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DEVELOPMENT OF A MODEL OF CONTACT SHOE BRAKE-DRUM INTERACTION IN THE CONTEXT OF A MINE HOISTING MACHINE

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РОЗРОБКА МОДЕЛІ КОНТАКТНОЇ ВЗАЄМОДІЇ КОЛОДКОВОГО ГАЛЬМА З БАРАБАНОМ ШАХТНОЇ ПІДЙОМНОЇ МАШИНИ

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ABSTRACT

Purpose. To develop models of contact brake shoe-drum interaction of a mine hoisting machine while braking, taking into account final bending stiffness of a beam and the effect of friction forces on the distribution of a contact pressure in it to make recommendations as for the rational design of a brake beam.

Methods. Laws of contact force distribution, forces within a vertical post, and braking moment arising in the braking process have been formulated with the help of exclusion method and Euler's method.

Findings. Areas to apply the hypotheses on absolute stiffness of a beam and the non-effect of friction forces on the distribution of contact pressure in it while calculating force of brakes of mine hoisting machines have been analyzed. Physical and mathematical models of contact interaction between a brake beam of a mine hoisting machine and a drum in the braking process have been developed.

Originality. For the first time, physical model of a brake lining in the form of a group of elastic non-interacting bodies of Winkler foundation has been developed. The bodies resist compression and transfer through themselves distributed friction forces arising between brake drum and brake shoe; the friction forces are meant for limiting balance state in accordance with Coulomb's law; physical model of a brake beam in the shape of uniform-section circular bar mounted on a vertical post and interacting with a brake drum through brake lining loaded with distributed normal and tangential load modeling contact brake shoe-drum interaction, and a vertical post has been modeled as a movable pivot point located in the medial part of the circular bar. For the first time, mathematical model to determine both tangential and normal forces acting on a brake beam has been formulated.

Practical implications. The developed recommendations concerning the use of different models of the braking process make it possible to generate the most rational model for force calculation of a brake beam using finite-element method.

Keywords: *physical and mathematical models of beam and lining, mine hoisting machine shoe brake, Euler's method, Coulomb's law*

1. INTRODUCTION

1.1. The problem definition

In the context of mining industry, hoisting machine is considered to be the most important link while mineral mining. Occurrence of emergency situations in the process of hoisting machine operations results not only in substantial material losses; they are also often dangerous for human life. Thus, braking system of hoisting equipment is the basic protective means against emergency situations (Zabolotny, Zhupiev, & Molodchenko, 2015; Zabolotnyi, Panchenko, & Zhupiev, 2017).

Decrease in contact pressure of shoe brakes is the topical technical problem as well as the determination of the required forces applied in brake beams and in vertical post, and calculation of the braking moment being developed (Cummings, 2009; Cummings, McCabe, Guelde, & Gosselin, 2009).

Analysis of the recent research demonstrated in publications of such well-known scientists as B.L. Davydov, Z.M. Fedorova, N.S. Karpyshev, A.J. Day, Y.M. Huang, J.S. Shyr, M. Tirovits, T.P. Newcomb, P.J. Harding, Z. Barecki, and S.F. Scieszka shows that the authors used pointwise calculations applying finite-element methods and other numerical approaches as well as not evaluated effect of basic parameters of shoe beam on the lining-brake rim contact interaction. As a rule, a method of calculation of braking mechanisms applied in mine hoisting machines (MHMs) is used as analytical model (Barkand & Helfrich, 1988; Nosko, 2017). The method, described in the papers by B.L. Davydov (1959), Z.M. Fedorova (1961), N.S. Karpyshev (1968), relies upon the hypothesis of absolute stiffness of a brake rim and a brake beam when friction forces do not effect the distribution of contact pressure.

Specifically, A.J. Day performed calculations concerning a specific case with the help of finite-element method representing a brake beam as a sequence of beams of various designs (Day, Harding, & Newcomb, 1979).

Y.M. Huang applied finite-element method without the analysis of basic parameters effecting on contact interaction (Huang & Shyr, 2002).

Z. Barecki and S.F. Scieszka considered the operation of a moving brake whereas MHMs are equipped with brakes with progressive motion of shoes (Barecki & Scieszka, 1989).

1.2. Determination of earlier unsolved issues being a part of a general problem

To some extent, calculation results concerning stress-strain state within MHM brake differ to compare with those described in scientific sources. For instance, nature of contact pressure values distribution along a brake beam is not sinusoidal with peak values relating to the shoe center; on the contrary, it has distinct boundary effect. In this context, topical scientific problem arises. The problem is to identify the factors effecting contact pressure distribution as well as to evaluate possibilities of applying the hypothesis of absolute beam stiffness and non-effect of friction forces on the distribution of contact pressure within it.

