УДК 623.746.5:629.7.045 https://doi.org/10.33271/crpnmu/75.119

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NUMERICAL SIMULATION OF AN EXTERNAL BALLISTIC PROBLEM USING ANALYTICAL APPROACH AND ATMOSPHERE FLOW VISUALIZATION BY FINITE ELEMENT METHOD

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ЧИСЕЛЬНЕ МОДЕЛЮВАННЯ ЗАДАЧІ ЗОВНІШНЬОЇ БАЛІСТИКИ З ВИКОРИСТАННЯМ АНАЛІТИЧНОГО ПІДХОДУ ТА ВІЗУАЛІЗАЦІЇ АТМОСФЕРНОГО ПОТОКУ МЕТОДОМ СКІНЧЕННИХ ЕЛЕМЕНТІВ

Purpose. Analysis of the dynamic characteristics of the cargo in the presence of the speed of its carrier, taking into account the influence of the external environment and the determination of factors that affect the operational characteristics and parameters for the dynamic system control.

Methodology. An approximate analytical approach to solving the nonlinear problem of external ballistics of a system with time-dependent parameters, which is implemented using the asymptotic perturbation method, and a numerical algorithm for modeling a dynamic process using a 3D software complex and the finite element method, which allows to visualize the nature of the flow around the object under wind load conditions.

Findings. An analytical approach to solving the nonlinear problem of external ballistics of a system with time-varying parameters is proposed, as well as a numerical model, solution and visualization of a dynamic process that can be applied in problems of mathematical physics and engineering calculations.

Originality. The use of an approximate analytical approach to solving the nonlinear problem of external ballistics, which includes a system of time-varying parameters, is a significant innovative step. For the first time, a three-dimensional distribution of the free fall parameters under atmospheric conditions was obtained depending on the time of free fall under the condition of an initial velocity different from zero. Obtaining the characteristics of the distribution, taking into account the aerodynamic quality of the object, made it possible to adapt the methods of analytical mechanics and differential equations with variable coefficients to the solution of the applied problem. The obtained characteristic three-dimensional surface allows to perform a study of the aerodynamic quality of the object under study using numerical methods and to visualize dynamic processes in three-dimensional space in order to obtain a qualitative picture of the perturbation of the object by the air flow. This integrated approach to the study of ballistic characteristics of cargo forms the scientific principle to perform engineering calculations to solve the problem of controlling dynamic systems.

Practical value. An approximate analytical approach and a calculation model of the dynamic process of a mechatronic unmanned system make it possible to increase its efficiency in the presence of an initial speed and ensure the accuracy of the application of ballistic cargo delivery systems in real operating conditions.

Keywords: *external ballistics, nonlinear system of differential equations with variable coefficients, 3-D and FEA-modeling, aerodynamic pressure, wind load, visualization of atmospheric flow.*

Introduction. The use of air vehicles for the cargo delivery in the conditions of man-made disasters consequences liquidation and the determination of the characteristics of the complex dynamic systems components, taking into account the aerodynamic parameters and the influence of the external environment, is relevant from the improving their operational characteristics point of view. One of the effective methods of delivering cargo to a given target is the free-falling method of dropping from low heights and flight speeds of the carrier [1, 2].

A significant number of publications in the field of research is related to the creation of numerical and analytical mathematical models built on the basis of the theory of external ballistics [3]. The peculiarity of the aerodynamic load is that its value significantly depends on the shape of the cargo flowing around the wind stream.

The construction of analytical models of the object's behavior in the wind flow, and even further analysis, encounter a few mathematical and computational difficulties. In view of this, mathematical modeling of dynamic behavior in software complexes that use 3D and FEA modeling [1, 3–6].

Main part. The use of air vehicles for the delivery of cargo to specified targets allows considering the task in the formulation "stop over the object" or according to the classic scheme [1]. In the first case, the load moves like a free falling body; in the second - the cargo, moving towards the drop as one unit with the air vehicle, will describe a certain curve during the fall (Fig. 1). At a low speed (up to 50 m/s) and height (up to 200 m) of the air vehicle flight, one of the primary problems of cargo delivery accuracy is related to the presence of wind.The vertical component of air resistance slows the fall, and the horizontal component makes the trajectory steeper compared to the parabola that would be described by the load in a vacuum, and creates lag. The fall speed of the load increases over time until the air resistance, which increases proportionally to the square of the speed, reaches a value equal to the weight of the load.

