UDC 531.2, 531.3, 004.94

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INVESTIGATION OF THE STEADY EQUILIBRIUM STATE OF THREE CHARGED BODY SYSTEM IN THE GRAVITATIONAL FIELD

Introduction. In 2014 the following problem was proposed on the All-Ukrainian physics Olympiad [1]. Three equally charged bodies are hanged on three threads of equal length. In equilibrium angles between the first two threads and the vertical line equals 20°, angle between the third thread and the vertical line equals 14°, see Fig. 1. The question is to describe the steady equilibrium state after the increasing of the charge of each particle in 2014 times. The author solution of the problem is an approximate one and it is based on the fact that after such increasing the Coulomb forces become much greater than the gravity ones. In this work we generalize this problem in the case where the charge of each particle increases in *n* times in comparison with the initial value, *n*>1 is an arbitrary number.

Problem generalization. We introduce the spherical coordinates of the bodies as follows, see Fig. 2 and Fig. 3. Obviously, $m_1 = m_2$, so $\varphi = \varphi_1 = -\varphi_2$ and $\theta_2 = \theta_1$. The radial coordinates of the particles are equal to the thread length l.

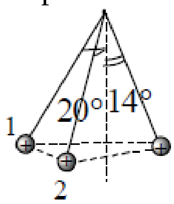

Figure 1 – Initial system

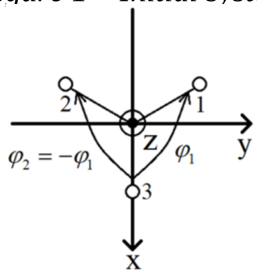

Figure 2 – φ coordinates

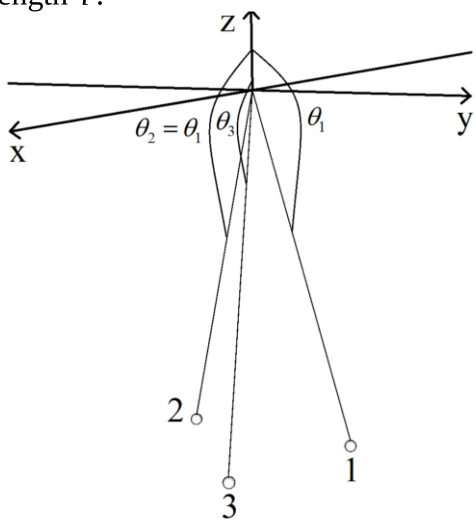

Figure 3 – θ coordinates

In this work in is shown that the system potential energy after the charge increasing is as follows:

$$U(\theta_1, \theta_3, \varphi) = (kn^2 q^2 / l) \cdot W(\theta_1, \theta_3, \varphi) \quad (1)$$

where the k is the constant in the Coulomb law, q is the initial body charge and

$$W(\theta_1, \theta_3, \varphi) = 2\frac{\delta}{n^2}\cos\theta_1 + \frac{\gamma}{n^2}\cos\theta_3 + \frac{1}{2\sin\theta_1 \sin\varphi} + $$
$$+2\cdot\left(\left(\sin\theta_1\cos\varphi - \sin\theta_3\right)^2 + \left(\sin\theta_1\sin\varphi\right)^2 + \left(\cos\theta_1 - \cos\theta_3\right)^2\right)^{-1/2}, \quad (2)$$
$$\gamma = m_3 g l^2 / (kq^2) \approx 24.578, \ \delta = m_1 g l^2 / (kq^2) \approx 16.660 .$$

We investigate the steady equilibrium state, so we seek for the local minimum of $U(\theta_1, \theta_3, \varphi)$ which leads to the search of the local minimum of the function $W(\theta_1, \theta_3, \varphi)$.

So, the first derivatives of W are equal to zero:

$$W'_{\theta_1} = 0, \ W'_{\theta_3} = 0, \ W'_{\varphi} = 0 \quad (3)$$

and additionally the following conditions for the second derivatives hold:

$$W''_{\theta_1,\theta_1} > 0,\ \det\begin{pmatrix} W''_{\theta_1,\theta_1} & W''_{\theta_1,\theta_3} \\ W''_{\theta_1,\theta_3} & W''_{\theta_3,\theta_3} \end{pmatrix} > 0,\ \det\begin{pmatrix} W''_{\theta_1,\theta_1} & W''_{\theta_1,\theta_3} & W''_{\theta_1,\varphi} \\ W''_{\theta_1,\theta_3} & W''_{\theta_3,\theta_3} & W''_{\theta_3,\varphi} \\ W''_{\theta_1,\varphi} & W''_{\theta_3,\varphi} & W''_{\varphi,\varphi} \end{pmatrix} > 0. \quad (4)$$

The solution is obtained both analytically and with the help of the programming in the Wolfram Mathematica package. It is shown that the steady state equilibrium coordinates depend on n as shown in Fig.4 – Fig.6; $n_t \approx 11.904$.