1.3. Objective of the research

The objective is to develop a model of contact interaction between a shoe brake of a mine hoisting machine and a drum in the braking process taking into consideration finite value of beam bending stiffness and effect of friction forces upon contact pressure distribution as well as further use of the model while developing recommendations for the selection of rational design of a brake beam.

2. MAIN PART

Novokramatorsk machine-building enterprise is one of the largest European industrial and research complexes. Up till now, the enterprise produces drum mine hoisting machines; their majority is equipped with shoe brakes. A number of well-known researchers were engaged in the design of such brake mechanisms for MHMs. However, many important problems connected with interaction between brake beam, lining, and brake rim were not solved due to limited capacities of computing facilities used at the time.

Mathematical model by B.L. Davydov concerning the calculation of distributed twisting and normal forces acting on a brake beam of a shoe brake of MHM relies upon following assumptions: distribution of normal internal forces, arising within a brake beam, does not depend on lining-drum friction and on binding stiffness of the beam (Davydov, 1959).

Author of the assumptions did not substantiate them; thus, the calculation results may contain essential errors while determining design loads of the shoe brake which may result in emergency situations.

The paper has developed mathematical model of contact interaction between shoe brake and a drum of MHM where friction forces as well as bending stiffness of a beam were taken into consideration.

Brake shoe (Fig. 1) consists of a brake beam *1* and brake linings *2* mounted on a vertical beam (post) *3*.

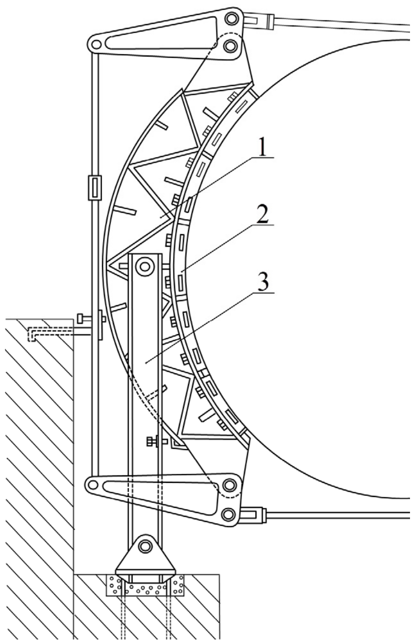

Figure 1. Fragment of a shoe brake of MHM

To solve the set problem, physical model of a brake lining in the form of a group of elastic non-interacting bodies of Winkler foundation was applied. The bodies resist compression and transfer through themselves distributed friction forces arising between a brake drum and a brake shoe; the friction forces are meant for limiting balance state in accordance with Coulomb's law (Fig. 2).

Two forces, arising within horizontal connection rods, and a force arising within a vertical post act on a brake beam mounted on the vertical post and interacting with the brake drum through a brake lining. Moreover, brake beam-brake rim interaction is modeled with the help of distributed normal load q and a tangent p resulting in braking moment M_T. It is assumed that the drum rotates clockwise.

The brake beam is represented as physical model in the form of a circular bar of uniform section on which inner part both contact pressure and distributed friction force act. Vertical post is modeled as a movable flapping hinge located in the central part of the circular bar.

In this case, brake beam is protected against vertical movements within the movable flapping hinge. Such a structure provokes origination of a wave of internal forces; thus, it is impossible to explain the balance with the help of one differential equation.

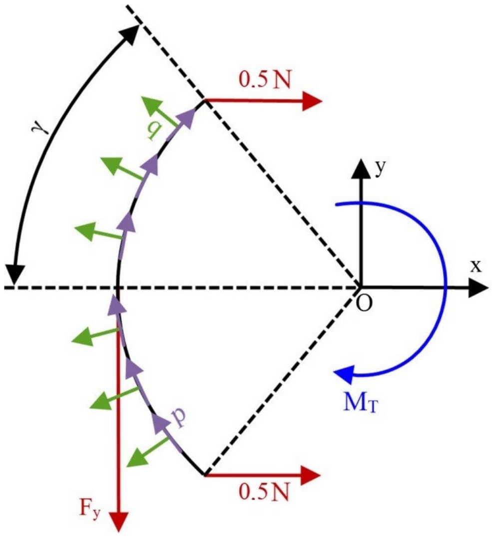

Figure 2. Calculation model of a brake beam of MHM

Write down the equations describing a balance of two elementary shares of the bar; specify them by following indices ($i = 0$) and ($i = 1$) respectively:

$$\begin{aligned}
&\frac{dT_i(\varphi)}{d\varphi}+Q_i(\varphi)=0;\ (i=0,1);\\
&\frac{dQ_i(\varphi)}{d\varphi}-T_i(\varphi)-q_i(\varphi)R=0;\ (i=0,1);\\
&\frac{dM_i(\varphi)}{d\varphi}+Q_i(\varphi)R=0;\ (i=0,1),
\end{aligned} \tag{1}$$

where:

$T_i(\varphi)$ – axial force;

$Q_i(\varphi)$ – shear force;

$M_i(\varphi)$ – bending moment;

φ – running angular coordinate;

R – radius of neutral line of brake beam.

It is possible to write down the equation to calculate distributed normal forces corresponding to a model of elastic foundation as follows:

$$q_i(\varphi)=\kappa w_i(\varphi);\ (i=0,1), \tag{2}$$

where:

$w_i(\varphi)$ – beam deflection;

$E_н$ – elasticity module of the lining material;

$H_н$ – the lining thickness;

B_T – the width of braking field;

κ – cross-sectional stiffness of the lining being determined as follows:

$$\kappa=E_н\frac{B_T}{H_н}. \tag{3}$$

The equation describing distributed friction force according to Coulomb's law is:

$$p_i(\varphi)=f\kappa w_i(\varphi);\ (i=0,1). \tag{4}$$

Formulate Hook's law to determine deflection moment, that is:

$$M_i(\varphi)=-\frac{EI}{R}\cdot\frac{d\theta_i(\varphi)}{d\varphi};\ (i=0,1), \tag{5}$$

where:

E – elasticity module of the beam material;

I – inertia moment of the beam cross-section;

θ_i – the beam rotation angle.

Consider following kinetic dependence:

$$\theta_i(\varphi)=\frac{1}{R}\left(\frac{dw_i(\varphi)}{d\varphi}+v_i(\varphi)\right);\ (i=0,1), \tag{6}$$

and inextensibility condition of the bar central line, that is:

$$w_i(\varphi)=\frac{dv_i(\varphi)}{d\varphi};\ (i=0,1). \tag{7}$$

Apply elimination method to solve the system of equation (1).

Substituting (2) – (6) expressions into equations (1), obtain following result:

$$\begin{aligned}
&\frac{EI}{R^3}\left(\frac{d^5w_i(\varphi)}{d\varphi^5}+\frac{d^3w_i(\varphi)}{d\varphi^3}\right)+\kappa R\frac{dw_i(\varphi)}{d\varphi}+\\
&+\frac{EI}{R^3}\left(\frac{d^3w_i(\varphi)}{d\varphi^3}+\frac{dw_i(\varphi)}{d\varphi}\right)+f\kappa Rw_i(\varphi)=0
\end{aligned}. \tag{8}$$

After expression (7) was substituted into equation (8), a differential equation to describe axial deformation of 6th order was obtained, i.e.:

$$\frac{d^6v_i(\varphi)}{d\varphi^6}+2\frac{d^4v_i(\varphi)}{d\varphi^4}+(1+\lambda)\frac{d^2v_i(\varphi)}{d\varphi^2}+f\lambda\frac{dv_i(\varphi)}{d\varphi}=0, \tag{9}$$

where:

λ – relative stiffness being equal to the ratio between cross-sectional stiffness of the lining and bending stiffness of the beam which can be determined with the help of the equation:

$$\lambda=\frac{E_HB_TR^4}{H_HEI}. \tag{10}$$

Euler's method should be applied to solve differential equation (9).

Characteristic equation describing a state of both parts of the bar is as follows:

$$n\left[n^5+2n^3+(1+\lambda)n+f\lambda\right]=0. \tag{11}$$

Represent the roots of the characteristic equation in a vector form, i.e.:

$$n=\begin{cases}0\\ r\\ \alpha_1+i\beta_1\\ \alpha_1-i\beta_1\\ \alpha_2+i\beta_2\\ \alpha_2-i\beta_2\end{cases}. \tag{12}$$