Complete air vehicle flight dynamics model together with an environmental estimation method is a multidimensional, highly complex nonlinear system of equations requires the development of high-level guidance algorithms. Therefore, it is more logical to consider low-order nonlinear equations of that model behavior with a closed control loop system [7].

The dynamics of the point of mass $m(t)$ system movement with an initial speed V_0 from a height h in the presence of a frontal flow V_B based on the nonlinear theory of ballistics is considered. At the first stage, the scheme of the cargo motion and the system of its loading by external forces at the instant point of the trajectory in the planar section, which is presented in Fig. 1, is considered.

Multiple coordinate systems are generally required to obtain and understand the dynamic behavior of an air vehicle and its associated cargo. At the same time, one coordinate system is transformed into another using two basic operations: rotation and shift.

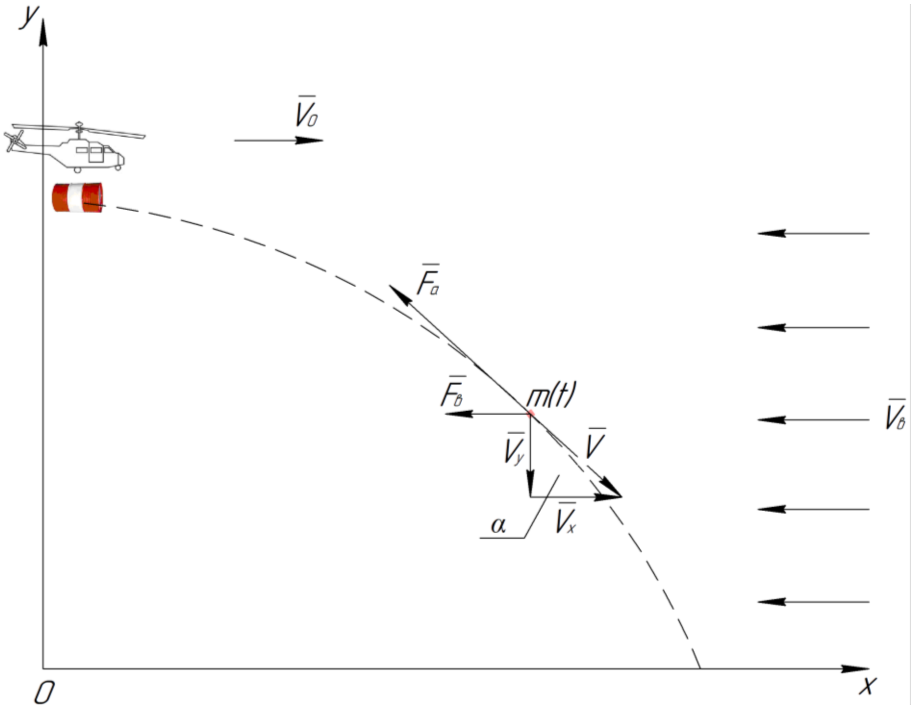

Fig. 1. Scheme of object motion and acting loads along the trajectory trace

The purpose of this work is a mathematical simulation and analysis of the dynamic characteristics of the load in the presence of the velocity of cargo carrier, taking into account the influence of the external environment and the determination of factors, that affect the operational characteristics and parameters of the system's dynamics control.

The main parameters of the studied system are determined by the following relations [8]:

$$F_c = C_x \cdot \frac{S_m \rho}{2} \cdot V^2(t) \quad F_v = C_x^* \cdot \frac{S_m^* \rho}{2} \cdot V_v^2, \tag{1}$$

where $\vec{F}_c, \vec{F}_v$ – aerodynamic resistance forces of the structure with stabilizer, including $\vec{F}_{cm}$, and wind load, respectively; C_x – drag coefficient (determined from experimental data in the direction of movement; C_x^* – the lateral resistance coefficient, which is calculated from the given initial conditions in the direction of the axis X; S_m, S_{xm}^* – area of the body middle along the axis of rotation of the body and the orthogonal direction; ρ – air density.

