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MATHEMATICAL MODEL OF MINE LOCOMOTIVE PULSATING BRAKING BY A DISC BRAKE

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МАТЕМАТИЧНА МОДЕЛЬ ПУЛЬСУЮЧОГО ГАЛЬМУВАННЯ ШАХТНОГО ЛОКОМОТИВА ДИСКОВИМ ГАЛЬМОМ

Purpose. Development, solution and analysis of a mathematical model to determine the dynamic and kinematic characteristics of a mine locomotive drive during braking with a pulsating braking torque.

The methods. Forced oscillations of the elements of the wheel-motor unit (WMU) of a mine locomotive during braking with a disc brake were investigated using the methods of differential calculus and mathematical modeling. The integration of the system of differential equations was performed using the Runge-Kutta method.

Findings. A mathematical model has been developed for braking a mine locomotive using a disc brake, which creates a pulsating braking torque on the axle of the wheelset, depending on its angular coordinate, taking into account the nonlinear dependencies of the adhesion coefficient on the relative slip in the braking mode of the mine locomotive for various track conditions, on the basis of which the braking torque parameters are established, allowing for improvement of braking characteristics. The friction characteristics of the interaction between the wheel and the rail during braking with a pulsating braking torque in mine conditions are theoretically substantiated. By means of mathematical modeling of the process of braking a mine locomotive with a disc brake, which creates a pulsating braking torque on the axle of the wheel pair, the parameters of the braking torque were established, ensuring high braking characteristics.

The originality. The dependences of the path and speed of the locomotive, the angular velocity and relative slip of one of its wheels, the coefficient of adhesion and the adhesion force of one of the locomotive wheels to the rail, the difference in the linear speeds of the locomotive and one of its wheels, the longitudinal force in one of the rubber-metal hinges of the wheel pair suspension on time during braking with a pulsating braking torque were obtained.

Practical implementation. A scientifically based engineering technique has been developed that allows, at the design stage, to select rational parameters of the disc brake of a mine locomotive, which creates a pulsating braking torque that ensures the implementation of the maximum braking force on the wheel-rail coupling during braking and determine the dynamic and kinematic characteristics of the mine locomotive drive during braking with a disc brake with a multi-sector disc under different initial data (rail track condition, train weight, initial locomotive speed).

Keywords: *mine locomotive, braking force, braking torque, adhesion coefficient, disc brake, friction pair.*

Introduction. The performance of underground mining depends on the performance of vehicles. One of the main modes of transport in coal and mine mines is locomotive haulage [1]. In mine trains, only locomotives are equipped with braking means. Therefore, the braking capabilities of the train in specific mine conditions are determined by the braking force, which is realized by the locomotive [2].

The main problem that arises during braking of mine locomotives with a wheel-block brake is the instability of the braking force generated by the brake shoe while reducing the locomotive's speed [3]. Impulse brake pressing used when braking the rolling stock of the main and industrial railway transport is a promising direction for improving the braking performance, but significantly complicates the design of the brake system [4]. The use of a pulsating braking torque due to the use of a disc brake with a multi-sector disc is a simple solution that improves the braking characteristics of mine shaft locomotives.

The monohrafiia [3] provides a methodology for choosing a constant braking torque applied to the axis of a wheel pair. In order to prevent clutch disruption and wheel use for mine electric locomotives, it is recommended to realize 80% of the maximum possible braking torque. In [5], a study was made of the braking process of a mine locomotive with a disk brake that creates a pulsating braking torque on the axis of the wheelset in order to realize the maximum possible coefficient of adhesion of wheels to rails. Recommendations are given on the analytical choice of braking torque for various rail conditions. Constructive conceptual solutions for the manufacture of a disk brake with a multi-sector disk that creates a pulsating braking torque are proposed.

The purpose of the article is development, solution and analysis of a mathematical model to determine the dynamic and kinematic characteristics of a mine locomotive drive during braking with a pulsating braking torque.

