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THEORETICAL SUBSTANTIATION OF EFFECTIVE FRACTURE TOUGHNESS IN METALLIC ALLOYS OF CYLINDRICAL SHELLS

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ТЕОРЕТИЧНЕ ОБҐРУНТУВАННЯ ЕФЕКТИВНОЇ ТРІЩИНОСТІЙКОСТІ МЕТАЛЕВИХ СПЛАВІВ ЦИЛІНДРИЧНИХ ОБОЛОНОК

Purpose. To establish the relationship between the microstructural features of cast materials, specifically porosity, segregation, grain size, and non-metallic inclusions and effective fracture toughness under high-velocity dynamic loading by determination of theoretical dependencies for calculating effective fracture toughness under defined parameters of macroscopic defects in the material.

Methodology. A combined analytical–numerical approach was applied to model the dynamic response of cylindrical shells under high-velocity loading, implemented through a system of dynamic differential equations that account for material quality criteria and the principles of linear elastic fracture mechanics.

Findings. A methodology for assessing the quality of cast materials has been developed based on an integrated mechanical–mathematical model and the principles of linear fracture mechanics. Quantitative relationships for evaluating crack initiation resistance were obtained with consideration of the microstructural characteristics of the material. The correlation between the crack initiation criterion and the critical defect size was established.

Originality. For the first time, a comprehensive approach is proposed to predict the behavior of cast cylindrical shells based on a mechanical–mathematical crack formation model incorporating internal defects. Relationships were established between the microstructural characteristics of metallic alloys-such as porosity, grain size, and non-metallic inclusions – and effective fracture toughness, enabling the formalization of conditions governing the transition from a local defect to a critical crack size. A methodology was proposed for quantitative reproduction of porosity- and cracking-risk zones, adapted to specific alloy compositions and casting technologies.

Practical value. The obtained results provide a scientifically grounded basis for optimizing casting processes of cylindrical shells with specified performance properties. The proposed methodology enables determination of permissible ranges of microstructural parameters, ensuring controlled fracture toughness and predictable service life of the structures.

Keywords: *metal alloy, crack formation, microstructural heterogeneity, porosity, casting defects.*

Introduction. Cylindrical shells play a key role in fragmentation front formation processes, as the energy of high-velocity dynamic loading is utilized to accelerate metallic elements to significant speeds. In scientific and engineering practice, several constructive approaches to designing such shells are distinguished.

In the case of natural fragmentation, intensive internal pressure leads to an expansion of the casing by 50–60% of its initial diameter, which is accompanied by considerable plastic deformation of the material. This, in turn, causes structural failure with the formation of a large number of fragments of varying sizes. The configuration of fragments (mass, geometry, quantity) is determined by the wall thickness of the shell, the type of energy carrier, and the physic-mechanical properties of the casing material.

Another approach is based on programmed fragmentation, in which zones of stress concentration with predetermined geometry are introduced into the design. This ensures the formation of fragments of relatively uniform size and enables a more predictable fragmentation pattern.

The most technologically advanced approach is considered to be the use of a preformed fragmented shell, in which fragmentary elements are integrated into the casing during production using polymeric or metallic matrices. This method ensures controlled characteristics of mass, shape, and quantity of fragments, providing a predictable distribution of the fragmentation field. At the same time, under conditions of high-velocity dynamic loading, a significant portion of the energy is consumed in accelerating gaseous products toward the end regions of the shell, thereby reducing its potential for effective fragment acceleration. As a result, the coefficient of energy utilization remains low: only 10–15% of the fragment mass reaches the target, while the overall energy efficiency coefficient is about 10–20% [1–3]. This highlights the need for optimization of both the structural parameters of the shell and the properties of the material from which it is produced.

The modern approach to designing such systems involves integrating analytical methods with computer modeling tools to accurately predict energetic characteristics and failure processes. Particular emphasis is placed on developing mathematical models capable of describing nonlinear regularities in crack formation in cast shell structures. This is due to the fact that casting-related microstructural defects – primarily porosity and segregation of alloying elements – directly determine the mechanisms of crack initiation and propagation under internal stresses.

