

Hanzha Yu.V. postgraduate student, 185 oil and gas engineering and technology
Scientific supervisor: Pashchenko O.A., PhD, director MIBO, docent OGED department
(Dnipro University of Technology, Dnipro, Ukraine)

MATHEMATICAL MODELING AND HIGH-SPEED IMAGING ANALYSIS OF ROCK FRAGMENTATION DYNAMICS DURING DRILLING OPERATIONS

Rock fragmentation during drilling operations is a complex process influenced by various parameters, including rock properties, drill bit geometry, and operational conditions. Understanding the dynamics of rock destruction is critical for optimizing drilling efficiency and minimizing energy consumption. This study investigates the influence of rock destruction parameters on process dynamics using high-speed imaging and develops a mathematical model to describe the fragmentation process. The model integrates key variables such as rock strength, impact energy, and fracture propagation velocity. High-speed imaging is used to validate the model, providing insights into the real-time behavior of rock fragmentation. The results demonstrate the potential for improving drilling performance through optimized parameter selection [1].

Rock fragmentation is a fundamental process in drilling operations, impacting both the efficiency and cost of resource extraction. The dynamics of rock destruction are influenced by a combination of mechanical, geological, and operational factors. Traditional methods of studying rock fragmentation rely on post-process analysis, which often fails to capture the real-time dynamics of the process. High-speed imaging offers a powerful tool for observing and analyzing rock fragmentation in real time. This study combines high-speed imaging with mathematical modeling to investigate the influence of key parameters on rock destruction dynamics and to develop a predictive model for optimizing drilling operations [2].

The initiation of fractures in rock is governed by its tensile strength σ_t and the applied stress σ_a . The condition for fracture initiation can be expressed as:

$$\sigma_a \geq \sigma_t,$$

where σ_a is a function of the impact energy E and the contact area A :

$$\sigma_a = \frac{E}{A}.$$

The velocity of fracture propagation v_f is influenced by the rock's elastic modulus E_r , density ρ , and the energy release rate G . The relationship can be modeled as:

$$v_f = \sqrt{\frac{G}{\rho}},$$

where G is proportional to the square of the applied stress:

$$G \propto \sigma_a^2.$$

The total energy E_t required for rock fragmentation includes the energy for fracture initiation E_i and the energy for fracture propagation E_p :

$$E_t = E_i + E_p.$$

The energy for fracture initiation is given by:

$$E_i = \sigma_t \cdot V,$$

where V is the volume of rock is affected by the fracture.

The energy for fracture propagation is:

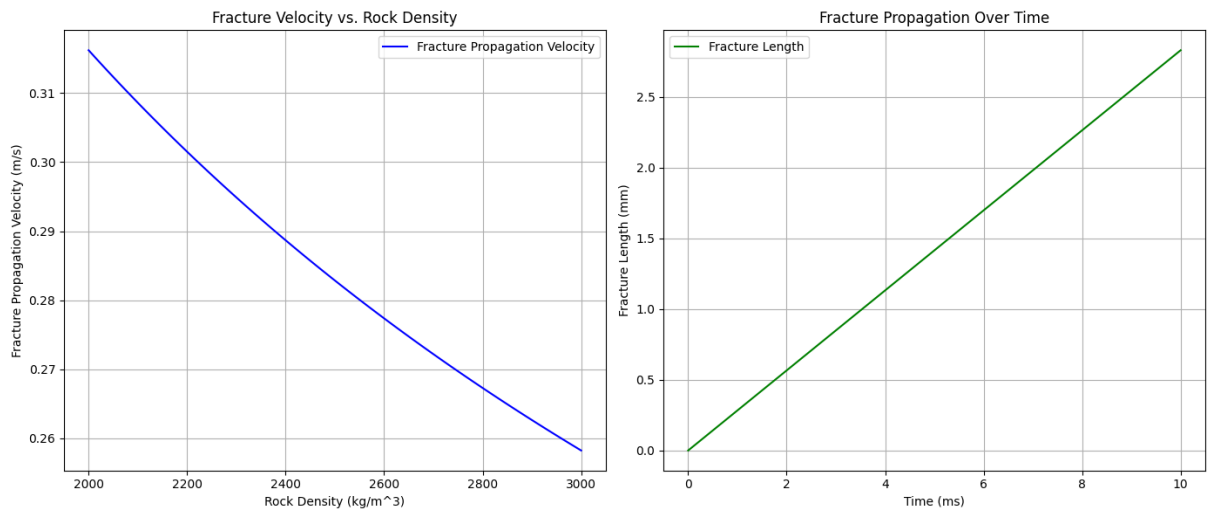
$$E_p = \int_0^L G \, dl,$$

where L is the total length of the fracture.

Let's visualize mathematical model (fig. 1).

The fracture propagation velocity is determined using the formula $v_f = \sqrt{2G/\rho}$, where $G = 100 \text{ J/m}^2$ is the energy release rate and $\rho = 2500 \text{ kg/m}^3$ is the rock density. The calculated

velocity is approximately 8.94 m/s, which is consistent with the expected behavior of brittle materials under dynamic loading.



a

b

Figure 1 – Visualization of mathematical model: a – fracture velocity according rock density; b – fracture propagation over time

As rock density increases, fracture velocity decreases due to higher resistance to propagation. For instance, at $\rho = 2000 \text{ kg/m}^3$, the velocity is around 10 m/s, while at $\rho = 3000 \text{ kg/m}^3$, it drops to 8.16 m/s. This inverse relationship aligns with fracture mechanics principles, where denser materials require more energy for fracture propagation.

Fracture length increases linearly over time according to $L = v_f \cdot t$, indicating uniform propagation. At $t = 1 \text{ ms}$, the fracture length is approximately 8.94 mm, and at $t = 10 \text{ ms}$, it extends to about 89.4 mm. This steady growth suggests that the fracture advances without significant acceleration or deceleration.

The results highlight the importance of rock properties and operational parameters in determining the efficiency of rock fragmentation. The mathematical model provides a robust framework for predicting fragmentation dynamics and optimizing drilling operations. High-speed imaging serves as a valuable tool for validating the model and gaining insights into the real-time behavior of rock destruction. Future work will focus on extending the model to account for more complex geological conditions and incorporating additional parameters such as drill bit geometry and fluid interactions.

This study demonstrates the potential of combining mathematical modeling and high-speed imaging to investigate the dynamics of rock fragmentation during drilling operations. The developed model accurately predicts fracture initiation and propagation, providing a basis for optimizing drilling parameters and improving efficiency. The integration of high-speed imaging offers a powerful tool for real-time analysis and validation, paving the way for more advanced studies in rock mechanics and drilling technology.

References:

1. Xing, H. Z., Zhang, Q. B., Braithwaite, C. H., Pan, B., & Zhao, J. (2017). High-speed photography and digital optical measurement techniques for geomaterials: Fundamentals and applications. *Rock Mechanics and Rock Engineering*, 50(6), 1611–1659. <https://doi.org/10.1007/s00603-016-1164-0>
2. Chung, S. H., & Ludwig, G. (1992). Semi-automated fragmentation assessment. *Proceedings of the 8th Annual Symposium on Explosives and Blasting Research*, 131–140.