Main part. Forced vibrations of the wheel-motor unit of a mine locomotive during braking with a disc brake on a straight horizontal section of a track, taking into account the nonlinear characteristics of the interaction of the wheel-rail friction pair, can be described by a system of six second-order differential equations, which is obtained using the Lagrange equation of the second kind [5].

$$
\left.\begin{aligned}
&\left(\frac{m_s}{4}-m_3-m_4\right)\ddot{y}=-\Big[C_{y3}\left(y-y_3\right)+\beta_{y3}\left(\dot{y}-\dot{y}_3\right)+\\
&\qquad+C_{y4}\left(y-y_4\right)+\beta_{y4}\left(\dot{y}-\dot{y}_4\right)+\left(\frac{m_s}{4}-m_3-m_4\right)g\sin\beta\Big],\\
&m_3\ddot{y}_3=C_{y3}\left(y-y_3\right)+\beta_{y3}\left(\dot{y}-\dot{y}_3\right)+F_3\left(S_3\right)-m_3\,g\sin\beta,\\
&m_4\ddot{y}_4=C_{y4}\left(y-y_4\right)+\beta_{y4}\left(\dot{y}-\dot{y}_4\right)+F_4\left(S_4\right)-m_4\,g\sin\beta,\\
&I_3\ddot{\varphi}_3=-\Big[C_{\varphi 3}\left(\varphi_3-\varphi_2\right)+\beta_{\varphi 3}\left(\dot{\varphi}_3-\dot{\varphi}_2\right)+rF_3\left(S_3\right)\Big],\\
&I_4\ddot{\varphi}_4=-\Big[C_{\varphi 4}\left(\varphi_4-\varphi_2\right)+\beta_{\varphi 4}\left(\dot{\varphi}_4-\dot{\varphi}_2\right)+rF_4\left(S_4\right)\Big],\\
&I_2\ddot{\varphi}_2=C_{\varphi 3}\left(\varphi_3-\varphi_2\right)+\beta_{\varphi 3}\left(\dot{\varphi}_3-\dot{\varphi}_2\right)+C_{\varphi 4}\left(\varphi_4-\varphi_2\right)+\\
&\qquad\qquad+\beta_{\varphi 4}\left(\dot{\varphi}_4-\dot{\varphi}_2\right)-uM_{\mathrm{t}}'/2,
\end{aligned}\right\}\tag{1}
$$

where m_s is train weight; m_3, m_4 are the specified mass of the corresponding wheels; y, y_3, y_4 are linear movements of the locomotive and corresponding wheels; $\dot{y}$, $\dot{y}_3$, $\dot{y}_4$ are linear speeds; $\ddot{y}$, $\ddot{y}_3$, $\ddot{y}_4$ are linear accelerations; C_{y3}, C_{y4} are coefficients of rigidity of the corresponding elastic elements; β_{y3}, β_{y4} are coefficients of viscous internal resistance of the corresponding elastic elements; $C_{\varphi 3}$, $C_{\varphi 4}$ are coefficients of rigidity of the corresponding half shafts of the wheel and motor block; $\beta_{\varphi 3}$, $\beta_{\varphi 4}$ are coefficients of viscous internal resistance of the corresponding half shafts of the wheel and motor block; β is a tilt angle of a way (positive at the movement on rise and negative at the movement on descent); $F_3 = \psi_3(S_3) m_l g/8$, $F_4 = \psi_4(S_4) m_l g/8$ are forces of adhesion of the corresponding wheels; $\psi_3 = k_1\left[\, th(k_2 S_3) - k_3 S_3 + k_4 S_3^3 \right]$, $\psi_4 = k_1\left[\, th(k_2 S_4) - k_3 S_4 + k_4 S_4^3 \right]$ are coefficients of coupling of the corresponding wheels (in the mode of braking accept negative values) [6]; k_1, k_2, k_3, k_4 are numerical coefficients of the mechanical characteristic of frictional couple; $S_3 = (\dot{\varphi}_3 r - \dot{y}_3)/\dot{y}_3$, $S_4 = (\dot{\varphi}_4 r - \dot{y}_4)/\dot{y}_4$ are relative slidings of the corresponding wheels; m_l is mass of the locomotive; g is acceleration of gravity; I_2 is the given moment of inertia of a reducer, a disk brake and the engine concerning an axis of wheel couple corresponding to one wheel couple (depends on the location of a disk brake); I_3, I_4 are the given moments of inertia of the corresponding wheels concerning an axis of wheel couple; φ_2, φ_3, φ_4 are angular coordinates of the gearbox output shaft (wheel pair axis) and the corresponding wheels; $\dot{\varphi}_2$, $\dot{\varphi}_3$, $\dot{\varphi}_4$ are angular speeds; $\ddot{\varphi}_2$, $\ddot{\varphi}_3$, $\ddot{\varphi}_4$ are angular accelerations; r is radius of a circle of swing of wheels; m_l is mass of the locomotive; g is acceleration of gravity; u is a gear ratio of a reducer; M_{t}' is the braking moment on an engine shaft.