In this context, one root is a zero one, another root is a real one, and four others are complex roots. Newton's method is proposed to determine numerical values of the roots. Analytical values of the model roots without the consideration of friction are selected as zero-order approximation, i.e.:

$$n_0 = \begin{cases} 0 \\ 0 \\ \alpha_0 + i\beta_0 \\ \alpha_0 - i\beta_0 \\ -\alpha_0 + i\beta_0 \\ -\alpha_0 - i\beta_0 \end{cases}, \quad (13)$$

where:

$$\alpha_0 = \sqrt{0.5\left(-1+\sqrt{1+\lambda}\right)}\,;\ \beta_0 = \sqrt{0.5\left(1+\sqrt{1+\lambda}\right)}. \quad (14)$$

Such initial approximation is used for r parameter:

$$r = -\frac{f\lambda}{1+\lambda}. \quad (15)$$

Taking into account vector form of the characteristic equation roots, we will determine tangential motions with the help of following ratio:

$$v_i(\varphi) = S_{i,0} + S_{i,1}e^{r\varphi} + \sum_{j=0}^{1}\left[e^{\alpha_j\varphi}\left(S_{i,2j+2}\cos(\beta_j\varphi) + S_{i,2j+3}\sin(\beta_j\varphi)\right)\right];\ (i=0,1), \quad (16)$$

where undefined $S_{i,j}$ coefficients can be defined taking into consideration boundary conditions given below.

Zero value of bending moments at the ends of the brake beam are ($\varphi = -\gamma$ and $\varphi = \gamma$), i.e.:

$$M_0(-\gamma) = 0\,;\ M_1(\gamma) = 0. \quad (17)$$

Equality of shear forces and axial forces corresponding to force projections within horizontal connection rods is:

$$Q_0(-\gamma) = \frac{N}{2}\cos(\gamma)\,;\ Q_1(\gamma) = -\frac{N}{2}\cos(\gamma)\,; \quad (18)$$

$$T_0(-\gamma) = \frac{N}{2}\sin(\gamma)\,;\ T_1(\gamma) = \frac{N}{2}\sin(\gamma). \quad (19)$$

Zero value of axial motions within a central point of the brake beams is:

$$v_0(0) = 0\,;\ v_1(0) = 0. \quad (20)$$

Continuity of radial motions, rotation angles, bending moments, and shear forces within the abovementioned point are:

$$w_0(0) = w_1(0)\,;\ \theta_0(0) = \theta_1(0)\,; \quad (21)$$

$$M_0(0) = M_1(0)\,;\ Q_0(0) = Q_1(0). \quad (22)$$

As it has already been mentioned, continuity condition for axial forces within the point is not met.

Uniting the solutions to determined parameters of each part of the brake beam, compile an expression describing axial motions, i.e.:

$$v(\varphi) = \begin{cases} v_0(\varphi), \varphi < 0 \\ v_1(\varphi), \varphi \ge 0 \end{cases}. \quad (23)$$

The same expressions may also be used to describe radial deflections of the beam, i.e.:

$$w(\varphi) = \begin{cases} w_0(\varphi), \varphi < 0 \\ w_1(\varphi), \varphi \ge 0 \end{cases}. \quad (24)$$

In this context, the following is applicable for each part of the beam:

$$w_i(\varphi) = S_{i,1}re^{r\varphi} + \sum_{j=0}^{1}\left[e^{\alpha_j\varphi}\left(S_{i,2+2}\left(-\beta_j\sin(\beta_j\varphi) + \alpha_j\cos(\beta_j\varphi)\right) + S_{i,2j+3}\left(\beta_j\cos(\beta_j\varphi) + \alpha_j\sin(\beta_j\varphi)\right)\right)\right];\ (i=0,1). \quad (25)$$

The expression determining rotation angles of the beam is:

$$\theta(\varphi) = \begin{cases} \theta_0(\varphi), \varphi < 0 \\ \theta_1(\varphi), \varphi \ge 0 \end{cases}. \quad (26)$$

In this context:

$$\theta_i(\varphi) = \frac{1}{R}\Big[S_{i,0} + S_{i,1}e^{r\varphi}(1+r)^2 + \sum_{j=0}^{1}\Big(e^{\alpha_j\varphi}\Big(S_{i,2+2}\big(\cos(\beta_j\varphi)(\delta_{1,j}+1) + \sin(\beta_j\varphi)(-\delta_{2,j})\big) + S_{i,2j+3}\big(\cos(\beta_j\varphi)(\delta_{2,j}) + \sin(\beta_j\varphi)(\delta_{1,j}+1)\big)\Big)\Big)\Big];\ (i=0,1)\,; \quad (27)$$

$$\delta_{1,j} = (\alpha_j)^2 - (\beta_j)^2\,;\ \delta_{2,j} = 2\alpha_j\beta_j. \quad (28)$$

