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ON THE APPLICABILITY OF THE USE OF THE KOLMOGOROV–WIENER FILTER FOR PREDICTION OF HEAVY-TAIL STATIONARY PROCESSES

This work is devoted to the investigation of the applicability of the use of the Kolmogorov–Wiener filter for prediction of heavy-tail stationary processes. The motivation of the investigation is as follows. The problem of telecommunication traffic forecasting is important for telecommunications. There are a plenty of rather sophisticated approaches to the telecommunication traffic prediction in different cases, see [1]. For example, the ARIMA models and the neural network approaches are used for traffic prediction for non-stationary traffic.

However, in several cases traffic is treated as a stationary random process, see [1]. One of the simple known approaches to the prediction of stationary random processes is the Kolmogorov–Wiener filter, see [2], however it is not sufficiently developed in the literature. This method is rather simple, so the use of this method may be of interest and its investigation may have scientific novelty. As is known [3], the telecommunication traffic in systems with data packet transfer is a self-similar process. Our investigation [4] is devoted to the calculation of the Kolmogorov–Wiener weight function in continuous case for different models. However, the corresponding applicability of the Kolmogorov–Wiener filter is still an open question.

In this paper the heavy-tail data is generated on the basis of the symmetric moving average approach which is described in detail in paper [5]. The generation of the data of a heavy-tail random process is made as follows:

$$X_i = \sum_{j=-q}^{q} a_{|j|} V_{i+j}\,,\; a_0 = \frac{\sqrt{(2-2H)\gamma_0}}{1.5-H}\,,\; a_j = \frac{a_0}{2}\left(j^{H+0.5} + (j-1)^{H+0.5} - 2j^{H+0.5} \right) \tag{1}$$

where V_i is a white noise with zero average and variance equal to 1, γ_0 is the process variance and H is the Hurst exponent.

The corresponding non-negative heavy-tail data, which may describe traffic in a real system, is

$$x_t = X_t + \left|\min(X)\right| + 10^{-3} \ge 0\,, \tag{2}$$

the term 10^{-3} is added to avoid an infinite MAPE. Then the centralized process is generated and the Kolmogorov–Wiener filter is in fact applied to the centralized process. The filter weight coefficients are as follows:

$$xc_i = x_i - \langle x \rangle\,,\; h = M^{-1} \cdot r\,,\; r_i = R(i+z)\,,\; M_{i,j} = R(|i-j|)\,,\; i, j = \overline{0,T} \tag{3}$$

where $T+1$ is the number of points on the basis of which the prediction is made, z is the number of points for which the forecast is made and R is the correlation function of the centralized process xc_i.

The prediction is made as follows. First of all, on the basis of the points $xc_1,\ xc_2,\ldots,xc_{T+1}$ we predict the values $x_{T+2},\ldots,\ x_{T+1+z}$. The predicted values are

$$\hat{x}_{t+z} = \langle x \rangle + \sum\nolimits_{j=0}^{t} h_j x_{t-j}^{\text{in}}\,,\; t = \overline{T+2-z, T+1} \tag{4}$$

where x_i^{in} are the filter input data: $x_0^{\text{in}} = xc_1$, $x_1^{\text{in}} = xc_2,\ldots,\ x_T^{\text{in}} = xc_{T+1}$. The MAPE is estimated as follows:

$$\text{MAPE} = \frac{1}{z} \sum_{j=T+2-z}^{T+1} \left| \frac{\hat{x}_{j+z} - x_{j+z}}{x_{j+z}} \right| \cdot 100\%\,. \tag{5}$$

Then, on the basis of the points xc_2, $xc_3,\ldots,xc_{T+2}$ we predict the values $x_{T+3},\ldots,$ x_{T+2+z} and so on around the whole array.

It is shown that the prediction for a non-smooth process does not work well, the corresponding MAPE is close to 25% even for $z=1$. So we investigate the applicability of the Kolmogorov–Wiener filter for a smooth process which is generated by a simple linear smoothing algorithm on the basis of the formula

$$\tilde{X}_i = \frac{1}{2l+1}\sum_{j=-l}^{l} X_{i+j}\,, \tag{6}$$

and the following using of (2)–(5) with the use of $\tilde{X}_i$ instead of X_i. The obtained results are given in Table 1, the average MAPE values over the whole array are indicated.

Table 1.

Average MAPE values for different l and z.

$T=100$

z \ l	1	2	3	4	5	6	7
1	9.11	6.23	4.85	3.92	3.37	2.98	2.68
2	15.05	10.19	7.86	6.13	5.30	4.73	4.24
3	20.29	13.84	10.34	8.08	6.97	6.35	5.66
4	20.97	17.17	13.24	10.01	8.64	7.74	7.02

$T=1000$

z \ l	1	2	3	4	5	6	7
1	9.00	6.14	4.78	3.84	3.28	2.92	2.58
2	14.93	9.96	7.72	5.95	5.11	4.59	4.06
3	20.22	13.58	10.16	7.80	6.64	6.11	5.40
4	20.89	16.93	13.00	9.69	8.25	7.45	6.73

The values for $z=1$ and $T=100$ are calculated in [6]. Obviously, the smoother the process is, the better the prediction is. The prediction accuracy is almost the same for $T=100$ and $T=1000$. The prediction accuracy is more than 90% for $z=1$ even for $l=1$, so the conclusion can be made that the Kolmogorov–Wiener filter may be applied for the short-term prediction of a heavy-tail stationary process if the process is smooth enough.

References

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