We take the number of sectors of the brake disc, made in turn from steel 45 HB 415 and gray cast iron SCh 15-32 HB 200, equal to eight. Brake pads in the form of an annular sector with a central angle of friction material 6KX-1 (cold forming press material). The friction coefficients for these pairs of disk materials and friction linings are respectively 0.535 and 0.41 [7].

For the selected number of sectors of the brake disc and the shape of the friction linings, the dependence of the pulsating braking torque on the motor shaft on the angular coordinate of the motor shaft can be described with a sufficient degree of accuracy by the expression [5]

$$M_{\mathrm{t}}' = 2\big(M_0 - A\sin(n\varphi_2)\big)/u = M_0' - A'\sin(n'\varphi_1) = M_0'\Big(1 - A^*\sin(n'\varphi_1)\Big) =$$

$$= M_0'\left(1 - \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\sin(n'\varphi_1)\right) \quad (\mu_1 > \mu_2), \tag{2}$$

where, M_0, M'_0 are the constant components of the braking moments, respectively, on the axis of the wheelset and on the motor shaft; n, n' are the number of periods of a sinusoid per revolution, respectively, of the axis of the wheel pair and the motor shaft; A, A' are the amplitude of the oscillations of the variable components of the braking moments on the axis of the wheelset and on the motor shaft; $A^* = A'/M'_0 = (\mu_1 - \mu_2)/(\mu_1 + \mu_2)$; μ_1, μ_2 are friction coefficients for two pairs of disc materials and friction linings.

Based on the selected parameters of a disk brake with a multi-sector disk, we set the values of the coefficients n' and A^*. In our case $n' = 4$; $A^* = 0.132$. Further, integrating the system of differential equations (1), taking into account formula (2), we determine the maximum value of the constant component of the braking torque on the engine shaft M'_{0max}, corresponding to the initial data (the state of the rail track, the mass of the train, the initial speed of the locomotive) [8]. Substitute in the formula (2) $M'_0 = 0{,}8 M'_{0max}$. Integrating the system of differential equations (1) taking into account formula (2) for given initial data, we obtain graphs of the linear and angular velocities of the links of the wheel-motor block, relative slip, coefficient and force of adhesion of the wheels to rails, and the braking moment created on the output shaft of the gearbox, and the braking torque on the wheel, the forces in the rubber mounts of the wheel pair suspension, the speed and the path of the locomotive from time to time, we determine the braking distance and the braking time. We will integrate the system of differential equations (1) using the Runge-Kutta method [9] with the standard «Mathematica» software package for four rail conditions (sprinkled with sand; covered with sand crushed as a result of a previous trip; wet, clean; covered with liquid coal mud). In numerical calculations we will use the geometric, weight, elastic-dissipative and stiffness characteristics of the elements of the mine electric locomotive E10. The mass of the composition is taken equal to $5 \cdot 10^4$ кг. The initial speed of the locomotive differs under various conditions of the rail track. Some of the graphs obtained for a rail track sprinkled with sand crushed as a result of a previous trip, with the mass of the composition $m_s = 5 \cdot 10^4$ kg and the initial locomotive speed $v_0 = 4{,}5$ m/s (dependence of the path and speed of the locomotive, the angular velocity of one of its wheels on time; dependence of the relative slip of one of the locomotive wheels on time; dependence of the adhesion coefficient and the adhesion force of one of the locomotive wheels on the rail on time; dependence of the difference in linear speeds of a locomotive and one of its wheels on time; dependence of the longitudinal force in one of the rubber-metal hinges of the wheel pair suspension on time) are shown below (fig. 1-5).

The angular speed of the wheel at the beginning of braking varies unevenly (fig. 1). For short periods of time, it can even increase. This is due to the presence of an oscillatory process in the wheel movement, characterized by an increase and decrease in the amount of wheel slippage relative to the rail.

