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## ON THE MEASUREMENT OF THE FREE FALL ACCELERATION ON THE BASIS OF THE ATWOOD MACHINE

**Introduction.** A well-known laboratory work dedicated to the measurement of the free fall acceleration on the basis of the Atwood machine is widely used in different universities, in particular, in the Dnipro University of Technology [1]. However, in [1] the block moment of inertia and the moment of frictional force in the block axis are neglected, which leads to understated measurement results. The mathematical model which takes into account the above-mentioned parameters of the Atwood machine is given in [2], but in [2] the problem of the free fall acceleration measurement is not considered. So, the aim the work is to investigate the accuracy of the measured results in the framework of the model which takes into account the block moment of inertia and to estimate the average moment of frictional force in the block axis.

**Measurements and estimates.** The Atwood machine is shown in Fig. 1. At the first stage, the motion is approximately uniformly accelerated because the load on the right-hand side of the string has the mass greater that the load on the left-hand side of the string. At the end of the first stage the ring  $\Delta m$  is separated from the load on the right-hand side of the string and the motion becomes approximately uniform.

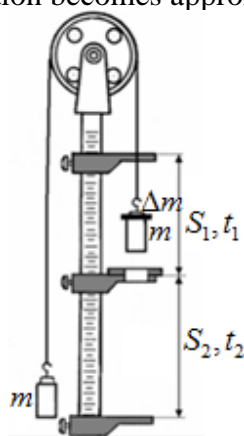


Figure 1 – Atwood machine

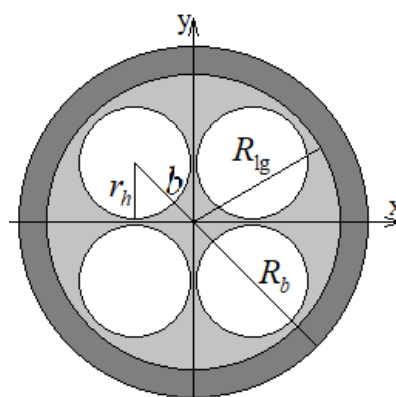


Figure 2 – Block of Atwood machine used in the Dnipro University of Technology

In the framework of the model [1] the design formula for the free fall acceleration is

$$g = \frac{2m + \Delta m}{\Delta m} \frac{S_2^2}{2S_1 t_2^2} \quad (1)$$

where the parameters in (1) are shown in Fig. 1. In the framework of the model which takes into account the block moment of inertia we derived the design formula for the free fall acceleration, which coincides with [2]:

$$g = \frac{S_2^2}{2S_1 t_2^2} \frac{(J/R_{lg}^2) + 2m + \Delta m}{\Delta m} \quad (2)$$

where  $J$  is the block moment of inertia. The measurements were carried out in the laboratory of the Physics department of the Dnipro University of Technology with the following

numerical parameters:  $m=60$  g,  $S_1=15$  cm,  $S_2=25$  cm. The block used in the measurements is shown in Fig.2, its moment of inertia is derived:

$$J = \frac{m_b}{\beta R_b^2 - 4r_h^2 + R_{lg}^2(1-\beta)} \left( \frac{\beta}{2} R_b^4 + \frac{1-\beta}{2} R_{lg}^4 - 2r_h^2(r_h^2 + 2b^2) \right) \approx 1.034 \cdot 10^{-5} \text{ кг} \cdot \text{м}^2 \quad (3)$$

where  $m_b=8.9$  g is the block mass,  $\beta=10/11$  is the ratio of surface densities of the “dark gray” and of the “light gray” parts in Fig. 2. The geometrical parameters in (3) are described in Fig. 2, their numerical values are as follows:  $R_b=4.25$  cm,  $R_{lg}=3.85$  cm,  $r_h=1.4$  cm,  $b=1.5\sqrt{2}$  cm. The obtained results are shown in Table 1, only the random error of the parameter  $t_2$  is taken into account. It should be stressed that the result (3) is confirmed by computer simulations in the Python programming language and all the errors presented in Table 1 are also obtained with the help of the Python.

Table 1. Obtained experimental results

$\Delta m$ , g	$t_2$ , s					$g$ , m/s <sup>2</sup> (1)	$g$ , m/s <sup>2</sup> (2,3)	$M_{fr}$ , 10 <sup>-4</sup> N·m
6.5	0.72	0.723	0.716	0.715	0.712	7.88 ± 0.12	8.32 ± 0.12	1.97 ± 0.16
8.5	0.616	0.607	0.610	0.619	0.620	8.34 ± 0.19	8.80 ± 0.19	1.7 ± 0.3
12	0.514	0.511	0.512	0.517	0.517	8.67 ± 0.12	9.13 ± 0.12	1.64 ± 0.29
15	0.459	0.463	0.460	0.467	0.455	8.83 ± 0.21	9.29 ± 0.23	1.5 ± 0.6
18.5	0.415	0.415	0.418	0.415	0.421	8.98 ± 0.14	9.43 ± 0.15	1.4 ± 0.5
20.5	0.396	0.400	0.399	0.391	0.396	9.08 ± 0.20	9.54 ± 0.21	1.1 ± 0.8
27	0.345	0.352	0.347	0.351	0.349	9.32 ± 0.19	9.77 ± 0.20	0.2 ± 1.0

As can be seen, the model [1] leads to the understated results. The model which takes into account the block moment of inertia leads to understated results for low masses  $\Delta m$ , but for rather high values of  $\Delta m$  this model leads to adequate results even without taking into account the moment of frictional force in the block axis. The design formula for the average moment of the frictional force in the block axis is derived in this work:

$$M_{fr} = \frac{2(J + 2mR_{lg}^2)^2}{R_{lg}t_2^2} \left( -\frac{S_2}{J + 2mR_{lg}^2} - \frac{2S_1}{J + R_{lg}^2(2m + \Delta m)} + \frac{\sqrt{2}}{J + R_{lg}^2(2m + \Delta m)} \times \right. \\ \left. \times \sqrt{2S_1^2 + \frac{S_1(J + (2m + \Delta m)R_{lg}^2)(2JS_2 + R_{lg}^2(4mS_2 + gt_2^2\Delta m))}{(J + 2mR_{lg}^2)^2}} \right). \quad (4)$$

The experimental results for  $M_{fr}$  are shown in Table 1. As can be seen, these results are rather accurate for low values of  $\Delta m$ , but for high values of  $\Delta m$  only the estimate by the order of magnitude is obtained for  $M_{fr}$  because of high errors.

**Conclusions.** The reliable results for the free fall acceleration may be experimentally obtained only for rather high values of  $\Delta m$  with taking into account the block moment of inertia. The average moment of the frictional force in the block axis is estimated. In particular, obtained results may be introduced to the educational process for the students of the physics specialty of the Dnipro University of Technology.

## References

1. Dnipro University of Technology, laboratory work devoted to Atwood machine, [https://physics.nmu.org.ua/ua/To\\_students/Day\\_mode\\_of\\_study/Methodical\\_instructions\\_to\\_laboratory\\_works/Mechanics/1\\_2\\_ua.pdf](https://physics.nmu.org.ua/ua/To_students/Day_mode_of_study/Methodical_instructions_to_laboratory_works/Mechanics/1_2_ua.pdf) (in Ukrainian).
2. Igor Sikorsky Kyiv Polytechnic Institute, laboratory work devoted to Atwood machine, [https://kzf.kpi.ua/wp-content/uploads/2021/09/Lab\\_rob\\_1\\_1-319.pdf](https://kzf.kpi.ua/wp-content/uploads/2021/09/Lab_rob_1_1-319.pdf) (in Ukrainian).