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ALGORITHM FOR EVALUATING STRESS-STRAIN STATE OF RIGID FIBERS IN COMPOSITE STAY ROPE

Abstract. A method for calculating a stress-strain state of a multilayer orthotropic composite rope is developed. The scientific novelty of research is in establishment of a dependency for distribution of internal loading forces in reinforcing rigid fibers on stay rope parameters. Practical value is in that the presented method allows determining the rope stress-strain state during the designing stage of a permanent structure.

Keywords: Stress-Strain State, Composite Stay Rope, Permanent Structure, Multilayer Orthotropic Rope, Rigid Fiber, Reinforcing Elements, Stress, Deformations.

Introduction. The ropes are used for suspension of spans in bridge structures with long spans. This solution is more technological and allows reducing the construction time [1]. In [2], a method of analytical calculation of a cable-stayed structure is suggested. The use of SCAD software is suggested for selection of rational parameters of elements of a cable-stayed bridge in [3]. For implementation of cable-stayed bridges with composite stay ropes, it is necessary to develop a methodology for their calculation. Its use allows solving the urgent problem of designing cable-stayed bridges with composite ropes.

Research material and results. The suggested orthotropic stay rope as a composite structure has a system of reinforcing fibers of the same diameter *d* regularly placed in two orthogonal planes. The regularity of placing the fibers when determining the character of their interaction allows considering the interaction of only four fibers with conditional numbers within the selected prism. Let's conditionally cut out from the composite a part of prismatic shape of an arbitrary i, j^{th} fiber together with a matrix of small length. There is a rectangle $a \times b$ at the base of the prism (Fig. 1).

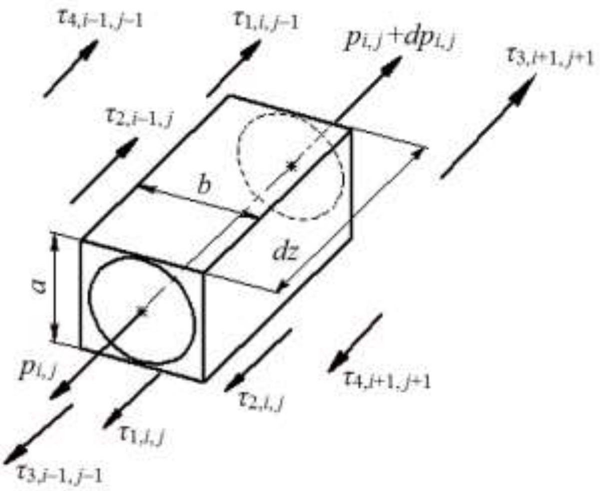

Figure 1 – Part of a composite matrix with a short fiber

Let's direct z-axis parallel to the fibers. Let's formulate the equilibrium condition of a matrix part

$$dP_{i,j}+\begin{pmatrix} b\left(\tau_{1,i,j-1}-\tau_{1,i,j}\right)+a\left(\tau_{2,i-1,j}-\tau_{2,i,j}\right)+ \\ +\left(2\sqrt{a^2+b^2}-d\right)\left(\tau_{3,i-1,j-1}-\tau_{3,i+1,j+1}+\tau_{4,i-1,j+1}-\tau_{4,i+1,j-1}\right)\end{pmatrix}dz=0, \quad (1)$$

where $\tau_{\rho,i,j}$ is average tangential stress in shell material between the fibers of one layer ρ = 1, 2 with the same numbers in adjacent layers of diagonally located fibers relative to the current i, j^{th} cable with ρ = 3, 4.

Due to matrix symmetry, its faces parallel to z-axis, which are flat before deformation remain flat after deformation – mutual displacement of fibers. We consider the fiber cross-section to be flat (unchanged) during the deformation process. Minimum distance between fibers in layers is $a-d$ and between the layers is $b-d$, between the fibers located on lines which are parallel to the diagonals of the prism.

Let's introduce the coefficient k_G of shear rigidity dependency on cross-sectional shape of the matrix. The distance between the fibers is conditionally assumed equal to the minimum. Hooke's law for mutual shear of fibers is

for side cables

$$\tau_{1,i,j} = \frac{Gk_{G_1}}{2b-d}\left(u_{i,j} - u_{i,j+1}\right); \quad \tau_{1,i,j-1} = -\frac{Gk_{G_1}}{2b-d}\left(u_{i,j} - u_{i,j-1}\right);$$
$$\tau_{2,i,j} = \frac{Gk_{G_2}}{2a-d}\left(u_{i,j} - u_{i+1,j}\right); \quad \tau_{2,i-1,j} = -\frac{Gk_{G_2}}{2a-d}\left(u_{i,j} - u_{i-1,j}\right); \qquad (2)$$

for non-side cables

$$\tau_{3,i-1,j-1} = \frac{Gk_{G_3}}{2\sqrt{a^2+b^2}-d}\left(u_{i,j} - u_{i-1,j-1}\right); \quad \tau_{3,i+1,j+1} = -\frac{Gk_{G_3}}{2\sqrt{a^2+b^2}-d}\left(u_{i,j} - u_{i+1,j+1}\right);$$
$$\tau_{4,i-1,j+1} = \frac{Gk_{G_4}}{2\sqrt{a^2+b^2}-d}\left(u_{i,j} - u_{i-1,j+1}\right); \quad \tau_{4,i+1,j-1} = -\frac{Gk_{G_4}}{2\sqrt{a^2+b^2}-d}\left(u_{i,j} - u_{i+1,j-1}\right);$$

(3)

where G is shear modulus of a matrix material.