For the case when the rails are covered with sand crushed as a result of a previous trip, the function $\psi = \psi(S)$ has an extremum at the point $S_0 = -0{,}025$ [6]. It is possible at the beginning of braking to achieve oscillations of relative slip around the extremum point by applying a pulsating braking torque to the axle of the wheelset, the average value of which is 80% of its maximum possible average value (fig. 2).

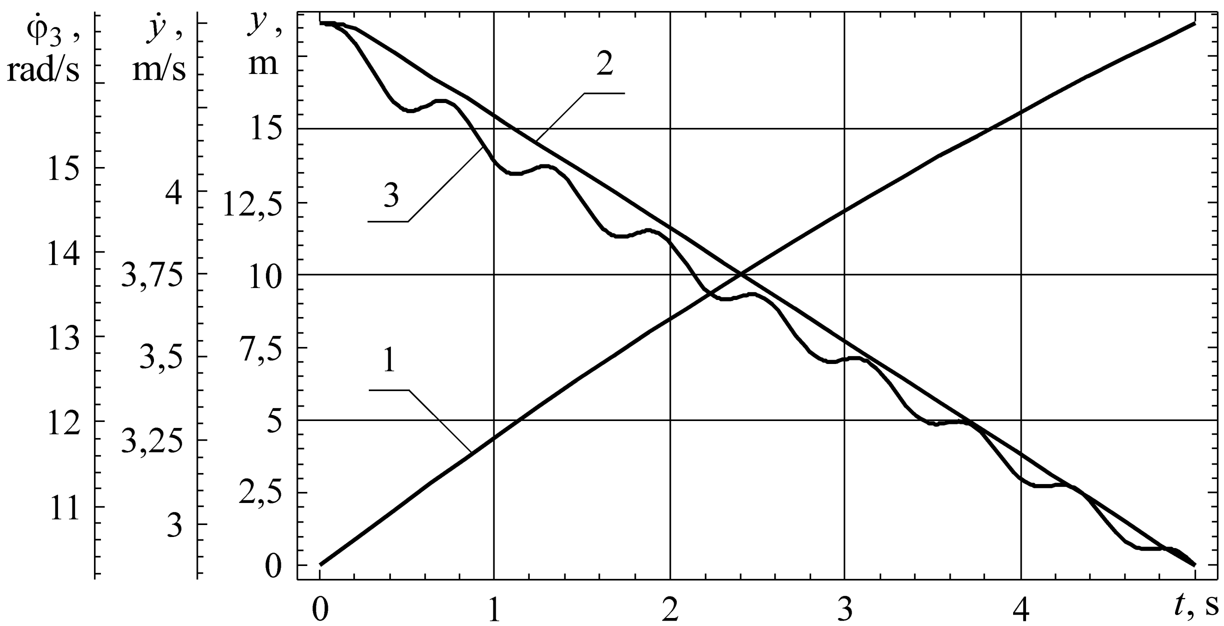

Fig. 1. Dependences of the path and speed of the locomotive, the angular velocity of one of its wheels on time: 1 is dependence of the path of the locomotive; 2 is locomotive speed dependence; 3 is dependence of the angular velocity of one of the wheels

