

## AN ADAPTIVE NEURO FUZZY INFERENCE SYSTEM TO MODEL THE UNIAXIAL COMPRESSIVE STRENGTH OF CEMENTED HYDRAULIC BACKFILL

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### ABSTRACT

**Purpose.** The purpose of this paper is to develop the models for predicting the uniaxial compressive strength (UCS) of cemented hydraulic backfill (CHB), a widely used technique for filling underground voids created by mining operations as it provides the high strength required for safe and economical working environment and allows the use of waste rock from mining operations as well as tailings from mineral processing plants as ingredients.

**Methods.** In this study, different modelling techniques such as conventional linear, nonlinear multiple regression and one of the evolving soft computing methods, adaptive neuro fuzzy inference system (ANFIS), were used for the prediction of UCS, the main criterion used to design backfill recipe.

**Findings.** Statistical performance indices used to evaluate the efficiency of the developed models indicated that the ANFIS model can effectively be implemented for designing CHB with desired UCS. As proved by the performance indicators ANFIS model gives more compatible results with the expert opinion and current literature than conventional modelling techniques.

**Originality.** In order to construct the models a very large database, containing more than 1600 UCS test results, was used. In addition to widely used conventional regression based modelling techniques, one of the evolving soft computing methods, ANFIS was employed. Numerical examples showing the implementation of constructed models were provided.

**Practical implementation.** As proved by the statistical performance indicators, the developed models can be used for a reliable prediction of the UCS of CHB. However, more accurate results can be achieved by expanding the database and by constructing improved models using the algorithm presented in this paper.

**Keywords:** cemented hydraulic backfill, adaptive neuro fuzzy inference system, multiple regression model, underground mining

### 1. INTRODUCTION

Waste rock and tailings from a mineral processing plant are the main waste streams of mining operations. Due to environmental and safety concerns, there is an increasing interest in using these waste rock and tailings as ingredients of backfill to fill the underground voids created by mining operations rather than storing them on surface. Cemented hydraulic backfill is composed of crushed waste rock, tailings, water and cement. As it allows the use of primary waste streams, CHB has been increasingly accepted as a component of underground mining operations.

During underground mining operations, CHB is placed into previously extracted stopes to provide a stable

working platform to work on, to support mined regions, to enhance pillar recovery and to reduce dilution. UCS is a primary parameter used for designing CHB. The factors affecting short and long term strength of backfill can be divided into two main groups, extrinsic and intrinsic factors. Extrinsic factors are mostly related to in situ conditions such as underground temperature and humidity conditions, drainage conditions, stope geometry, ground water conditions: such factors can be considered as uncontrollable parameters. Intrinsic factors, however, are related to physical, chemical and mineralogical properties of the main components of backfill materials and mixing properties. Unlike extrinsic factors, intrinsic factors can be controlled to obtain backfill with the desired laboratory strength. Density (D), cement dosage (CD), coarse and

fine aggregates ratio (ATR), and curing time (CT) are widely known intrinsic factors controlling UCS (Belem & Benzaazoua, 2007). Density (or solid content) is a key factor controlling not only the flowability but also the resulting UCS of backfill (Choudhary & Kumar, 2013). The adjustment of the density is considered the most practical way of increasing the UCS. There is also a strong positive correlation between the cement dosage and strength of backfill (Belem, Benzaazoua, & Bussiere, 2000; Choudhary & Kumar, 2013). On the other hand, it is known that the cement cost is the main component of overall backfill cost. The combined grading of coarse aggregate, crushed waste rock, fine aggregate, and processing plant tailings is represented by the aggregate tailings ratio (ATR). High tailings addition decreases the ratio and the UCS because the surface area of the particles requiring cement coating and binding is increased (Clark, 1988; Wang & Villaescusa, 2000). Regarding the curing time (CT), when fly ash or furnace slag are used as cement replacement material, the UCS may continue to increase even after 28 days (Elchalakani, Basarir, & Karrech, 2017), whereas when ordinary Portland cement is used as a binder, like other concrete structures, cemented backfill gains most of its strength within 28 days (Clark, 1988; Wang & Villaescusa, 2000; Lee, 2003; Potvin, Thomas, & Fourie, 2005).

Carrying out an experimental program for every potentially viable recipe is an expensive and time-consuming task. Therefore, modelling studies are widely used to predict UCS values. In most of the modelling studies, the above-mentioned main factors are considered as independent variables for the prediction of UCS. For cement-like mixtures such as CHB, conventional regression modelling has been widely used (Tsilivilis & Parisakis, 1995; De Siqueira Tango, 1998; Kheder, 2003; Akkurt, Tayfur, & Can, 2004) to predict UCS. Recently, evolutionary soft computing methods such as artificial neural networks (ANN) and adaptive neuro fuzzy inference systems (ANFIS) have been used for similar purposes (Akkurt, Tayfur, & Can, 2004; Özcan, Atiş, Karahan, Uncuoğlu, & Tanyildizi, 2009).