Hooke's law for fiber tension

$$P_{i,j} = E\ F\frac{du_{i,j}}{dz}, \qquad (4)$$

where E, F are reduced modulus of elasticity and cross-section area of an individual fiber.

Consider the expressions of Hooke's law Eq. 2 - 4. Obtain the equilibrium Eq. 1 in an expanded form

$$\frac{d^2u_{i,j}}{dz^2} + \frac{G}{E\ F}\left(\begin{array}{c} \frac{k_{G_a}a}{(b-d)}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) + \\ + \frac{k_{G_b}b}{E\ F\,(a-d)}\left(u_{i,j-1} - 2u_{i,j} + u_{i,j+1}\right) + \\ + u_{i-1,j-1} - 4u_{i,j} + u_{i+1,j+1} + u_{i-1,j+1} + u_{i+1,j-1} \end{array}\right) = 0, \qquad (5)$$

$1 < i < M \quad \wedge 1 < j < N.$

The distribution of forces between fibers is

$$p_{i,j} = E\ F\left(\begin{array}{c}\sum_{m=1}^{M-1}\sum_{n=1}^{N-1}\left[\begin{array}{c}\left(A_{m,n}e^{\beta_{m,n}z} - B_{m,n}e^{-\beta_{m,n}z}\right)\beta_{m,n}\times \\ \times\cos\left(\mu_m\left(i-0.5\right)\right)\cos\left(\chi_n\left(j-0.5\right)\right)\end{array}\right]+ \\ +\sum_{m=1}^{M-1}\left(Am_m e^{\beta m_m z} - Bm_m e^{-\beta m_m z}\right)\beta m_m z\cos\left(\mu_m\left(i-0.5\right)\right)+ \\ +\sum_{n=1}^{N-1}\left(An_n e^{\beta n_n z} - Bn_n e^{-\beta n_n z}\right)^{\beta n_n z}\cos\left(\chi_n\left(j-0.5\right)\right)\end{array}\right)+P, \qquad (6)$$

where $A_{m,n}$, $B_{m,n}$, Am_m, Bm_m, An_n, Bn_n are arrays of constant coefficients; δ is displacement of a tractive element as a rigid body,

$$\beta_{m,n}=\sqrt{\frac{2G}{E\ F}\left(\begin{array}{c}\frac{k_{G_a}a}{(b-d)}\left(\cos^2\left(\mu_m\right)-1\right)+\frac{k_{G_b}b}{(a-d)}\left(\cos^2\left(\chi_n\right)-1\right)+ \\ +2\left(\cos\left(\mu_m\right)\cos\left(\chi_n\right)-1\right)\end{array}\right)},$$

$$\beta m_m=\sqrt{\frac{2G}{E\ F}\left(1-\cos\left(\mu_m\right)\right)\left(\frac{a}{b-d}+2\right)},\ \beta n_m=\sqrt{\frac{2\ G}{E\ F}\left(1-\cos\left(\chi_n\right)\right)\left(\frac{b}{a-d}+2\right)}.$$

Expressions in Eq. 6, and a possibility of determining tangential stresses in an elastic matrix based on the known values of coefficients, make it possible to determine a stress-strain state of a stay rope taking into account its composite structure. Since the mechanical properties and stay rope design are different and regular in orthogonal directions, the stay rope as a composite structure is orthotropic.

Conclusion. Orthotropic ropes are more suitable for operating conditions of cable-stayed bridges. The considered studies of ropes in permanent structures, in particular cable-stayed bridges, do not allow determining a stress-strain state of a multilayer tractive element (stay rope), taking into account a character of its interaction with the structure.

A model is created, a method for calculating a stress-strain state of an orthotropic composite tractive element is developed, with comprehensive consideration of its design, mechanical properties of its components, and given conditions for connection to the structure. The method allows determining its stress-strain state at a design stage of the permanent structure based on structural parameters and mechanical properties of composite rope components and conditions of its interaction with the structure, thereby increasing reliability and operational efficiency of the structure, in particular, a cable-stayed bridge.

The ability to provide continuous automatic control of cable breakages allows increasing reliability of a cable-stayed structure by making technical decisions justified for this situation.

It is established that the zone of disturbance of a stress-strain state along the rope length depends on a ratio of shear rigidity of elastic shell material located between the cables and tensile rigidity of cables. The character of disturbance zone distribution across the stay rope cross-section depends on the amount of reinforcing elements and their relative location.

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