Accordingly, the requirements for materials consist in ensuring structural homogeneity, controlled grain size, absence of critical shrinkage defects, and stable mechanical properties. Such a combination of characteristics is essential for improving the reliability of shell systems and ensuring the predictability of their performance response under complex mechanical and thermal loading conditions.

Main part. As shown by recent studies, an adequate reproduction of shell behavior is possible only through comprehensive mechanical-mathematical models that account for the variability of material properties and the interaction of the shell with the gaseous medium. In this context, particular importance is attached to quality prediction criteria for cast materials, especially the Niyama criterion, which enables the assessment of susceptibility to shrinkage porosity and cracking. The application of this criterion in combination with numerical modeling of loading processes provides the possibility of developing effective engineering methodologies that ensure the compliance of the operational characteristics of shell structures with their functional purpose [4–6].

Taking these considerations into account, it is advisable to introduce a formalized mathematical description of the process, based on a system of differential equations of dynamics, which allows for a quantitative reproduction of the influence of microstructural and thermodynamic factors on the stability of the shell.

It should be noted that in the problems of nonlinear dynamics of systems with time-dependent parameters and external loading on the investigated system, the analytical solution requires the presence of a particular solution for further reduction to the Bernoulli equation, which in many cases represents a rather complex task. Therefore, in such cases, approximate analytical methods are generally employed, based on asymptotic approaches and hybrid techniques (perturbation, phase integrals in combination with the Galerkin principle) [7], in order to obtain approximate analytical solutions suitable for both "small" and "large" expansion parameters, or direct numerical methods of computation.

The results of thermohydrodynamic simulation of metallic alloy solidification provide the domain of Niyama criterion values N_{y}. The N_y map makes it possible to identify areas with an increased risk of shrinkage porosity and macro/microdefects and, after calibration for a specific alloy and technology, to optimize these values for the assessment of local porosity $\varphi(x)$ and associated defects.

The specified technological predictors are transformed into materials science equivalents: the effective fracture toughness $K_{Ic,eff}(\varphi)$, the yield strength $\sigma_{y,eff}(\varphi,d)$, and the elastic modulus $E_{eff}(\varphi)$, where d is the characteristic grain size.

Subsequently, the determination of impulse internal pressure (impact, blast-like, or another type of dynamic loading) can be computed as a stress field in the shell $\sigma(x,t)$, taking into account geometry and rate effects.

The phase of crack initiation and growth is described in terms of Linear Elastic Fracture Mechanics (LEFM) for a surface/edge crack with a rod (or plate) equivalent of K_I. The critical defect a_c defines whether an initial flaw will propagate under a given instantaneous stress. For cyclic or long-term loading, the Paris–Erdogan law may be applied to determine the crack growth rate, but for short impulses, the dominant factor is the ratio of the stress intensity factor K_I to the effective fracture toughness $K_{Ic,eff}$. In this case, the critical defect a_c is defined by the relation:

$$a_c = \frac{1}{\pi} \; \frac{K_{Ic,eff}}{Y\,\sigma}^{\;2}, \tag{1}$$

where Y – geometrical stress coefficient, σ – Stress in the defect region, Pa.

The stress distribution in an isotropic shell is expressed by the following relation, assuming the absence of initial defects:

$$\sigma_\theta\left(r_y,t\right) = p(t)\frac{r_0^2+r_i^2}{r_0^2-r_i^2}, \sigma_z(t) = p(t)\frac{r_i^2}{r_0^2-r_i^2}, \ \sigma_r(r_i,t) = -p(t), \tag{2}$$

where σ_θ – circumferential stress, Pa; σ_z – longitudinal stress, Pa; σ_r – radial stress, Pa; r_i – inner radius of the cylindrical shell, m; r_o – outer radius, m; t – wall thickness, m; L – shell length, m; $p(t)$ – Internal pressure as a function of time, Pa;

Existing dynamic models of cylindrical shells are predominantly simplified in nature, since when calculating the stress state over a wide range of harmonics they do not account for the full spectrum of real loading conditions as well as characteristic structural and technological factors. Such approaches are based on static computational schemes and, in essence, do not allow an adequate assessment of the material and structural resistance to damage evolution, particularly to crack formation.