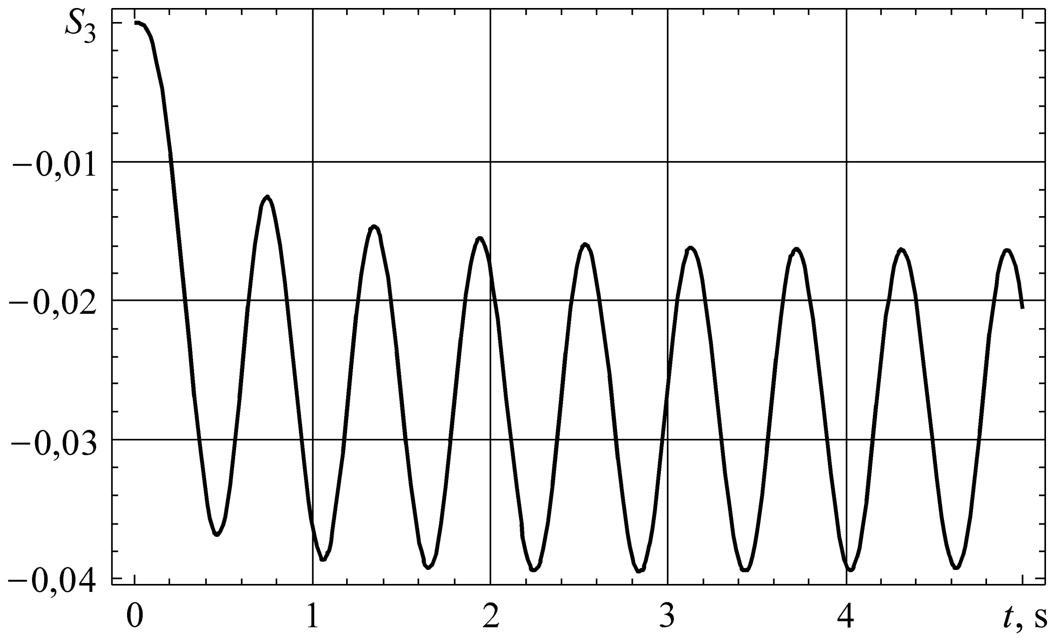

Fig. 2. The dependence of the relative slip of one of the wheels of the locomotive on time

The coefficient of adhesion at the beginning of braking fluctuates around its maximum possible value (fig. 3). The upper peaks correspond to a relative slip in absolute value that is smaller $|S_0|$, and the lower peaks to a relative slip that is larger $|S_0|$.

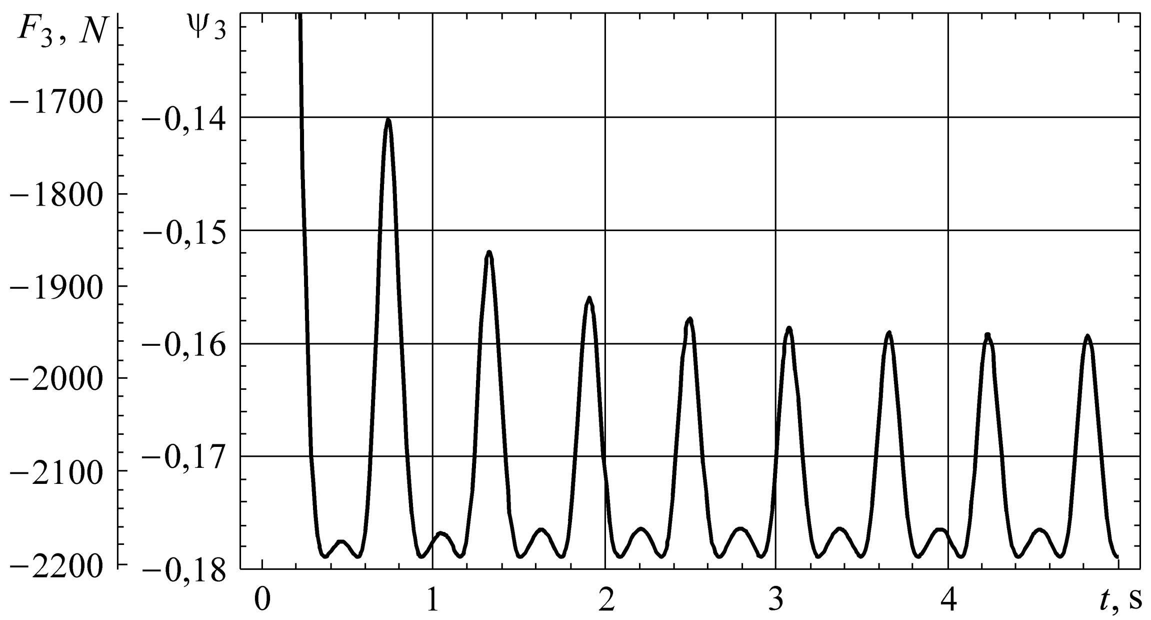

Fig. 3. Dependences of the coefficient of adhesion and the adhesion force of one of the wheels of a locomotive with a rail on time

The coefficient of adhesion approaches the value $-0{,}18$, and the adhesion force tends to -2200 N. The linear speed of one of the locomotive wheels (fig. 4), starting from 0.3 s, differs from the linear speed of the locomotive by no more than 0.0011 m/s.