Jinfeng underground gold mine is located in Southwest Guizhou province of China. The annual production of the mine is around 0.5 Mt and will be increased to 0.75 Mt/year. Such production will increase not only the amount of waste rock but also the amount of tailings; almost 95 – 98% of feed ore, from the processing plant will be generated. The inclination and thickness of the orebody ranges from 55 to 85° and 3 to 20 m respectively. Hangingwall and footwall contacts are mostly located within fault and fissure zones. For such conditions the most appropriate mining method was selected as the overhand cut and fill mining method, requiring high strength backfill in order to prevent collapse and provide a strong working floor for mechanised mining equipment. Considering relevant environmental issues, the selected mining method and corresponding backfill requirements, the company preferred the use of CHB as the support method. For the development of an effective backfill program, the company started a comprehensive research and testing program as explained in the following section.

In this paper, the results of the research program are used to construct linear regression, nonlinear regression and ANFIS models to predict the UCS of different CHB recipes at certain curing times. The developed models can be used to design CHB recipes considering the desired UCS. In the models, accepted intrinsic factors such as D, ATR, CD and CT are considered as independent variables given that the other factors such as chemistry of tailings and mixing water properties have fixed values for the studied mine.

## 2. FIELD AND LABORATORY STUDIES

The main components of CHB are cement, crushed waste rock from both open pit and underground mines, processing plant tailings and water. The specific gravity, bulk density and void ratio of crushed waste rock are 2.57 g/cm<sup>3</sup>, 1.05 g/cm<sup>3</sup> and 0.57 respectively. The particle size distribution of the crushed waste rock is given in Figure 1.

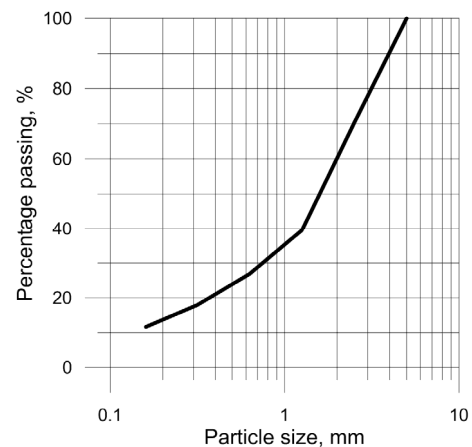


Figure 1. Particle size distribution of crushed waste rock

The chemical and physical properties of the Portland cement, PC 32.5, used at the mine are given in Table 1.

Table 1. Chemical and mechanical properties of cement used in backfill

MgO, %	2.15
SO <sub>3</sub> , %	2.02
Alkali (NaO + 0.658 K <sub>2</sub> O), %	0.76
Chloride content, %	0.009
Loss on ignition, %	5.26
Insoluble residue, %	12.71
+0.08 mm, %	17.50
Specific surface, cm <sup>2</sup> /g	3580
Soundness, mm	62.3
Initial setting time, min	172
Final setting time, min	204
3 days Compressive strength, MPa	14.77
7 days Compressive strength, MPa	21.5
28 days Compressive strength, MPa	—

The specific gravity, bulk density and osmotic coefficient of the tailings are 2.7 g/cm<sup>3</sup>, 1.3 g/cm<sup>3</sup> and 0.28 cm/h respectively. The grain size distribution of tailings is given in Figure 2.

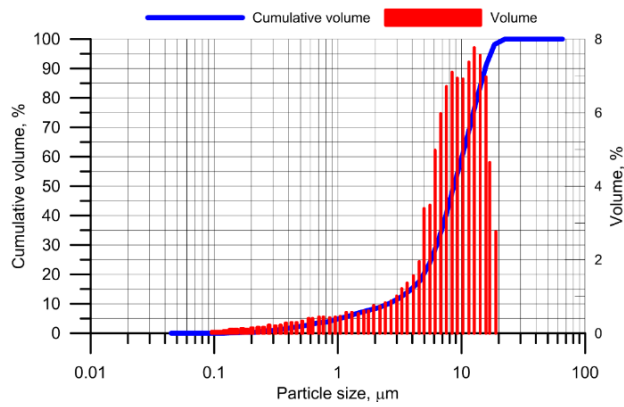


Figure 2. Grain size distribution of tailings

An experimental program was designed considering the main parameters D, CD, ATR and CT, affecting the strength of CHB as independent variables. The histograms and brief statistical information of the variables are shown in Figure 3.

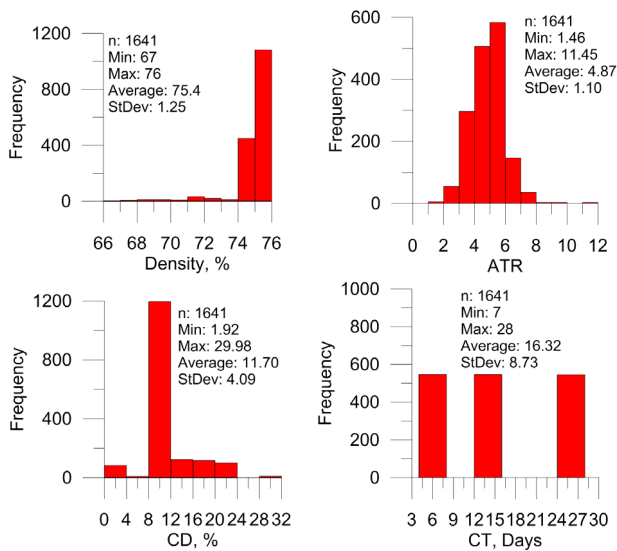


Figure 3. Histograms of D, ATR, CD, and CT

The program was operated over three years and 1641 UCS tests were conducted on cylindrical samples. The prepared samples were poured into cylindrical plastic moulds and cured in a humidity chamber maintained at 90% humidity and  $23 \pm 2^\circ\text{C}$  temperature for a period of 7, 14 and 28 days. At the end of the curing time, the specimens were cut into  $80 \times 160$  mm size and loaded using a 100 kN capacity universal testing loading machine.

