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INVESTIGATION OF THE RESISTANCE OF AN INFINITE PARALLELEPIPED CIRCUIT

Introduction. At the all-Ukrainian Physics Olympiad in 2016 I. M. Gelfgat proposed the following problem [1]. An infinite circuit is given (see Fig. 1) where the resistance of each horizontal segment is equal to 1 Ω, and the resistance of each vertical segment is equal to 3 Ω.

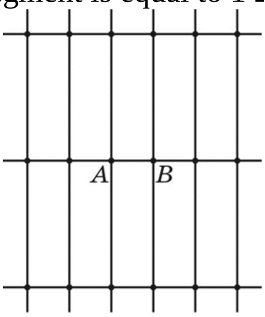

Figure 1 – Infinite circuit investigated in [1]

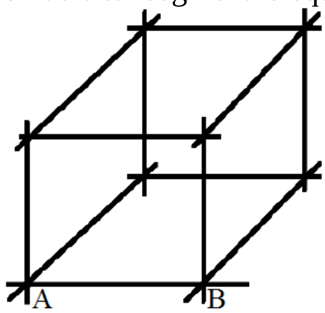

Figure 2 – Infinite circuit under investigation

The task was to estimate the general resistance of the given infinite circuit between the terminals *A* and *B*. In this work we propose the following generalization of the problem. We investigate general resistance of an infinite parallelepiped circuit (see Fig. 2) between the terminals *A* and *B*, each segment parallel to *Ox* has the resistance R_x, each segment parallel to *Oy* has the resistance R_y and each segment parallel to *Oz* has the resistance R_z.

Problem solution and generalization. First of all, a segment *AB* is considered (0-layer circuit, see Fig. 3), the corresponding resistance is R_x. Then a layer is added at all sides of the circuit (1-layer circuit, see Fig. 4). Once again a layer is added at all sides (2-layer circuit, see Fig. 5), and so on.

Figure 3 – 0-layer circuit

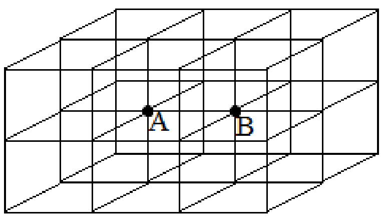

Figure 4 – 1-layer circuit

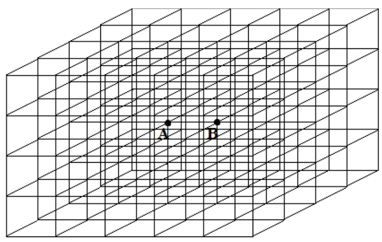

Figure 5 – 2-layer circuit

The general resistance of an *n*-layer circuit is denoted as R_n, the general resistance of an infinite circuit is equal to the following limit:

$$R = \lim_{n \to +\infty} R_n \,. \quad (1)$$

The program which calculates R_n is written in the Wolfram Mathematica package on the basis of the first and the second Kirchhoff's circuit laws. For example, obtained results for R_x=1 kΩ, R_y=2 kΩ and R_z=3 kΩ are shown in Table 1. The calculation of R_n contains the solving of an algebraic linear system of equations which contains $3n^3+7n^2+5n$ equations, so we restrict ourselves to the calculation of the 17-layer circuit, which requires 16847 equations. As can be seen, the more layers are taken, the less is the change of R_n. As can be seen, the values R_n rounded off to 3 significant digits are equal to 489 Ω if $n \geq 6$. So, one can conclude that the corresponding result for the infinite circiut rounded off to 3 significant digits is R=489 Ω.

Table 1. The results for R_x=1 kΩ, R_y=2 kΩ and R_z=3 kΩ

n	R_n, Ω	n	R_n, Ω	n	R_n, Ω	n	R_n, Ω
0	1000	5	489.525	10	489.042	15	488.984
1	516.968	6	489.305	11	489.022	16	488.979
2	494.931	7	489.186	12	489.008	17	488.976
3	491.120	8	489.116	13	488.998		
4	489.982	9	489.072	14	488.990		

It is shown that the infinite circuit resistance under consideration is as follows:

$$R = R_x f(R_y/R_x, R_z/R_x) \tag{2}$$

where the function $f(y,z)$ obeys the following functional equation:

$$f(y,z) + f(1/y, z/y) + f(1/z, y/z) = 1. \tag{3}$$

The following properties of $f(y,z)$ are proved:

$$f(1,1) = 1/3,\; f(y,z) = f(z,y),\; \lim_{z\to\infty} f(y,z) = g(y),\; \lim_{z\to 0} f(y,z) = 0 \tag{4}$$

where $g(y)$ obeys the functional equation: $g(y) + g(1/y) = 1$. The following expression for the function $f(y,z)$ is proposed:

$$f(y,z) = y^{\xi} z^{\xi} / \left(y^{\xi} + z^{\xi} + y^{\xi} z^{\xi}\right) \tag{5}$$

where the coefficient $\xi \approx 0.65$ is found on the basis of the least-squares method. The graphic comparison of the values of $f(y,z)$ calculated on the basis of (5) with $\xi = 0.65$ and of the computer calculation based on the 10-layer circuit is given on Fig. 6. The surface on Fig. 6 is built on the basis of (5), the points are the results of the 10-layer computer calculation.

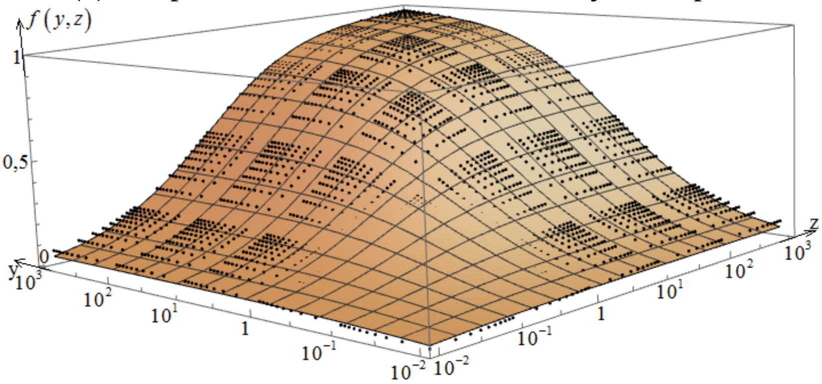

Figure 6 – Graphic comparison of the results

Conclusions. The problem of the calculation of the resistance of an infinite parallelepiped circuit with different resistances in three directions is considered. A program for the corresponding approximate calculation is written and an approximate analytical expression for the resistance under investigation is proposed. In the case where $R_x = R_y = R_z$ our results coincide with the results [2]. In [2] the result for different resistances in three directions is derived in terms of complex integrals, however the numerical data is given only for $R_x = R_y = R_z$. We propose the method which is based on the Kirchhoff's circuit laws and does require such a complicated technique as the complex integrals. The numerical comparison of our results with the results [2] for different resistances in three directions may be a plan for the future.

References

1. All-Ukrainian physics olympiad 2016, 9 grade. Problems and author solutions https://upho.org.ua/national/national-2016-09-theory-solutions.pdf

2. D. Atkinson and F. J. van Steenwijk, “Infinite resistive lattices”, American Journal of Physics, 67 (6), 1999, p. 486–492, doi: 10.1119/1.19311.