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INVESTIGATION OF THE RESISTANCE OF AN INFINITE RECTANGULAR CIRCUIT

Introduction. The following problem was proposed in 2016 at the all-Ukrainian Physics Olympiad [1]. An infinite circuit is given (see Fig. 1) where the resistance of each horizontal segment is equal to 1 Ω, and the resistance of each vertical segment is equal to 3 Ω.

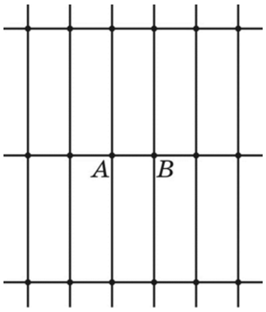

Figure 1 – Infinite circuit under consideration

The task was to estimate the general resistance of the given infinite circuit between the terminals *A* and *B*. The author solution [1] leads to the result $R = 0,65\Omega \pm 0,05\Omega$. However, the problem occurs to calculate the general resistance under consideration with higher accuracy.

Problem solution and generalization. We propose the following method in order to obtain rather accurate result for the general resistance under consideration. Lee us first of all consider a segment *AB* (0-layer circuit, see Fig. 2). The corresponding resistance is equal to 1 Ω. By adding a layer at all sides of the circuit, we obtain the 1-layer circuit (see Fig. 3). Once again by adding a layer at all sides, we obtain the 2-layer circuit (see Fig. 4), and so on.

Figure 2 – 0-layer circuit

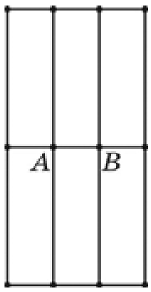

Figure 3 – 1-layer circuit

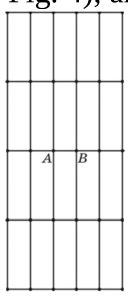

Figure 4 – 2-layer circuit

Let us denote the general resistance of an *n*-layer circuit as R_n. Obviously, the general resistance of an infinite circuit is the following limit:

$$R = \lim_{n \to +\infty} R_n \qquad (1)$$

An analytical expression for this limit can hardly be calculated, but its numerical value can be estimated by calculation of the members of sequence R_n. The program which calculates the corresponding members is written in the Wolfram Mathematica package on the basis of the first and the second Kirchhoff's circuit laws. The obtained results are shown in Table 1. The calculation of R_n contains the solving of an algebraic linear system of equations which contains $2n^2 + 3n$ equations, so we restrict ourselves to the calculation of the 90-layer circuit, which requires 16470 equations. As can be seen, the more layers are taken, the less is the change of R_n.

Table 1. The results for the the members of sequence R_n

n	R_n, Ω	n	R_n, Ω	n	R_n, Ω	n	R_n, Ω
0	1	5	0.670420	10	0.667736	15	0.667166
1	0.714286	6	0.669382	11	0.667562	16	0.667108
2	0.684276	7	0.668724	12	0.667427	17	0.667060
3	0.675725	8	0.668280	13	0.667321	⋮	⋮
4	0.672209	9	0.667966	14	0.667236	90	0.666682

As can be seen, the values R_n rounded off to 3 decimal places are equal to 0.667 Ω if $n \geq 12$. So, one can conclude that the rounded off to 3 decimal places result for the infinite circiut resistance is $R = 0.667$ Ω.

A generalization of the problem under consideration is proposed. Let us consider a similar circuit where the resistance of each horizontal segment is equal to R_x, and the resistance of each vertical segment is equal to R_y. It is shown that in such a case the infinite circiut resistance is as follows:

$$R = R_x f(R_y / R_x) \tag{2}$$

where the function $f(x)$ obeys the following functional equation:

$$f(x) + f(1/x) = 1. \tag{3}$$

The following solution is proposed

$$f(x) = x^{\alpha} / (1 + x^{\alpha}). \tag{4}$$

where the coefficient $\alpha \approx 0.6$ is found on the basis of the least-squares method. The corresponding graphic comparison of the values of $f(x)$ calculated on the basis of (4) with $\alpha = 0.6$ and of the computer calculation based on the 50-layer circuit is given on Fig. 5.

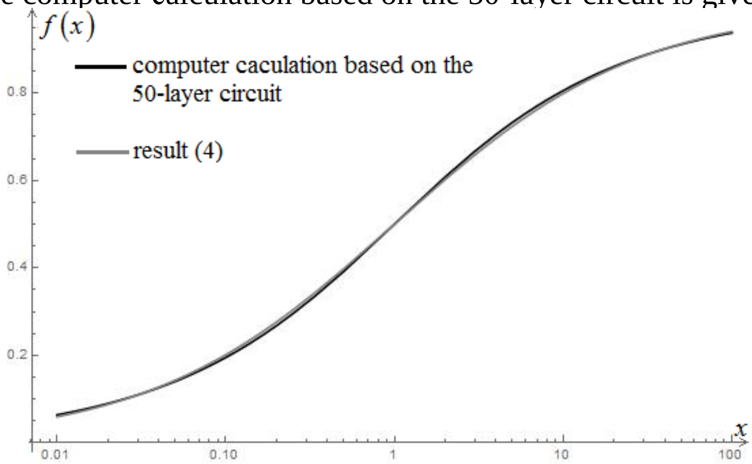

Figure 5 – Graphic comparison of the results

In the case where $R_x = R_y$ our results coincide with the well-known results [2].

Conclusions. The infinite circuit resistance in the framework of the problem [1] is calculated to rounded off to 3 decimal places, the accuracy of this result is much higher than the accuracy of the author solution result [1]. The problem is also generalized in the case of arbitrary horizontal and vertical resistances.

References

1. All-Ukrainian physics olympiad 2016, 9 grade. Problems and author solutions https://upho.org.ua/national/national-2016-09-theory-solutions.pdf

2. D. Atkinson and F. J. van Steenwijk, "Infinite resistive lattices", American Journal of Physics, 67 (6), 1999, p. 486–492, doi: 10.1119/1.19311.