### 3. MODELLING STUDIES

For model development, the database was initially randomly divided into two parts, being the training and checking data sets to be used in the modelling phase. The statistical properties of training, checking and all datasets are presented in Table 2.

Conventional linear (LMR), nonlinear (NMR) multiple regression and ANFIS modelling techniques were used to construct the UCS prediction models. The models were trained using the training dataset and the performance of trained model is checked using the checking dataset.

Table 2. Statistical properties of training, checking and all datasets

		D, %	ATR	CD, %	CT, days	UCS, MPa
Training dataset	No of data	1149	1149	1149	1149	1149
	Max	76	11.45	29.98	28	6.34
	Min	67	1.46	1.92	7	0.03
	Range	9	9.99	28.06	21	6.31
	Average	75.41	4.86	11.80	16.36	0.90
	Std. Dev.	1.27	1.11	4.18	8.75	0.63
Checking dataset	No of data	492	492	492	492	492
	Max	76	9.02	29.14	28	4.09
	Min	68	1.46	1.93	7	0.04
	Range	8	7.56	27.21	21	4.05
	Average	75.42	4.91	11.46	16.23	0.86
	Std. Dev.	1.20	1.05	3.86	8.70	0.58
All datasets	No of data	1641	1641	1641	1641	1641
	Max	76	11.45	29.98	28	6.34
	Min	67	1.46	1.92	7	0.03
	Range	9	9.99	28.06	21	6.31
	Average	75.4	4.87	11.70	16.32	0.89
	Std. Dev.	1.25	1.10	4.09	8.73	0.61

A Pearson correlation table was used to survey the independence of the variables relative to each other. Exceeding the limits of correlation between the variables can force the equation to produce unfaithful outputs due to multi-collinearity. The Symmetrical Pearson Table shown in Table 3 shows the one-to-one statistical relationships between the variables. It is undesirable to have the variables with a correlation value higher than 0.7 in the equation (Pallant, 2010). All values in the table are less than 0.7 and no correlation is detected between the variables.

Table 3. Pearson's correlation table

	D	ATR	CD	CT	UCS
D	1	0.45	0.01	0.00	0.09
ATR		1	-0.35	0.00	-0.20
CD			1	0.00	0.69
CT				1	0.46
UCS					1

Through modelling studies, a number of LMR, NMR and ANFIS models were constructed using different randomly generated training sets. The best performing models were selected and presented in the following sections.

#### 3.1. Multiple regression modelling

Application of multiple regression analyses using a statistical package SPSS 20 (SPP20 IBM, 2011) yielded the following linear (LMR) and nonlinear (NMR) models:

$$UCS = -0.0144D + 0.0316ATR + 0.1091CD + 0.0335CT; \quad (1)$$

$$UCS = \exp(0.0835D + 0.0373ATR + 0.0910CD + 0.0352CT - 8.3578). \quad (2)$$

For the LMR and NMR models the coefficient of correlation ( $R$ ) values were 86 and 90% respectively. The analysis of variance (ANOVA) tables for linear and non-linear models are given in Table 4. F-test was used to confirm the validity of the overall model. The calculated probability value ( $Prob(F)$ ) demonstrates a very high significance and confirms the adequacy of the models.

The significance level of the test is specified as the commonly accepted value of 0.05.

The cross-correlation graphs showing the observed and predicted UCS for training, checking and overall data sets from LMR and NMR models together with the 1:1 correspondence line are given in Figure 4, together with the coefficient of correlation  $R$ .

Table 4. ANOVA tables for linear and non-linear multiple regression models

	Source	Degree of freedom	Sum of squares	Mean square	$F$ , ratio	$R^2$	$Prob(F)$
LMR	Regression	3	332.33	110.78	1072.38	74	0
	Error	1145	118.28	0.103			
	Total	1148	450.61				
NMR	Regression	4	364.28	91.07	1206.72	81	0
	Error	1144	86.34	7.55			
	Total	1148	450.62				