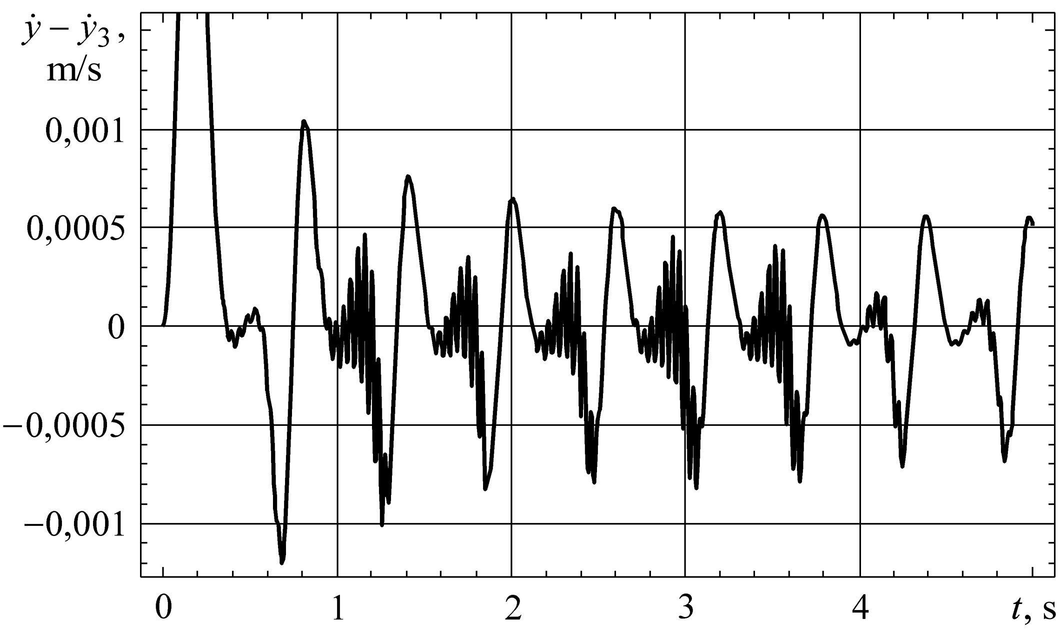

Fig. 4. The dependence of the difference between the linear speeds of the locomotive and one of its wheels on time

It can be concluded that the longitudinal force in one of the rubber-metal hinges of the wheel pair suspension does not reach 2150 N in absolute value (fig. 5). At the same time, its average value, starting from the second second, is approximately 2000 N in absolute value. Based on this and similar dependencies for other conditions of the track and with other initial data, it is possible to establish the characteristics of rubber-metal hinges, make their choice and determine their durability, which is necessary when designing a wheel-motor unit.

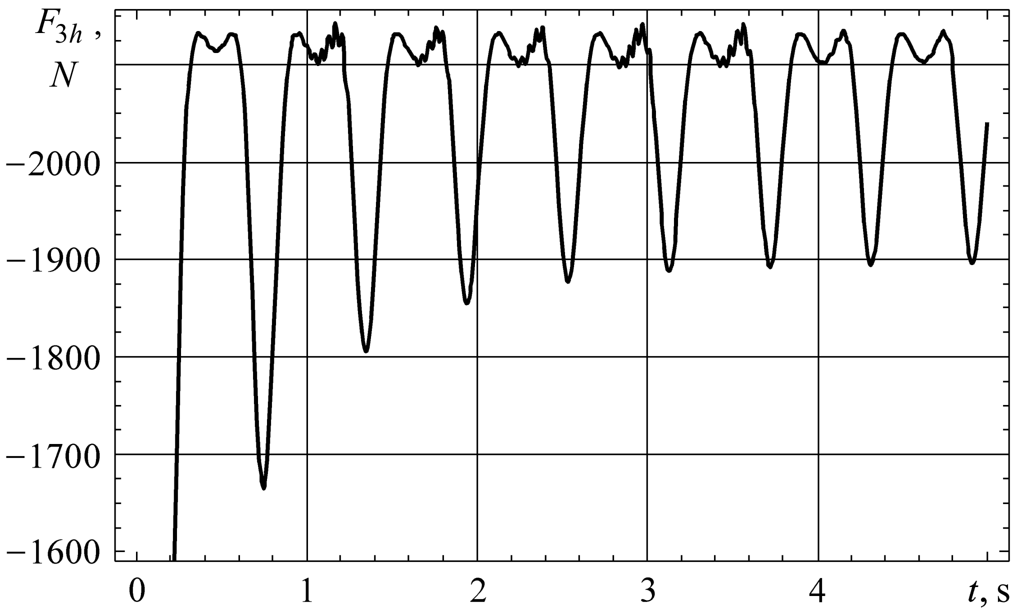

Fig. 5. The dependence of the longitudinal force in one of the rubber-thallic hinges of the wheel pair suspension on time