Therefore, the development of generalized numerical models integrated with criterion-based approaches, grounded on the concept of effective fracture toughness, provides a foundation for solving the complex scientific problem of predicting the quality of cast shell products and controlling the evolution of cracking at the design stage. This approach enhances the efficiency of material development, shortens the design cycle, and ensures the stability of the operational performance of the finished components.

For thin-walled shells ($t \ll r_i$), a simplified approximation is applied for determining stresses in the loading zones:

$$\sigma_\theta \approx \frac{p\,r_i}{t}, \sigma_z \approx \frac{p\,r_i}{2t}. \tag{3}$$

Additional stress components, in particular the thermal stress that reduces the fracture toughness reserve, are determined by the following relation:

$$\sigma_{th} = \frac{E\alpha_T \Delta T}{1-\vartheta} \tag{4}$$

where ΔT – The difference in temperature between the inner and outer surfaces of the shell, K; E – modulus of elasticity, Pa; α_T- coefficient of linear thermal expansion, 1/К; ΔT– temperature difference, К; ϑ – Poisson's ratio

Thus, the effective hoop stress of the cylindrical shell is determined as follows:

$$\sigma_{\theta,eff} = \sigma_\theta + \sigma_{th} + \sigma_{res,\theta} \tag{5}$$

The local nominal stress in the defect zone is amplified by the stress concentration factor K_t:

$$\sigma_{loc} = K_t\,\sigma_{\theta,eff}, \tag{6}$$

$$K_t = 1 + 2\sqrt{\frac{a}{\rho}} \tag{7}$$

where a is the depth of the defect or crack, m; ρ is the radius of curvature at the crack tip, m.

The stress intensity factor under elastic behavior at the macro level is defined as the plastic zone near the crack tip (LEFM conditions):

$$K_I = Y\sigma_{loc}\sqrt{\pi\, a}\,, \tag{8}$$

where Y is the geometric factor (dimensionless), which depends on the shape and location of the crack, Y=1.1…1.8; σ_{loc} is the local stress in the defect zone, Pa.

The effective fracture toughness considering the structure is determined by the following relation:

$$K_{\mathrm{IC},eff} = K_{\mathrm{IC}0} \cdot \varphi_T(T) \cdot \varphi_\varepsilon(\varepsilon) \cdot \varphi_P(P) \cdot \varphi_d(d) \cdot \varphi_i(N_i) \tag{9}$$

where $\varphi_T(T)$ – temperature correction factor for the reduction of fracture toughness at low values of T; $\varphi_\varepsilon(\varepsilon)$ – strain rate correction factor; $\varphi_P(P)$ – porosity influence coefficient:

$$\varphi_P(P) = 1 - \beta_P P^{m_P}\,, \tag{10}$$

where P - porosity level, m_P- Weibull modulus; $\varphi_d(d)$ – grain size influence coefficient, defined as follows:

$$\varphi_d(d) = \left(1 + \beta_d d^{-\frac{1}{2}}\right)^{-\eta_d}, \tag{11}$$

where d – defect size, m; β_d – material sensitivity coefficient; η_d – exponent of material property degradation with increasing defect size; $\varphi_i(N_i)$ non-metallic inclusion influence factor

$$\varphi_i(N_i) = (1 + \beta_i N_i)^{-\eta_i}, \tag{12}$$

where N_i – is the quantitative characteristic of defects of a certain type.

Based on the above, graphs were constructed to illustrate the dependence of effective fracture toughness on the material porosity P for different grain sizes (Fig.).