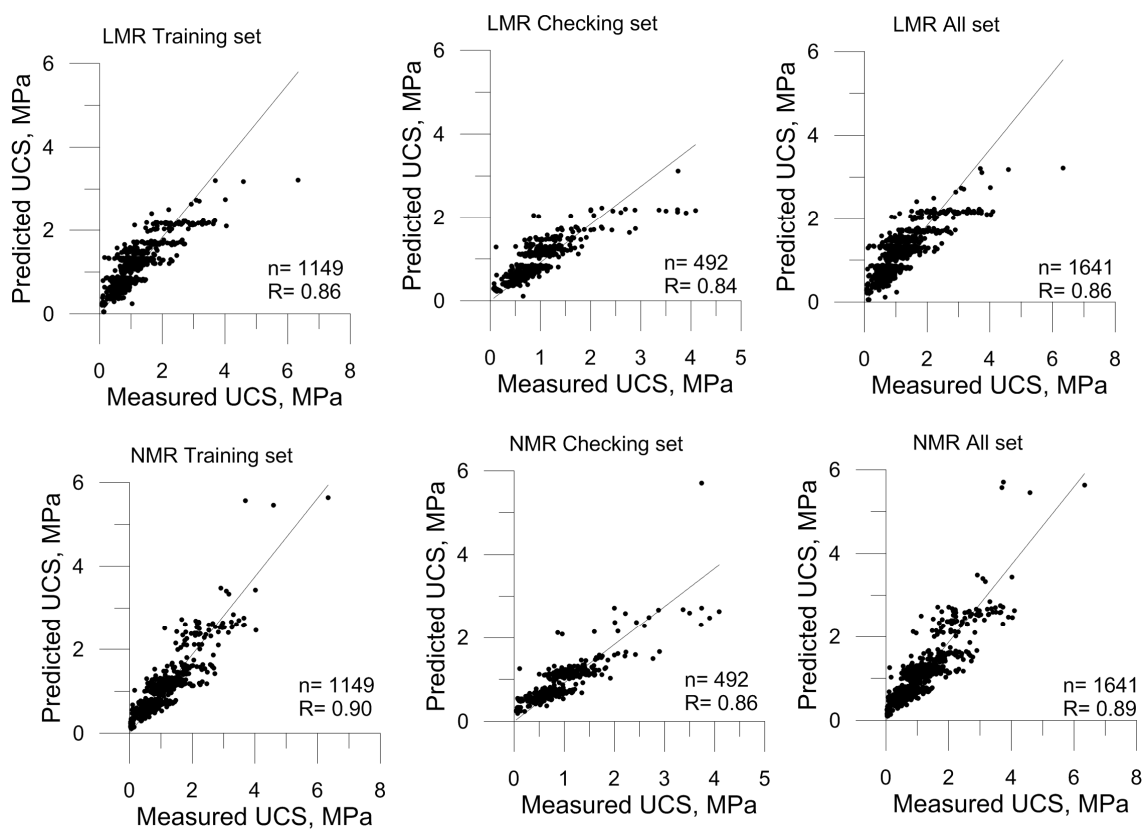


Figure 4. Cross correlation graph for LMR and NMR models

### 3.2. ANFIS modelling

Artificial neural network (ANN) and fuzzy inference system (FIS) have been increasingly used as soft modelling techniques. Each technique has its own advantages and disadvantages. The main advantage of ANN are pattern recognition and adaption to a changing environment, whereas, FIS has the advantage of incorporating human knowledge and expertise to deal with uncertainty and imprecision. As a hybrid modelling technique, adaptive neuro fuzzy inference system (ANFIS) has the advantages of both methods and is widely used in practical cases involving high uncertainty (Asrari, Shahriar, & Ataeeppour, 2013; Bilgehan & Kurtoğlu, 2015; Fattahi, 2016; Basarir & Dincer, 2017; Basarir, Wesseloo,

Karrech, Paternak, & Dyskin, 2017). A brief introduction to ANFIS modelling is given in Appendix A.

The same training and checking datasets with regression modelling were used for constructing the ANFIS model. MATLAB (MATLAB, 2011) was used to train the ANFIS model incorporating a Sugeno-type FIS and ANN structure. Due to its inherent advantages, a hybrid learning algorithm combining a least square estimator and gradient descent method was preferred (Jang, 1993; Jang, Sun, & Mizutani, 1997; Nayak, Sudheer, Rangan, & Ramasastri, 2004). As shown in the sequential network structure given in Figure 5, the inputs and output of the ANFIS model are  $D$ ,  $ATR$ ,  $CD$ ,  $CT$  and  $UCS$ , respectively.

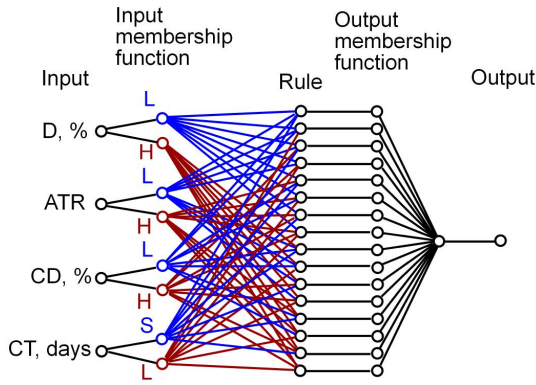


Figure 5. Sequential network structure of ANFIS model

The explanations of the notations used are provided in Figure 6, showing the trained membership functions. In ANFIS modelling, the number of membership functions is defined by the number of clusters. In this study, the number of clusters, two for all inputs, were determined experimentally by developing various modelling.