It is possible to represent bending moments originating within the beam as follows:

$$M(\varphi) = \begin{cases} M_0(\varphi), \varphi < 0 \\ M_1(\varphi), \varphi \ge 0 \end{cases}. \quad (29)$$

In this context:

$$M_i(\varphi) = -\frac{EI}{R}\Big[S_{i,1}e^{r\varphi}r\left(1+r^2\right) + \sum_{j=0}^{1}\Big[e^{\alpha_j\varphi}\Big(S_{i,2j+2}\big(\cos(\beta_j\varphi)(\delta_{5,j}) + \sin(\beta_j\varphi)(\delta_{6,j})\big) + S_{i,2j+3}\big(\cos(\beta_j\varphi)(-\delta_{6,j}) + \sin(\beta_j\varphi)(\delta_{5,j})\big)\Big)\Big]\Big];\ (i=0,1)\,; \quad (30)$$

$$\delta_{3,j} = (\alpha_j)^3 - 3\alpha_j(\beta_j)^2\,;\ \delta_{4,j} = (\beta_j)^3 - 3(\alpha_j)^2\beta_j\,;\ \delta_{5,j} = \delta_{3,j} + \alpha_j\,;\ \delta_{6,j} = \delta_{4,j} - \beta_j. \quad (31)$$

Shear forces are:

$$Q(\varphi) = \begin{cases} Q_0(\varphi), \varphi < 0 \\ Q_1(\varphi), \varphi \ge 0 \end{cases}. \quad (32)$$

In this context:

$$Q(\varphi) = \frac{EI}{R^3}\Big[S_{i,1}e^{r\varphi}r^2\left(1+r^2\right) + \sum_{j=0}^{1}\Big[e^{\alpha_j\varphi}\Big(S_{i,2j+2}\big(\cos(\beta_j\varphi)(\delta_{7,j}) + \sin(\beta_j\varphi)(\delta_{8,j})\big) + S_{i,2j+3}\big(\cos(\beta_j\varphi)(-\delta_{8,j}) + \sin(\beta_j\varphi)(\delta_{7,j})\big)\Big)\Big]\Big];\ (i=0,1). \quad (33)$$

$$\delta_{7,j} = \delta_{5,j}\alpha_j + \delta_{6,j}\beta_j\text{ ; }\delta_{8,j} = \delta_{6,j}\alpha_j - \delta_{5,j}\beta_j. \quad (34)$$

Axial forces are:

$$T(\varphi) = \begin{cases} T_0(\varphi), \varphi < 0 \\ T_1(\varphi), \varphi \ge 0 \end{cases}, \quad (35)$$

where:

$$\begin{aligned} T_i(\varphi) &= S_{i,1}e^{r\varphi}\left[\frac{EI}{R^3}r^3\left(1+r^2\right)+kRr\right]+ \\ &+\sum_{j=0}^{1}\left[e^{\alpha_j\varphi}\left(S_{i,2j+2}\left(\cos\left(\beta_j\varphi\right)\left(\delta_{9,j}\right)+\sin\left(\beta_j\varphi\right)\left(\delta_{10,j}\right)\right)+\right.\right. \\ &\left.\left.+S_{i,2j+3}\left(\cos\left(\beta_j\varphi\right)\left(-\delta_{10,j}\right)+\sin\left(\beta_j\varphi\right)\left(\delta_{9,j}\right)\right)\right)\right];\left(i=0,1\right) \end{aligned} \quad (36)$$

$$\begin{aligned} \delta_{9,j} &= kR\alpha_j + \frac{EI}{R^3}\left(\delta_{7,j}\alpha_j + \delta_{8,j}\beta_j\right); \\ \delta_{10,j} &= -kR\beta_j + \frac{EI}{R^3}\left(-\delta_{7,j}\beta_j + \delta_{8,j}\alpha_j\right) \end{aligned}. \quad (37)$$

Distributed normal forces in the beam correspond to following expression:

$$q(\varphi) = \frac{w(\varphi)k}{B_T}. \quad (38)$$

Equations (23), (24), (26), (29), (32), (35) are mathematical model to identify tangential and normal forces acting on the brake beam; the model involves values of the parameters of friction forces and bending stiffness of the beam.