According to Fig. 1, the angle of the direction of the body's velocity is determined by relations:

$$\sin\alpha = \frac{V_x}{V}, \quad \cos\alpha = \frac{V_y}{V}. \tag{2}$$

In the formula for body velocity:

$$V(t) = \sqrt{V_x^2 + V_y^2} = V_y\sqrt{1 + \frac{V_x^2}{V_y^2}} \approx V_y(t). \tag{3}$$

Rule exists, if obeys the following relation:

$$\frac{V_x^2}{V_y^2} < 1. \tag{4}$$

Flight dynamics, including, flying object trajectories, as well as issues of stability and controllability during its movement. The study of trajectory tasks is carried out under the assumption that a flying object is a material point that moves under the action of applied forces. While studying the stability and controllability of a flying object, it is considered as a material body moving under the action of moments of these forces.

Most mechanical systems occur nonlinear properties under certain parameters of external disturbance. In this work, a flat nonlinear system of forces is considered, taking into account the nonlinear components of the motion of a material point with time-varying parameters. According to Newton's second law, the basic system of differential equations of the external ballistics of the dynamic process under study follows from the projections acting on the mass $m(t)$ forces, respectively, on the axis of the coordinate system X and Y. That is, the projection on the abscissa axis:

$$\sum X : \frac{d}{dt}\left[m(t)\left(V_x + V_0 \mp V_v\right)\right] = -F_c \sin\alpha - F_v \tag{5}$$

on the ordinate axis:

$$\sum Y : \frac{d}{dt}\left\{ m(t)\left[V_0 + V_y(t)\right]\right\} = F_c \cos\alpha - m(t)g. \tag{6}$$

To assess the nature of the dynamic behavior of the studied system, the following parameters are introduced:

$$b_0 = \frac{S_{\mathrm{m}}\rho}{m_0}, \tag{7}$$

$$\varepsilon = \frac{C_x}{2}, \tag{8}$$

$$\frac{d_x}{d_t} = \dot{x}, \tag{9}$$

$$a(t) = \dot{\bar{f}}(t) \tag{10}$$

$$m(t) = m_0 f(t). \tag{11}$$

The system of equations in the projections on the axis in the form (5) and (6) is reduced to the form:

$$\begin{cases} \dot{V}_x(t) + a(t)\cdot V_x(t) + \varepsilon b_0 \dfrac{1}{f(t)} V_x(t) V_y(t) = -a(t)\cdot\left(V_o \mp V_v\right) - \bar{F}_v \\ \dot{V}_y(t) + a(t) V_y(t) = \varepsilon V_o \dfrac{1}{f(t)} V_y^2(t) - g \end{cases}, \tag{12}$$

where: $$\bar{F}_v = \frac{C_x^*}{2} \cdot \frac{1}{f(t)} \cdot \frac{S_m^* \rho}{m_0}. \quad (13)$$

The final differential equation of the problem in the projection on the horizontal axis *X*:

$$\dot{V}_x(t) + a(t) \cdot V_x(t) + \varepsilon b_0 \frac{1}{f(t)} V_x(t) V_y(t) = \varnothing(t), \quad (14)$$

where: $$\varnothing(t) = -a(t) \cdot (V_o \mp V_v) - \bar{F}_v, \quad (15)$$

where ε – small parameter.

For time-independent system parameters accepted as:

$$a(t) = 0, \quad f(t) = 1. \quad (16)$$

The corresponding system of nonlinear differential equations (12) is transformed into the form:

$$\begin{cases} \dot{V}_x(t) + \varepsilon \cdot b_0 \cdot V_x(t) \cdot V_y(t) = \varnothing_0 \\ \dot{V}_y(t) = \varepsilon b_0 V_y^2(t) - g \end{cases}. \quad (17)$$

To obtain an approximate analytical solution of the nonlinear differential equations system with variable coefficients (17), the asymptotic method of perturbation with a small parameter, or a hybrid asymptotic approach based on the method of phase integrals and the Galerkin orthogonalization principle can be applied [3].

In order to determine the aerodynamic drag forces that act on a body during a fall, it is necessary to indirectly consider the shape. However, taking into account the heterogeneity of the environment, namely turbulence, non-laminar flow etc., the analytical determination of air resistance can introduce a high degree of integral error. The force of air resistance cannot be expressed by a simple and accurate formula obtained on the basis of theoretical conclusions. Air resistance is usually determined experimentally – by blowing the body in a wind tunnel. However, the use of numerical methods of modeling and analysis of the obtained results allows to increase the accuracy of the obtained data and provide multi-iteration modeling of the *i*-th number of geometric parameters of the body under different initial conditions. When modeling, it is possible to determine the coefficient of aerodynamic resistance, as a value proportional to the speed pressure and the force of air friction along the aerodynamic surfaces using turbulent flow models.