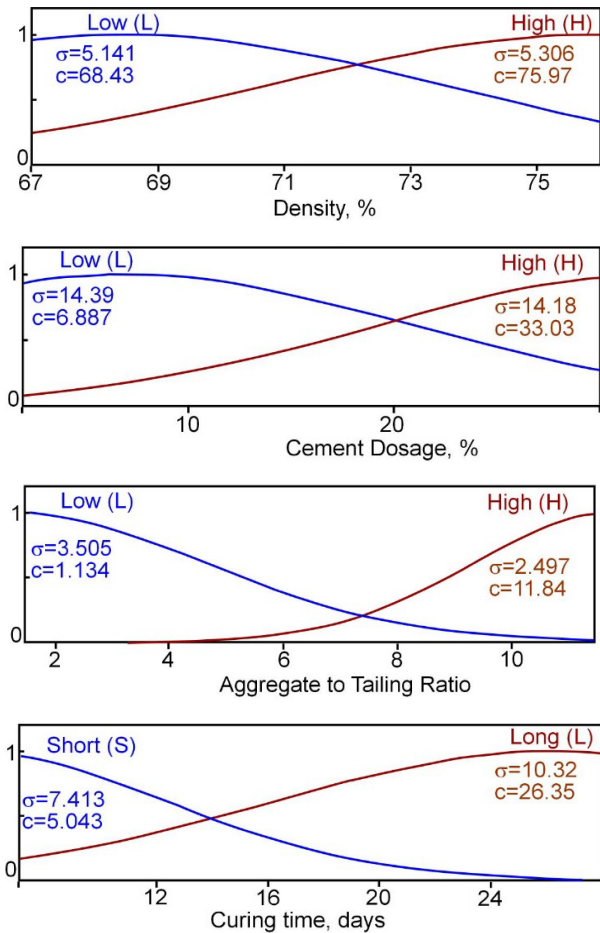


Figure 6. Trained membership functions and Gaussian membership function parameters

For each input a linguistic expression was assigned for each of the membership functions. In this study, a Gaussian membership function was selected due to its smoothness, concise notation and non-zero output at all points. The trained Gaussian type membership functions (Equation (1)), controlling parameters ( $\sigma$  and  $c$ )

and names for each input membership function are shown in Figure 5:

$$f(x; \sigma; c) = e^{-\frac{(x-c)^2}{2\sigma^2}} \quad (3)$$

A rule-based mechanism, consisting of the rules extracted from data statistics, determines the relationship between input and output variables. Having four input variables with two membership functions, 16 rules, each of which yield different outputs, were derived. The type of output can be in the form of a linear equation (or a constant) depending on the order of fuzzy model used. The derived rules and corresponding constant output values for the constructed zero order Sugeno-type ANFIS model are shown in Table 5.

Table 5. Derived ANFIS rules and corresponding output values

No.	Rule	Output, $f$
1	If D is L and ATR is L and CD is L and CT is S	then 0.240
2	If D is L and ATR is L and CD is L and CT is L	then -0.166
3	If D is L and ATR is L and CD is H and CT is S	then -0.377
4	If D is L and ATR is L and CD is H and CT is L	then 0.898
5	If D is L and ATR is H and CD is L and CT is S	then 4.483
6	If D is L and ATR is H and CD is L and CT is L	then -2.735
7	If D is L and ATR is H and CD is H and CT is S	then -17.320
8	If D is L and ATR is H and CD is H and CT is L	then 24.510
9	If D is H and ATR is L and CD is L and CT is S	then -0.260
10	If D is H and ATR is L and CD is L and CT is L	then -0.298
11	If D is H and ATR is L and CD is H and CT is S	then 2.835
12	If D is H and ATR is L and CD is H and CT is L	then 7.324
13	If D is H and ATR is H and CD is L and CT is S	then -2.065
14	If D is H and ATR is H and CD is L and CT is L	then -1.615
15	If D is H and ATR is H and CD is H and CT is S	then 9.883
16	If D is H and ATR is H and CD is H and CT is L	then 7.431

The final stage of the constructed ANFIS model is the de-fuzzification stage. At this stage a crisp value, expressed as a combination of outputs is obtained. For a certain set of D, ATR, CD, CT each rule yields different UCS values. Finally, combining the calculated UCS values, a final crisp UCS value is determined.

An example showing the practical application of the constructed model is given in Appendix B.

Plots of the predicted vs observed UCS values with 1:1 correspondence line, and coefficient of correlation ( $R$ ) values for training, checking and overall data set are given in Figure 7.



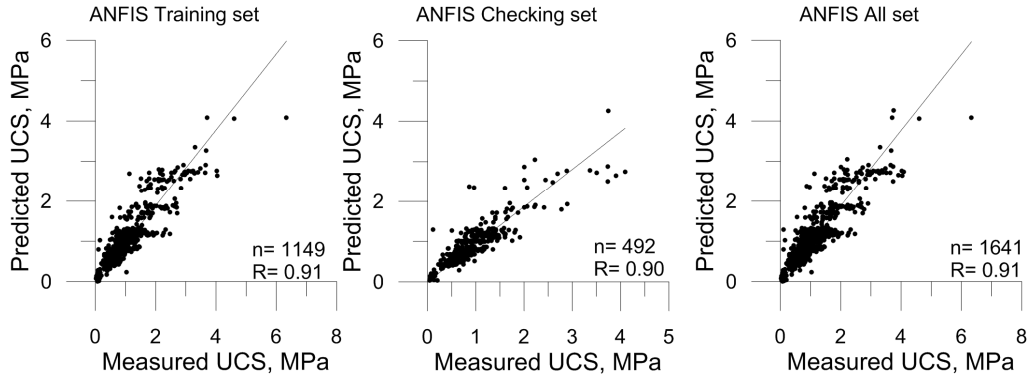


Figure 7. Measured and predicted UCS by ANFIS model

4. MODELLING PERFORMANCE

For model performance assessment the most widely used performance indicators, variable account for (VAF) and mean absolute percentage error (MAPE), are used. VAF performance indices are used to investigate to what degree the model can explain the variance in data. VAF is used to verify the correctness of a model, by comparing the measured values (y) with the estimated output of the model (y<sub>est</sub>). In other words, the higher the VAF the better the model performs. If the measured and predicted values are exactly the same, VAF will be equal to 100%. The accuracy of a fitted model can be measured using mean absolute percentage error (MAPE), expressing accuracy as a percentage:

$$VAF = \left( 1 - \frac{\text{var}(y - y_{est})}{\text{var}(y)} \right) \cdot 100; \tag{4}$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{y_i - y_{est_i}}{y_i} \right| \cdot 100, \tag{5}$$

where:

- var – variance;
- n – the number of samples;
- y – measured;
- y<sub>est</sub> – predicted UCS.

The constructed models and corresponding statistical performance indicators are presented in Table 6. It can be seen that the ANFIS model presented better performance than LMR and NMR models. The ANFIS model has the highest R<sup>2</sup>, VAF and lowest MAPE values for all datasets.

Table 6. Statistical performance indicators for LMR, NMR and ANFIS models

	VAF			MAPE		
	Training	Checking	All	Training	Checking	All
LMR	73.75	71.26	73.11	48.70	44.17	47.34
NMR	80.87	74.54	79.18	37.03	40.26	38.00
ANFIS	82.88	80.13	82.16	23.93	24.12	23.99

In order to check the model sensitivity to sampling variation, the overall data set was randomly divided into separate training and checking data sets 10 more times. In each case, the training set and checking data sets were composed of 1149 and 492 records, respectively. For each training and checking data set, the developed models were

applied, VAF and MAPE values were calculated and presented in Figure 8. For each case, higher VAF and lower MAPE values are observed for the ANFIS model.

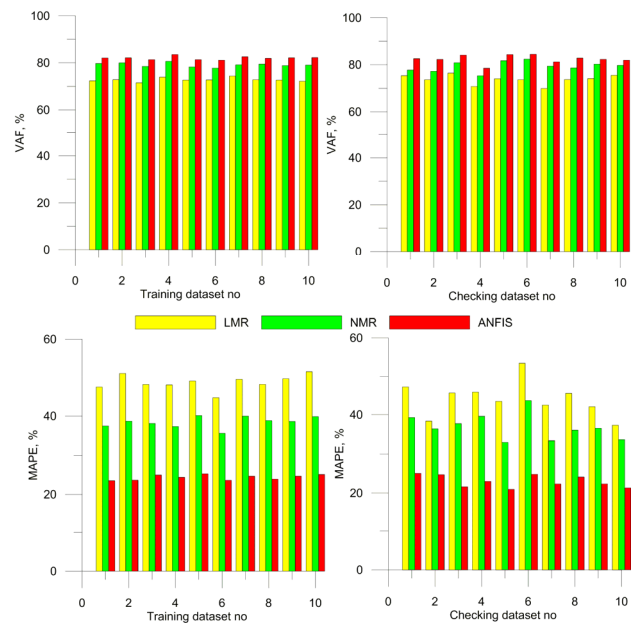


Figure 8. Statistical performance indicators for randomly generated training and checking datasets

In order to compare the relative importance of each parameter for the constructed models, a sensitivity analysis was conducted using the selected values (Fig. 9). The effect of each parameter was analysed by changing one variable in a specified range, as others were kept constant. Both ANFIS and NMR models indicated that the most dominant parameter controlling the UCS is CD followed by ATR and D, respectively. However, LMR models reveal that CT is the most effective parameter followed by CD.

D is the least significant parameter according to LMR and ANFIS models, for NMR model ATR is the least significant parameter. Unlike the LMR model, both ANFIS and NMR models indicate a strong positive correlation between D and UCS, compatible with current literature and expert opinion. All the models reveal that UCS increases with increasing CD and ATR. Such a result is also compatible with observations, i.e. it is expected that as the amount of coarse aggregate increases, UCS increases.

