_j x_j} \to \min \\ \sum_{j=1}^{N} x_j = 1 \\ x_j \geq 0,\ j = 1,..,N \end{cases}$$

To compare the effectiveness of the models data of NYSE trades are used. Calculation and comparison was conducted for the results obtained by the models of Sharpe, Pistunov-Sitnikov and Markowitz with a given income and a given risk. Especially for this comparison criterion of relative riskiness was developed, that is calculated according the formula:

$$Vr = R/M,$$

Where $R$ – risk, a $M$ – rate of return of asset.

The criterion of the relative riskiness shows the effectiveness of the model, since it determines the ratio of risk to the income portfolio. The smaller $Vr$ – the more effectively investment is distributed.

Calculations were made on the energy sector, as this stock market segment is unstable vibrations of this segment are typical. The portfolio is formed of six companies of the energy sector (Table 1).

Estimation on described optimization models defines different structure of optimal portfolios (shown in Table 1.). The developed integrated model shows the lowest relative riskiness $Vr = 0.000702$ and the highest income $Mp = 64.37$. The second performance is proved to be model Pistunov-Sitnikov with estimated relative riskiness $Vr = 0{,}00077$, and rate of return $Mp = 62{,}5$.

Analysis of existing approaches to optimal investment portfolio formation allowed us to develop the model of optimal investment portfolio structure. The model was tested on real data of equities of the energy sector, traded on the New York Stock Exchange. The effectiveness of the model was proved by the comparison with the

results of calculations on existing optimal portfolio models: Markowitz, Sharpe and Pistunov-Sitnikov. The optimal investment portfolio was formed of equities of six companies: Ameren, Amirican Electric, DTE Energy, Edison International, Enbridge, Public Service Enterprice Group.

The obtained results indicate that the developed integrated model is the most effective among models considered on the rate return criteria. This conclusion is supported with the criterion of relative riskiness that was developed specially for this study. The relative riskiness (0.000702) is minimal for the portfolio formed on integrated model.

Table 1

Comparison of calculations on optimization models

| Model | A | AE | DTEE | EI | En | PSEG | *Mp* | *rp* | *Vr* |
|---|---|---|---|---|---|---|---|---|---|
| Developed integrated model | 22,8% | 33,8% | 22,6% | 6,6% | 1,7% | 12,5% | 64,3 | 0,0452 | 0,000702 |
| Pistunov-Sitnikov model | 22,7% | 42,6% | 24,7% | 3,1% | 0,0% | 6,9% | 62,5 | 0,04814 | 0,0008 |
| Sharpe model | 0,0% | 0,0% | 22,7% | 52,0% | 25,3% | 0,0% | 59,3 | 2,8 | 0,0472 |
| Markowitz (risk minimisation) | 91,6% | 8,5% | 0,0% | 0,0% | 0,0% | 0,0% | 48,6 | 4,999 | 0,102801 |
| Markowitz (profit maxomisation) | 20,6% | 79,4% | 0,0% | 0,0% | 0,0% | 0,0% | 59,9 | 6,2144 | 0,1036 |

Legend for corporations: A – Ameren, AE – American electric, DTEE – DTE Energy, EI – Edison international, En – Enbridge, PSEG – Public Service Enterprice Group,

## References

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