Use of the equations describing distributed normal forces within the beam helps formulate the expression to determine braking moment, i.e.:

$$M_T = fB_TR^2\int_{-\gamma}^{\gamma}q(\varphi)d\varphi. \quad (39)$$

Hence, forces within horizontal connection rods are:

$$N_x = B_TR\int_{-\gamma}^{\gamma}q(\varphi)\cos(\varphi)d\varphi. \quad (40)$$

Forces within a vertical post are:

$$N_y = B_TR\int_{-\gamma}^{\gamma}q(\varphi)\sin(\varphi)d\varphi. \quad (41)$$

As an example, consider force calculation of shoe brake MHM CR-4×3/0.7 having following parameters:

– radius of neutral line of a brake beam is $R = 2260$ mm;
– width of braking field is $B_T = 400$ mm;
– half of contact arc is $\gamma = 50°$;
– thickness of brake beam is $H = 400$ mm;
– thickness of brake lining is $H_H = 80$ mm;
– elasticity module of a beam material is $E = 2.1\cdot10^{11}$ Pa;
– elasticity module of a lining material is $E_H = 3\cdot10^8$ Pa;
– friction coefficient between a lining and a drum is $f = 0$; 0.3.

Assume that force within horizontal connection rod used in calculations according to B.L. Davydov model applicable to the machine is 699 kN (Davydov, 1959).

Figures 3 – 6 represent graphic interpretation of the calculation results. Red line is a curve of the calculation results according to the model where friction effect on the distribution of contact pressures was not involved; blue line is a curve of the calculation results involving friction; and green line is a graph of results obtained with the help of a model by B.L. Davydov.

As it is obvious, distribution of radial deflections when friction is not taken into consideration (Fig. 3) is a symmetric function. Consideration of friction shows that the share of the brake beam moving forward is slower than that moving back. Calculations involving the model by B.L. Davydov (with the assumption of absolute stiffness of brake beam and noneffect of friction forces) give qualitatively incorrect result reflected within the graph by means of maximum in the central part and with no boundary effect. Error in the process of maximum motion determination and, hence, contact forces is 148.7%.

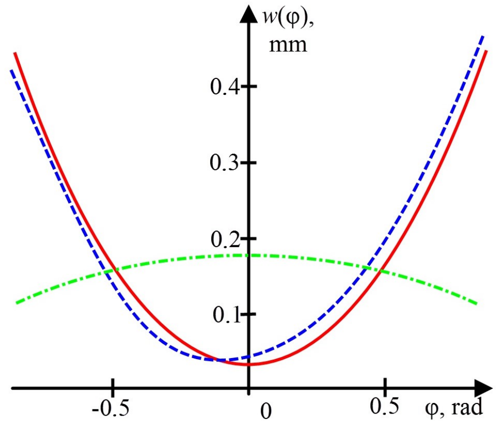

Figure 3. Graph of distribution of radial deflections along a brake beam

A function of bending moments distribution developed without consideration of friction is also symmetric (Fig. 4); in this context, the curve has two similar minimums $-1.1\cdot10^5$ N·m and one extremum corresponding to the beam center and being equal to $-7.1\cdot10^4$ N·m.

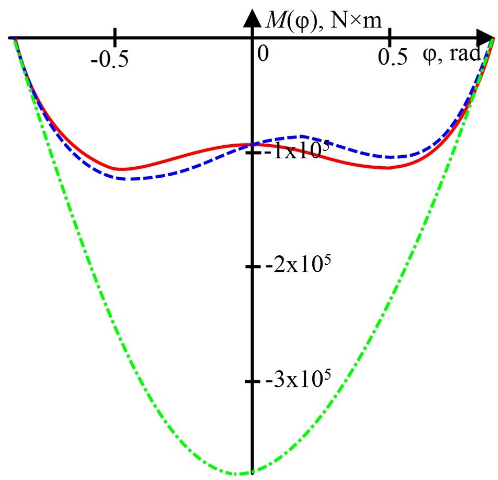

Figure 4. Graph of distribution of bending moments along a brake beam

Use of the model involving friction means that contrary to the above considered graphs, absolute values of a bending moment within the beam share moving forward exceeds the moment value within the beam share moving back. Calculation involving the model by B.L. Davydov gives three times overstated values of bending moments.

Results of shear force distribution not involving friction are antisymmetric function with zero value in a point corresponding to the beam centre (Fig. 5). If friction is involved, then the graph of shear force has a break corresponding to the beam centre. In this context, the force value is 39.5 kN. Calculation with the use of the model by B.L. Davydov produces monotonically decreasing function which curve passes through 0.