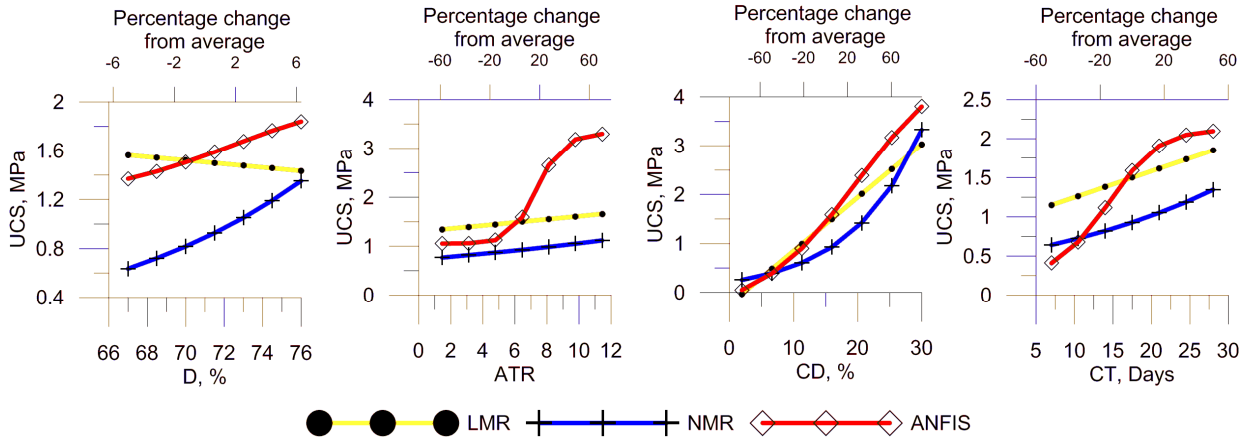


Figure 9. Sensitivity analysis results

Regarding curing time, it is well known that for concrete-like structures most of the strength is gained within 28 days of curing time, unless cement replacement materials such as fly ash or furnace slag are used (Elchalakani, Basarir, & Karrech, 2017). Only ANFIS models predict such behaviour, since both LMR and NMR show that the strength will continue to increase even after 28 days of curing time.

5. CONCLUSIONS

In this study conventional LMR, NMR and ANFIS modelling techniques were employed to construct predictive UCS models to be used for designing CHB recipe. The intrinsic parameters affecting the strength of CHB such as D, ATR, CD, CT were selected and used as input variables for the developed models. Some other variables affecting the strength such as chemistry of tailings and the content of water were not considered as independent variables for the studied mine since they were kept unchanged.

All of the constructed models represented acceptable prediction performance indicating that the models constructed in this study can conveniently be used for the initial estimation of UCS of CHB. The ANFIS model yielded the best performance considering the statistical performance indicators such as MAPE and VAF.

In order to analyse the effect of random dataset construction processes and model sensitivities, 10 more datasets were randomly constructed and model performances were checked again. The results proved that the ANFIS model yields the best performance and the most consistent results for all randomly generated datasets.

The sensitivity analysis showed that the ANFIS model is more sensitive to the changes in the input parameters than the regression models; the model reflects even a small change in the input variables. Moreover, the results of the ANFIS model seem to be more compatible with the expert opinion and current literature. One of the reasons behind high performance and compatibility is the learning capacity of the technique as proved by the better performance indicators for checking datasets. Unlike regression modelling, ANFIS does not require pre-defined mathematical equations for the relationship between input and output variables, and it uses the provided data set for determining the structure of the model effectively.

Through modelling, a special recipe suitable for a desired CHB application can be designed. Therefore, the cost and time of site and laboratory testing can be reduced. Although the performance of the models are satisfactory and acceptable, their accuracy can be further improved by enriching the database with additional experimental results. For the cases where the chemical properties of tailings are different and thus affect the achievable strength, new models can be derived using the algorithm presented in this study.

APPENDIX A

In this paper Takagi, Sugeno, & Kang fuzzy inference system was used to construct ANFIS. A common figure (Fig. 10) is used to explain the method through a simple model containing two rules (Sugeno & Kang, 1988). The model involves premise and consequent parts (Jang, Sun, & Mizutani, 1997). The inference system consists of five layers, each of them involves several nodes, described by node functions.

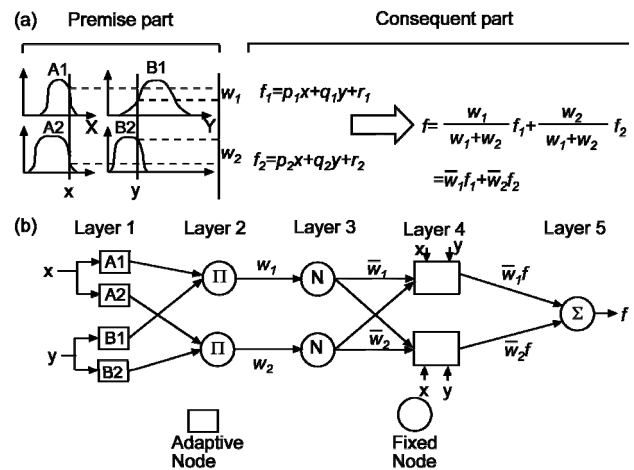


Figure 10. An adaptive network and a fuzzy inference system

The system has two input variables  $x$  and  $y$  and an output variable  $f$ . In fuzzy systems every input variable is described by fuzzy sets. In the example,  $A_1$  and  $A_2$  are the fuzzy sets for the  $x$  variable and  $B_1$  and  $B_2$  are the fuzzy sets for the  $y$  variable.

A fuzzy set  $A(x)$  is represented by the constituent elements  $x$  and their associated membership values  $\mu_A(x)$  (the degree of belongingness):

$$A(x) = \{(x, \mu_A(x)), x \in X\}, \quad (6)$$

where:

$X$  – the universal set consisting of all possible elements;

$\mu_A(x)$  – the membership function, ranges from 0 to 1.

In the ANFIS model the relationship between input and output is expressed by means of if-then rules. The explained example model involves 2 fuzzy “if-then” rules as follows.