Based on similar calculations made for the other three conditions of the track (sand-sprinkled; wet, clean; covered with liquid coal mud) and an analysis of the obtained graphical dependencies, it was established that with the number of sectors of a multi-sector brake disc, made alternately from two different materials, equal to eight and the linings of the friction brake shoes in the form of a ring sector with a central angle $\alpha = \pi/4$ the ratio of the difference in the friction coefficients for two pairs of disc and friction lining materials to their sum should be from 0.1 to 0.15.

Conclusions. Based on mathematical modeling of the braking process of the E10 mine locomotive with a disk brake with a multi-sector disk, the dynamic and kinematic characteristics of its drive are determined for given initial data.

It has been established that due to the oscillatory process, characterized by an increase and decrease in the value of wheel slippage relative to the rail, at the beginning of braking, the angular velocity of the wheel can increase over short periods of time.

It has been shown that when using a disc brake with a multi-sector disc made of two alternating materials and friction linings of the brake pads in the form of an annular sector, the ratio of the difference in the friction coefficients for two pairs of disc materials and friction linings to their sum should be from 0.1 to 0.15. Rational parameters of a disc brake with a multi-sector brake disc have been substantiated and selected for the E10 mine locomotive.

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АНОТАЦІЯ

Мета. Розробка, розв’язання та аналіз математичної моделі для визначення динамічних та кінематичних характеристик приводу шахтного локомотива під час гальмування пульсуючим гальмівним моментом.

Методика. Вимушені коливання елементів колісно-моторного блоку (КМБ) шахтного локомотива в процесі гальмування дисковим гальмом досліджені методами диференціального числення та математичного моделювання. Інтегрування системи диференціальних рівнянь виконано методом Рунґе-Кутта.

Результати. Розроблено математичну модель гальмування шахтного локомотива дисковим гальмом, що створює на осі колісної пари пульсуючий гальмівний момент, який залежить від її кутової координати, з урахуванням нелінійних залежностей коефіцієнта зчеплення від відносного ковзання в режимі гальмування шахтного локомотива для різних станів рейкової колії, на базі якої встановлено параметри гальмівного моменту, що дозволяють поліпшити гальмівні характеристики. Теоретично обґрунтовано фрикційні характеристики взаємодії колеса та рейки при гальмуванні пульсуючим гальмівним моментом в умовах шахти. Шляхом математичного моделювання процесу гальмування шахтного локомотива дисковим гальмом, що створює на осі колісної пари пульсуючий гальмівний момент, встановлені параметри гальмівного моменту, які забезпечують високі гальмівні характеристики.

Наукова новизна. Отримано залежності шляху та швидкості локомотива, кутової швидкості та відносного ковзання одного з його коліс, коефіцієнта зчеплення та сили зчеплення одного з коліс локомотива з рейкою, різниці лінійних швидкостей локомотива та одного з його коліс, поздовжньої сили в одному з гумометалевих шарнірів підвіски колісної пари від часу при гальмуванні пульсуючим гальмівним моментом.

Практична значимість. Розроблено науково обґрунтовану інженерну методику, яка дозволяє на стадії проектування вибрати раціональні параметри дискового гальма шахтного локомотива, що створює пульсуючий гальмівний момент, який забезпечує реалізацію максимальної гальмівної сили по зчепленню колеса з рейкою в процесі гальмування і визначити динамічні та кінематичні характеристики приводу шахтного локомотива під час гальмування дисковим гальмом із багатосекторним диском при різних вихідних даних (стан рейкової колії, маса складу, початкова швидкість локомотива).

Ключові слова: *шахтний локомотив, гальмівна сила, гальмівний момент, коефіцієнт зчеплення, дискове гальмо, фрикційна пара.*

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