The numerical determination of the specified parameter will provide a ballistic assessment of the quality of the body, in particular the characteristic time.

The use of a numerical approach based on FEA makes it possible to estimate the influence of the specified parameters (7–11) on the dynamic behavior of the system under study. A pressure difference between the windward and leeward sides occurs in the air flow field relative to the centerline of the load. This difference creates aerodynamic control forces and moments relative to its center of mass (Fig. 2), which allows to achieve a vertical position at the moment of approach to the target and minimize deviations in the horizontal plane.

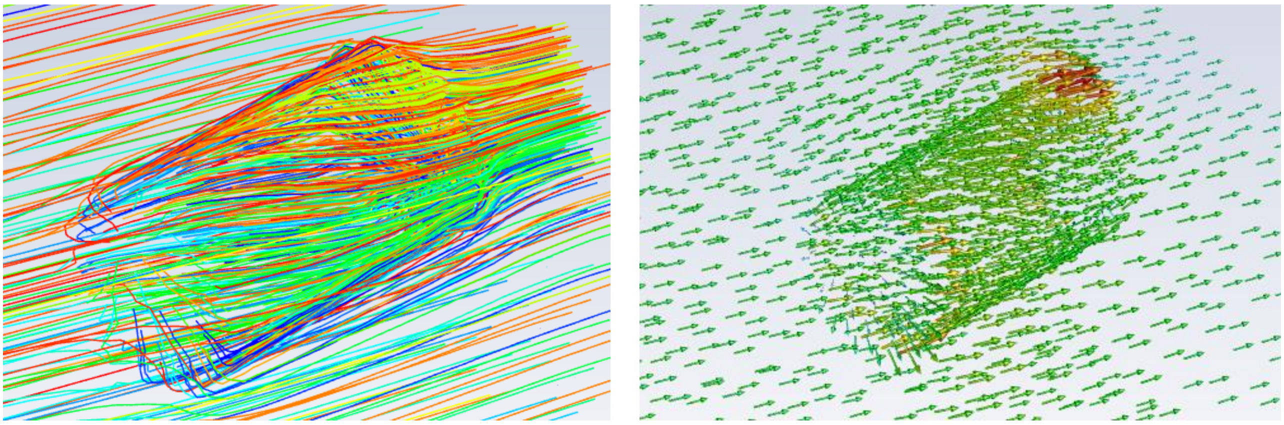

Fig.2. Pathlines and vectors of FEA calculated air flow while cargo free falling, obtained via numerical simulation

The control forces and moments created by the surfaces of the planes of the load must be sufficient to move it into such a position as to have minimal aerodynamic resistance to the oncoming flow.

The static stability of the load can be defined as the tendency to return to a given flight path after being dropped [9]. Determine the static stability of the load and the location of the center of mass and the center of pressure on its floor. In free fall, the wind force and the moment arm formed by the center of mass and the center of pressure cause rotational motion around the center of mass and in the planes of available degrees of freedom.

The margin of static stability of the load is defined as the ratio between the distance from the center of mass to the center of pressure and the diameter of the case (if it has an axis of symmetry). As a rule, a high margin of static stability (≥ 1) provides more stable indicators of rectilinear movement with increased resistance to external forces to change the trajectory of movement.

The absence of turbulent flows, which is a condition for high accuracy of hitting the target, is confirmed by the calculation in Fluid Flow. To formulate the condition of cargo stabilization after dropping using Fluid Flow, the stabilizing moment around the center of mass, which is formed by the forces on the surface of the aerodynamic planes, was determined (if the cargo has a complex geometric shape).

To check out the developed approach correctness, the model has been benchmarked with example parameters.

The dynamics of a material point with parameters are considered to represent the mass and geometry parameters of the cargo to be delivered.

In fig. 3 the dependence of the time fall on the parameters of the studied system and the height of the fall is presented.

Thus, on above mentioned, the control system allowed to increase the drop-delivery with higher accuracy, using the benchmarked relations.