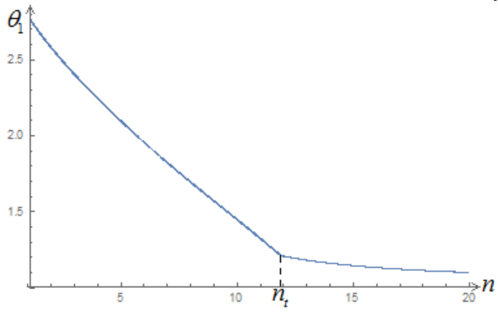

Fig. 4 – equilibrium steady state dependence of θ_1 on n

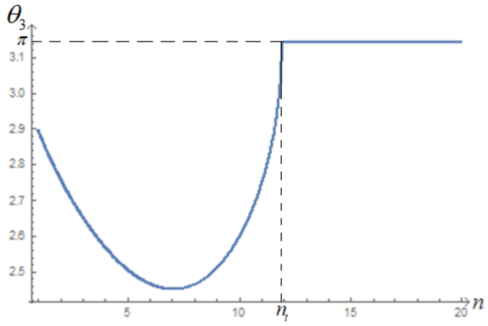

Fig. 5 – equilibrium steady state dependence of θ_3 on n

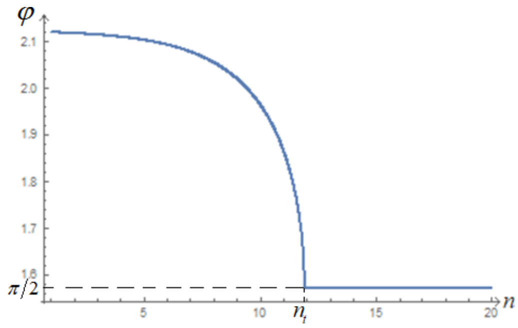

Fig. 6 – equilibrium steady state dependence of φ on n

In particular, in the case where $n > n_t$ it is shown that in the steady equilibrium state

$$\varphi = \pi/2,\ \theta_3 = \pi,\ \theta_1 \approx \pi/3 + 6\delta/(5n^2 - 2\sqrt{3}\delta) \quad (5)$$

where the blue term is our correction to the author result [1].

An algorithm of the oscillation modeling in the vicinity of the steady equilibrium state is developed on the basis of the programming in the Wolfram Mathematica package. It is shown that if $n > n_t$ in the case where the initial velocities are zero ones and $\theta_1 - \theta_{1\text{eq}} = \theta_2 - \theta_{2\text{eq}} = \theta_0$, $|\theta_0| \ll 1$, $\varphi_1 = \varphi_{1\text{eq}} = \pi/2$, $\varphi_2 = \varphi_{2\text{eq}} = -\pi/2$, $\theta_3 = \theta_{3\text{eq}} = \pi$ oscillations of the first two bodies are harmonic ones with the period

$$T = \frac{4\pi}{n}\sqrt{\frac{\delta l}{g}}\left(-\frac{4\delta\cos\theta_{1\text{eq}}}{n^2} + \frac{\sqrt{2}\cos\theta_{1\text{eq}}}{(1+\cos\theta_{1\text{eq}})^{3/2}} + \frac{1}{\sin\theta_{1\text{eq}}} + \frac{2\cos^2\theta_{1\text{eq}}}{\sin^3\theta_{1\text{eq}}} + \frac{3\sin^2\theta_{1\text{eq}}}{\sqrt{2}(1+\cos\theta_{1\text{eq}})^{5/2}}\right)^{-1/2} \quad (6)$$

where the subscript eq indicates the corresponding equilibrium value. The modeling results coincide with the analytical result (6).

Conclusions. The equilibrium steady state is described in the case of arbitrary charge increasing in the framework of the problem under consideration. The algorithm of small oscillation modeling is developed, the oscillation period in the case of symmetric initial conditions is derived.

References

1. All-Ukrainian Physics Olympiad 2014. Problems and author solutions https://upho.org.ua/national/national-2014-09-theory-solutions.pdf (in Ukrainian).