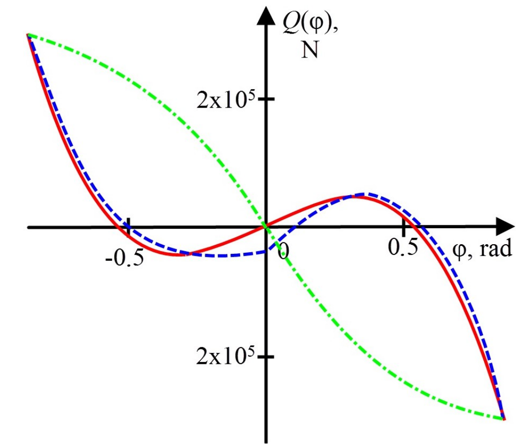

Figure 5. Graph of distribution of shear forces along a brake beam

Without consideration of friction, distribution of axial force looks like a symmetric function relative to other parameters (Fig. 6). Axial forces calculated for both models coincide at the ends of the beam. In the central part, a curve of distribution function involving friction has a break corresponding to 383 kN; it is equal to a force within a vertical post. Calculation involving the model by B.L. Davydov gives 39% understated step in value of axial force being equal to 275 kN; in this context, minimal of them decreases by 34%.

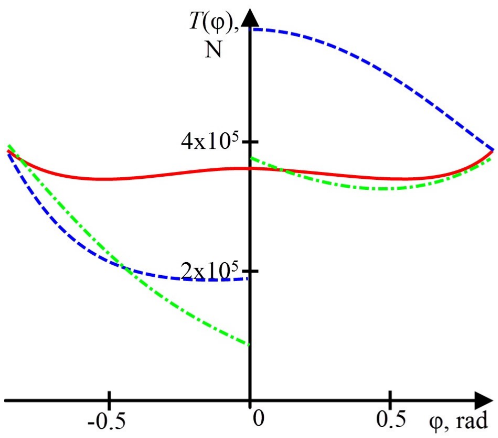

Figure 6. Graph of distribution of axial forces along a brake beam

Following formula is used to calculate braking moment:

$$M_T = fR^2 \int_{-\gamma}^{\gamma} \kappa w(\varphi) d\varphi . \tag{42}$$

Value of the parameter with the use of frictionless model is $7.72 \cdot 10^5$ N·m; if friction is involved, it is $7.87 \cdot 10^5$ N·m; if model by B.L. Davydov is applied, the value is $6.98 \cdot 10^5$ N·m. Deviation of the braking moment value towards its decrease with the use of simplified model is 1.94%; in terms of the model by B.L. Davydov, it is 11.3%.

Following formula is applied to determine forces within a vertical post with the use of the complete model:

$$F_y = R \int_{-\gamma}^{\gamma} \kappa w(\varphi)\left(f \cos(\varphi) + \sin(\varphi)\right) d\varphi . \tag{43}$$

Like in previous case of the models use, the parameter is: $2.75 \cdot 10^5$ N, $3.48 \cdot 10^5$ N, and $2.75 \cdot 10^5$ N respectively. Error of the force determination with the use of two models (i.e. simplified model and the model by B.L. Davydov) is 26.5%.

The obtained results help analysts and designers select the most appropriate model of contact interaction in the process of braking to provide its rational design by means of finite-element method.

CONCLUSIONS

1. For the first time, physical model of a brake lining in the form a group of elastic non-interacting bodies of Winkler foundation was developed. The bodies resist compression and transfer through themselves the distributed friction forces arising between a brake drum and a brake shoe; the friction forces are meant for limiting balance state in accordance with Coulomb's law.

2. For the first time, physical model of a brake beam in the shape of uniform-section circular bar has been developed. The bar is mounted on a vertical post; its interacts with brake drum through brake lining loaded with the distributed normal and tangential load modeling contact brake shoe-drum interaction, and a vertical post has been modeled as a movable pivot point located in the central part of a circular bar.

3. For the first time, mathematical model to determine both tangential and normal forces acting on a brake beam has been formulated. The model involves six balance equations of elementary sections of a bar mounted on the elastic basis and experiencing the action of distributed friction forces calculated according to Coulomb's law; equations describing Hook's law as for a bending moment; a condition of the bar's central line inelasticity; twelve boundary conditions according to which values of shear and longitudinal force, bending moment within the bar ends, continuity of all motions and forces except longitudinal one (equal to zero) within the area of movable pivot point mounting.

4. Exclusion method and Euler's method were applied to calculate both tangential and normal forces acting on a brake beam; the forces are used to determine braking moment as well as forces in connection rods and a post.