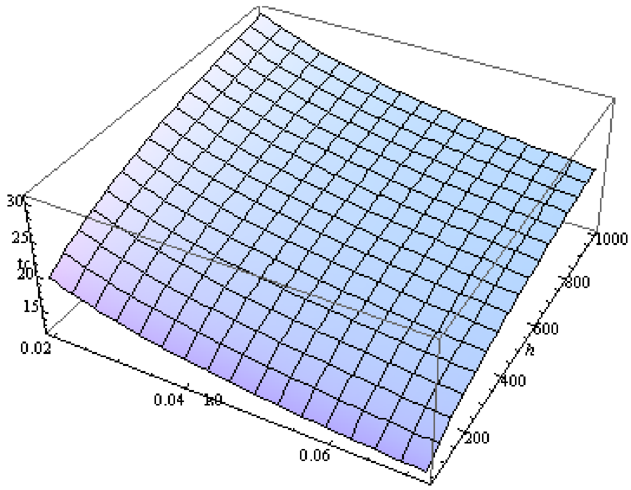

Fig. 3. Three-dimensional surface of the time fall dependence on parameters of the studied system

Conclusion. A mathematical model of the external ballistics of the cargo is proposed, taking into account the geometric characteristics and material in the presence of the speed of the carrier. The results of numerical calculations based on computer algebra, using 3D and FEA modeling methods for analyzing the dynamics of the studied system according to the proposed parameters are given. The proposed approach proves that the geometry of the shape-forming surface of the load, related to the aerodynamic and initial parameters of the system under study, are significant factors affecting the operational characteristics and control parameters of the dynamics of the system.

To check out the developed approach correctness, the model has been benchmarked with example parameters of system. The dependence of the cargo drop time on the parameters of the studied system and the drop height was obtained.

The obtained research results can be applied for the further formulation of the analytical-numerical approach of the system with variable properties and the environment.

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АНОТАЦІЯ

Мета. Аналітичний підхід і чисельне моделювання динамічного процесу задачі зовнішньої балістики вантажу за наявності початкової швидкості і впливу повітряного середовища з метою визначення факторів, що впливають на параметри керування досліджуваною системою.

Методика. Наближений аналітичний підхід до розв'язку нелінійної задачі зовнішньої балістики системи з параметрами, залежними від часу, який реалізується із застосуванням асимптотичного методу збурення, та чисельний алгоритм моделювання динамічного процесу з використанням 3D програмного комплексу і методу скінченний елементів, що дозволяє візуалізувати характер обтікання об'єкту в умовах вітрового навантаження.

Результати. Запропоновано аналітичний підхід до розв'язку нелінійної задачі зовнішньої балістики системи із змінними у часі параметрами, і чисельна модель, розв'язок і візуалізація динамічного процесу які можуть бути застосовані в прикладних задачах математичної фізики та інженерних розрахунках.

Наукова новизна. Використання наближеного аналітичного підходу до вирішення нелінійної задачі зовнішньої балістики, яка включає систему змінних параметрів у часі, є значним інноваційним кроком. Вперше було отримано тривимірний розподіл параметру падіння вантажу в умовах атмосферного впливу в залежності від часу вільного падіння при умові початкової швидкості, відмінної від нуля. Отриманні характеристики розподілу із урахуванням аеродинамічної якості об'єкту дозволили адаптувати методи аналітичної механіки та диференціальних рівнянь зі змінними коефіцієнтами до вирішення прикладної задачі. Отримана характеристична тривимірна поверхня дозволяє виконувати дослідження аеродинамічної якості досліджуваного об'єкта із використанням чисельних методів та візуалізувати динамічні процеси у тривимірному просторі з метою отримання якісної картини збудження об'єкта потоком повітря. Даний інтегрований підхід до дослідження балістичних характеристик вантажів формує науковий принцип виконання інженерних розрахунків для вирішення задачі керування динамічними системами.

Практична значимість. Наближений аналітичний підхід і розрахункова модель динамічного процесу мехатронної безпілотної системи дозволяють підвищити її ефективність при наявності початкової швидкості і забезпечення точності застосування балістичних систем доставки вантажів в реальних умовах експлуатації.

Ключові слова: *зовнішня балістика, нелінійна система диференціальних рівнянь із змінними коефіцієнтами, 3-D та МСЕ-моделювання, аеродинамічний тиск, вітрове навантаження, візуалізація атмосферного потоку.*