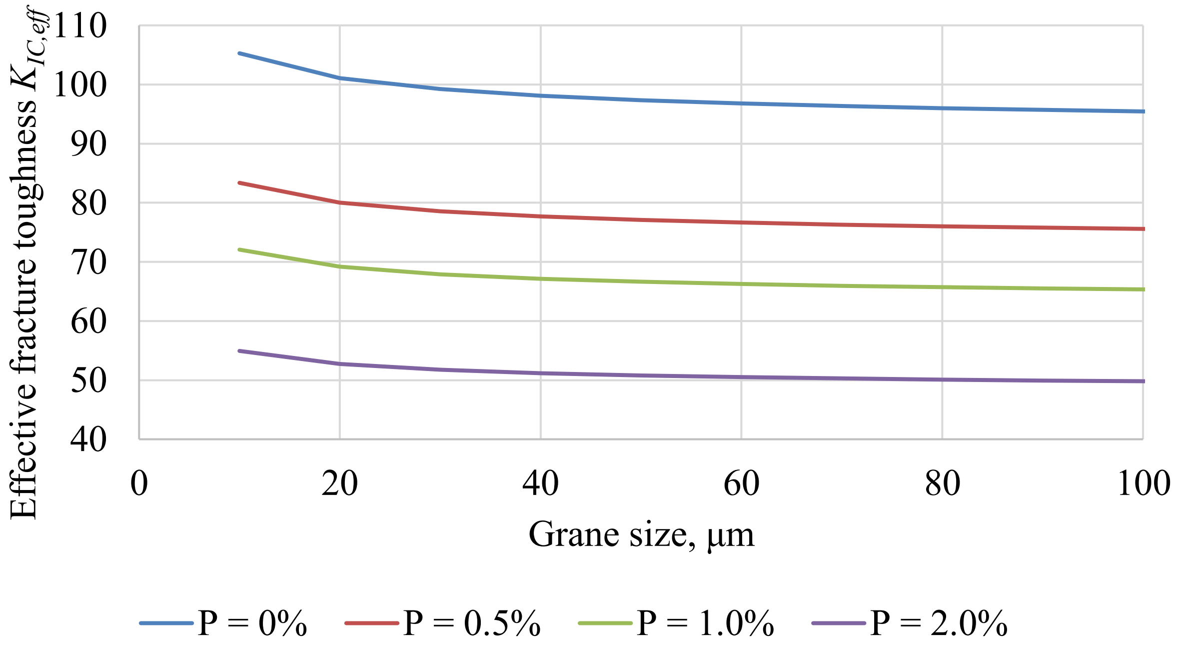

Fig. Influence of grain size structure and porosity on effective fracture toughness

The graph shows the effective fracture toughness $K_{IC,eff}$ as a function of the average grain size d for several levels of porosity P , at a fixed fraction of non-metallic inclusions N_i=0,2 and under quasi-static loading conditions ($\varphi_T(T) = 1$, $\varphi_\varepsilon(\varepsilon) = 1$).

The toughness change character in the graph is monotonically decreasing with both porosity and grain size. At low porosity (P = 0%), fracture toughness starts high (~105 MPa√m) and gradually decreases with grain size but remains significantly above the curves for porous materials. With increasing porosity (P = 0.5%, 1.0%, 2.0%), the initial toughness is progressively lower, and the rate of decline with grain size is more pronounced. At P = 2%, toughness is minimal (~50 MPa√m) and shows only slight reduction as grain size increases, indicating that porosity dominates over grain refinement at higher levels. Thus, porosity has the primary degrading effect, while grain size refinement plays a secondary but still notable role in sustaining higher toughness values. Thus, the physical meaning corresponds the theoretical approach.

