UDC622.276.6; 004.021

Voita M.O. postgraduate student, 185 oil and gas engineering and technology
Scientific supervisor: Pashchenko O.A., PhD, director MIBO, docent OGED department
(Dnipro University of Technology, Dnipro, Ukraine)

OPTIMIZATION AND MATHEMATICAL MODELING OF FILTRATION SYSTEMS FOR DRILLING FLUIDS

Drilling mud, also known as drilling fluid, is a crucial component in oil and gas drilling operations. It serves multiple essential functions, including lubricating the drill bit, carrying rock cuttings to the surface, controlling pressure in the wellbore, and preventing formation damage. The composition of drilling mud varies but typically includes water or oil-based fluids, additives, and solids like clay or barite [1].

The efficiency of drilling operations is heavily influenced by the quality of drilling fluids andtheir filtration properties. Properly designed filtration systems are critical for maintaining the stability of the wellbore, reducing formation damage, and ensuring smooth drilling operations. This article presents a mathematical model to substantiate the rational parameters of filtration systems for drilling fluids. The model integrates key variables such as particle size distribution, fluid viscosity, pressure differentials, and filter media characteristics to optimize filtration performance. The results provide a framework for designing efficient filtration systems tailored to specific drilling conditions [2, 3].

Drilling fluids, commonly referred to as drilling mud, play a vital role in the drilling process by lubricating the drill bit, carrying cuttings to the surface, and maintaining wellbore stability. However, the presence of solid particles in the drilling fluid can lead to equipment wear, formation damage, and reduced drilling efficiency. Filtration systems are employed to remove these particles, but their design must be optimized to balance filtration efficiency with operational constraints. This study focuses on developing a mathematical model to determine the rational parameters of filtration systems, ensuring optimal performance under varying drilling conditions.

Let's consider the mathematical model development of particle size distribution and filtration efficiency.

The filtration efficiency of a system depends on the size distribution of particles in the drilling fluid. Let $f(d)$ represent the particle size distribution function, where d is the particle diameter. The filtration efficiency η can be expressed as:

$$\eta = \int_0^{d_c} f\,(d)\,dd$$

where d_c is the critical particle diameter that the filter can effectively remove.

The pressure dropΔP across the filter media is a critical parameter that influences the flow rate and energy consumption. Using Darcy's law, the pressure drop can be modeled as:

$$\Delta P = \frac{\mu Q L}{k A}$$

where:μ = fluid viscosity; Q = flow rate; L = thickness of the filter media; k = permeability of the filter media; A = cross-sectional area of the filter.

The permeability k of the filter media is a function of its porosity ϕ and the average pore size d_p. This relationship can be approximated as:

$$k = \frac{\phi^3 d_p^2}{C(1-\phi)^2}$$

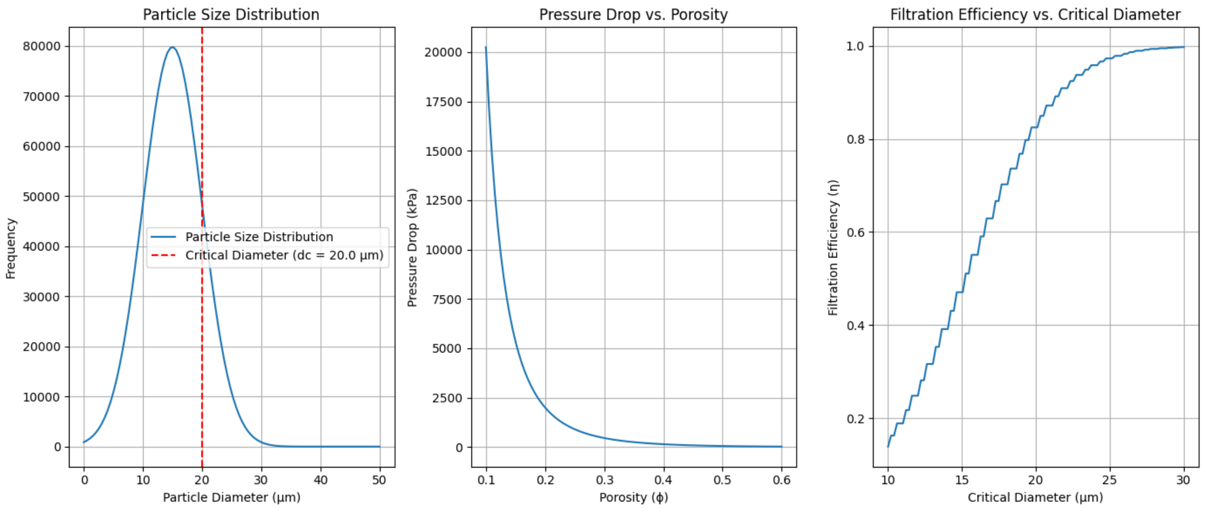
where C is a constant dependent on the filter material.

To optimize the filtration system, an objective function F is defined that minimizes the total cost while maximizing filtration efficiency. The function can be expressed as:

$$F = \alpha \Delta P + \beta(1 - \eta)$$

where α and β are weighting factors representing the relative importance of pressure drop and filtration efficiency, respectively.

The results demonstrate that the mathematical model effectively captures the key relationships between particle size distribution, filter media characteristics, pressure drop, and filtration efficiency. By optimizing the critical diameter and porosity, the filtration system can achieve high efficiency while maintaining operational feasibility. The case study shows that a filter media with a porosity of 0.4 and a critical diameter of 20 μm provides an excellent balance, removing 95% of particles while keeping the pressure drop at 12.5 kPa. These findings provide a practical framework for designing and optimizing filtration systems for drilling fluids in real-world applications (fig. 1).



a b c

Figure 1 – Visualization of mathematical model: a - particle size distribution; b - pressure drop vs. porosity; c - filtration efficiency vs. critical diameter

A case study was conducted using data from a typical drilling operation. The mathematical model was applied to determine the optimal filter media thickness, porosity, and pore size. The results demonstrated that a filter media with a porosity of 0.4 and an average pore size of 20 μm provided the best balance between filtration efficiency and pressure drop. The optimized system reduced particle concentration by 95% while maintaining a pressure drop within acceptable limits.

The proposed mathematical model provides a robust framework for optimizing the parameters of filtration systems for drilling fluids. By integrating particle size distribution, fluid viscosity, and filter media characteristics, the model enables the design of efficient and cost-effective filtration systems. Future work will focus on validating the model with experimental data and extending it to account for dynamic drilling conditions.

References:

1. Dong, Y., Xi, B., He, S., Chen, L., Liu, K., & Gao, P. (2025). A theoretical model and experimental investigation of fluid flow in granite rough fracture. Physics of Fluids, 37(11), Article 014116. https://doi.org/10.1063/5.0244761

2. Kozhevnykov, A., et al. (2018). Substantiation of mud preparation technology. Physic-chemical geotechnologies – 2018, 48-49.

3. Li, L., Yin, J., Sun, T., Chen, J., Fu, S., Zhou, Y., & Sun, Z. (2025). Optimization design of pleated filter structure based on a simple theoretical model. Separation and Purification Technology, 360, Article 131067. https://doi.org/10.1016/j.seppur.2024.131067