5. Analysis of a shoe brake of machine CR-4×3/0.7 was applied to prove that distribution of normal force acting on a brake beam calculated according to B.L. Davydov method as well as the approach developed by the authors has following principal difference: in case one it is in the form of sinusoid; in case two described by the paper it is a parabola with the distinct boundary effect. In this context, a value of a maximum contact pressure calculated according to B.L. Davydov model has appeared to be 2.5 times understated.

6. Minimum value of longitudinal force calculated according to B.L. Davydov model is 34% lower; values of bending moment are three times higher; and forces in a vertical post are 26.5% less to compare with the research results.

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ABSTRACT (IN UKRAINIAN)

Мета. Розробка моделі контактної взаємодії колодкового гальма шахтної підйомної машини з барабаном при гальмуванні з урахуванням кінцевої згинальної жорсткості балки та впливу сил тертя на розподіл контактного тиску задля створення рекомендацій до проектування раціональної конструкції гальмівної балки.

Методика. За допомогою методу виключення і методу Ейлера сформульовано закони розподілу контактного зусилля, зусилля у вертикальній стійці та гальмівного моменту, котрі мають місце у процесі гальмування.

Результати. Досліджено можливості застосування в силовому розрахунку гальм шахтних підйомних машин гіпотез щодо абсолютної жорсткості балки та відсутності впливу сил тертя на розподіл контактного тиску. Розроблені фізична і математична моделі контактної взаємодії гальмівної балки шахтної підйомної машини з барабаном у процесі гальмування.

Наукова новизна. Вперше розроблена фізична модель гальмівної накладки у вигляді масиву пружних тіл типу вінклеровскої основи, які працюють на стиск і передають через себе розподілене дотичне навантаження (сили тертя), що виникає між гальмівним барабаном і гальмівною колодкою, розраховане на граничний стан рівноваги відповідно до закону Кулона; фізична модель гальмівної балки у вигляді кругового бруса постійного перерізу, який встановлений на вертикальній стійці та взаємодіє з гальмівним барабаном через гальмівну накладку, навантажену розподіленим нормальним і дотичним навантаженням, що моделює контактну взаємодію гальмівної колодки і барабана, а вертикальна стійка змодельована як рухливий шарнір, розташований посередині кругового бруса. Вперше сформульована математична модель визначення дотичних і нормальних зусиль, що діють на гальмівну балку.

Практична значимість. Розроблені рекомендації щодо використання різних моделей процесу гальмування дозволяють створити найбільш раціональну модель для силового розрахунку гальмівної балки методом кінцевих елементів.

Ключові слова: *фізична та математична моделі балки і накладки, шахтна підйомна машина, колодкове гальмо, метод Ейлера, закон Кулона*

ABSTRACT (IN RUSSIAN)

Цель. Разработка модели контактного взаимодействия колодочного тормоза шахтной подъемной машины с барабаном при торможении с учетом конечной изгибной жесткости балки и влияния сил трения на распределение в ней контактного давления для создания рекомендаций к проектированию рациональной конструкции тормозной балки.

Методика. С помощью метода исключения и метода Эйлера сформулированы законы распределения контактного усилия, усилия в вертикальной стойке и тормозного момента, которые возникают в процессе торможения.

Результаты. Исследованы границы применения в силовом расчете тормозов шахтных подъемных машин гипотез об абсолютной жесткости балки и невлиянии сил трения на распределение в ней контактного давления. Разработаны физическая и математическая модели контактного взаимодействия тормозной балки шахтной подъемной машины с барабаном в процессе торможения.

Научная новизна. Впервые разработана физическая модель тормозной накладки в виде массива упругих тел типа винклеровского основания, которые работают на сжатие и передают через себя распределенную касательную нагрузку (силы трения), возникающую между тормозным барабаном и тормозной колодкой, рассчитанную на предельное состояние равновесия в соответствии с законом Кулона; физическая модель тормозной балки в виде кругового бруса постоянного сечения, который установлен на вертикальной стойке и взаимодействует с тормозным барабаном через тормозную накладку, нагруженную распределенной нормальной и касательной нагрузкой, моделирующей контактное взаимодействие тормозной колодки и барабана, а вертикальная стойка смоделирована как подвижный шарнир, расположенный посередине кругового бруса. Впервые сформулирована математическая модель определения касательных и нормальных усилий, действующих на тормозную балку.

Практическая значимость. Разработанные рекомендации к использованию разных моделей процесса торможения позволяют создать наиболее рациональную математическую модель для силового расчета тормозной балки методом конечных элементов.

Ключевые слова: *физическая и математическая модели балки и накладки, шахтная подъемная машина, колодочный тормоз, метод Эйлера, закон Кулона*

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