The determination of the critical crack size, at which further propagation and fracture occur, is established by the crack initiation criterion K_I. The condition for crack initiation is defined as follows:

$$K_I \geq K_{IC,eff} \tag{13}$$

The determination of the critical defect size a_c will be carried out using the formula:

$$a_c = \frac{1}{\pi} \left(\frac{K_{IC,eff}}{Y \, \sigma_{loc}} \right)^2 \tag{14}$$

If the actual defect $a_0 \geq a_c$, crack growth becomes possible. Thus, the integration of quality prediction criteria for cast materials, in particular the Niyama criterion, with comprehensive mechanic-mathematical models and the principles of linear elastic fracture mechanics provides a foundation for the quantitative assessment of the stress-strain state of shells, taking into account microstructural characteristics. At the same time, determining the critical defect size enables the establishment of permissible ranges of the Niyama criterion during finite element modeling of the casting process of cylindrical shells.

Conclusions. Thus, the behavior of cylindrical shells under high-velocity loading is determined both by their structural parameters and by the microstructural characteristics of the material. The low energy utilization efficiency highlights the need for design improvements and material selection strategies that combine stable mechanical properties with a controlled microstructure.

The analysis of current approaches has demonstrated that adequate prediction of crack initiation and fragmentation in cast shell structures is possible only through comprehensive mechanic-mathematical models integrated with material quality prediction criteria. Assessing the risk of shrinkage porosity and cracking by linking casting process factors with materials science equivalents - such as effective fracture toughness, yield strength, and elastic modulus - makes it possible to define an effective design space for material development and the associated manufacturing process. As a result, the influence of microstructural defects on the operational performance of shells can be reproduced with high accuracy.

Determining the critical defect size as the key parameter controlling crack propagation provides a basis for defining the permissible range of the Niyama criterion in finite element simulations. This approach establishes a new level of control over the casting process of cylindrical shells, enabling quality prediction at the design stage, shortening the design cycle, and enhancing reliability under real operating conditions.

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АНОТАЦІЯ

Мета. Встановити взаємозв'язок між мікроструктурними особливостями литих матеріалів, зокрема пористістю, сегрегацією, розміром зерна та неметалевими включеннями, і ефективною тріщиностійкістю в умовах дії високошвидкісних динамічних навантажень шляхом визначення теоретичних залежностей для розрахунку ефективної тріщиностійкості за заданих параметрів макроскопічних дефектів у матеріалі.

Методика. Застосовано комбінований аналітико-чисельний підхід до моделювання динамічного відгуку циліндричних оболонок за умов високошвидкісного навантаження, що реалізується через систему диференціальних рівнянь динаміки з урахуванням критеріїв якості матеріалу та положень лінійної пружної механіки руйнування.

Результати. Розроблено методику оцінки якості литих матеріалів на основі комплексної механіко-математичної моделі та положень лінійної механіки руйнування. Отримано залежності кількісної оцінки стійкості до тріщиноутворення із урахуванням мікроструктурних характеристик матеріалу виготовлення. Встановлено відношення критерію тріщиноутворення до критичного розміру дефекту.

Наукова новизна. У роботі вперше запропоновано комплексний підхід до прогнозування поведінки циліндричних оболонок литої конструкції на основі механіко-математичної моделі тріщиноутворення із урахуванням внутрішніх дефектів. Встановлено залежності між мікроструктурними характеристиками металевого сплаву, таких як пористість, розмір зерна, вміст неметалевих включень, та ефективною тріщиностійкістю, що дозволяє формалізувати умови переходу від локального дефекту до критичного розміру тріщини. Запропоновано методологію кількісного відтворення зон ризику пористості та тріщинуватості, адаптованих до конкретних складів сплавів і технологій лиття.

Практична цінність. Отримані результати створюють науково обґрунтовану основу для оптимізації процесів лиття циліндричних оболонок із заданими експлуатаційними властивостями. Запропонована методика дозволяє визначати область допустимих значень мікроструктурних параметрів, що забезпечує контрольованість тріщиностійкості і прогнозованість довговічності конструкцій.

Ключові слова: *металевий сплав, тріщиноутворення, мікроструктурна неоднорідність, пористість, дефекти лиття.*

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