Rule 1: if  $x$  is  $A_1$  and  $y$  is  $B_1$ ; then:

$$f_1 = p_1x + q_1y + r_1. \quad (7)$$

Rule 2: if  $x$  is  $A_2$  and  $y$  is  $B_2$ ; then:

$$f_2 = p_2x + q_2y + r_2, \quad (8)$$

where:

$p_1, q_1, r_1, p_2, q_2, r_2$  – the consequent parameters;

$A_1, B_1, A_2, B_2$  – the linguistic labels, which are represented by fuzzy sets as shown in Figure 8;

$\{p_1, q_1, r_1\}, \{p_2, q_2, r_2\}$  – the parameter sets.

Each layer containing the node functions is described below.

*Layer 1: fuzzification layer.* In this layer, every node is an adaptive node with a node function. In other words, the antecedents of the fuzzy rules are represented by nodes in this layer. The parameters of these nodes control the shape and the center of each fuzzy set. The Gaussian type membership function, one of the most widely used membership functions, is adapted in this study and the membership function is given by:

$$O_{1,i} = \mu_{A_i}(x) = f(x; \sigma; c) = e^{-\frac{(x-c)^2}{2\sigma^2}}, \quad (9)$$

where:

$O_{1,i}$  – the output functions;

$x$  – the input node  $i$ ;

$A_i$  – the linguistic label associated with this node (i.e. low, moderate, high etc.);

$\{\sigma, c\}$  – the parameter set that changes the shape of the membership function;

$\sigma, c$  – referred to as premise parameters.

*Layer 2: rule layer.* The nodes in this layer are fixed and their output is the product of all the incoming signals. In this layer each node calculates the firing strength of each rule via multiplication. Each rule is assigned as firing strength measuring the degree to which the rule matches the inputs. The number of nodes in this layer equals the number of if-then rules i.e. 2 for the explained example:

$$O_{2,i} = w_i = \mu_{A_i}(x) = \mu_{B_i}(y) \quad i = 1, 2, \quad (10)$$

where:

$O_{2,i}$  – the output of layer 2;

$w_i$  – the firing strength.

*Layer 3: normalization layer.* The nodes in this layer are the fixed nodes as in layer 2. The ratio of the  $i^{th}$  rule's

firing strength to the sum of all rules firing strength is calculated by the nodes in this layer:

$$O_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2, \quad (11)$$

where:

$O_{3,i}$  – the output of this layer called as normalized firing strength ( $\bar{w}_i$ ).

*Layer 4: defuzzification layer.* Node function is given by:

$$O_{4,i} = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i), \quad i = 1, 2, \quad (12)$$

where:

$\bar{w}_i$  – a normalized firing strength from layer 3;

$\{p_i, q_i, r_i\}$  – the consequent parameter set of this node;

$f_i$  can either be first order polynomial as shown in the example or predefined constant.

*Layer 5: output layer.* The single node in this layer is a fixed node labeled as  $\Sigma$ . The overall output, as the summation of all input from layer 4, is computed by a fixed node. Overall output is given by:

$$O_{5,i} = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i}, \quad i = 1, 2. \quad (13)$$

## APPENDIX B

It is assumed that the used recipe is as follow; D is 71.5, ATR is 6.454, CD is 15.95 % and curing time is 14 days.

*Layer 1: fuzzification.* In this layer for each input variable the membership degrees are calculated by substituting the presented membership function parameters  $\sigma$  and  $c$  (Table 5) into Equation (7).

For diameter (D) for the value of 71.5 mm the membership degrees of Low ( $\mu_{D_L}$ ) and High ( $\mu_{D_H}$ ) are calculated as follow:

$$\begin{aligned} \mu_{D_L}(71.5) &= f(71.5; 5.141; 68.43) = \\ &= e^{-\frac{(71.5-68.48)^2}{2 \cdot 5.141^2}} = 0.837; \end{aligned} \quad (14)$$

$$\begin{aligned} \mu_{D_H}(71.5) &= f(71.5; 5.306; 75.97) = \\ &= e^{-\frac{(71.5-75.97)^2}{2 \cdot 5.306^2}} = 0.701. \end{aligned} \quad (15)$$

For assumed 6.454 ATR the membership degrees of Low ( $\mu_{ATR_L}$ ) and High ( $\mu_{ATR_H}$ ) are calculated:

$$\begin{aligned} \mu_{ATR_L}(6.454) &= f(6.454; 3.505; 1.134) = \\ &= e^{-\frac{(6.454-1.134)^2}{2 \cdot 3.505^2}} = 0.316; \end{aligned} \quad (16)$$

$$\begin{aligned} \mu_{ATR_H}(6.454) &= f(6.454; 2.497; 11.84) = \\ &= e^{-\frac{(6.454-11.84)^2}{2 \cdot 2.497^2}} = 0.0098. \end{aligned} \quad (17)$$



The degree of membership functions cement dosage – Low ( $\mu_{CD_L}$ ) and High ( $\mu_{CD_H}$ ); and curing times – Short ( $\mu_{CT_S}$ ) and Long ( $\mu_{CT_L}$ ) are calculated:

$$\begin{aligned} \mu_{CD_L}(15.95) &= f(15.95; 14.39; 6.887) = \\ &= e^{-\frac{(15.95-6.887)^2}{2 \cdot 14.39^2}} = 0.820; \end{aligned} \quad (18)$$

$$\begin{aligned} \mu_{CD_H}(15.95) &= f(15.95; 14.18; 33.03) = \\ &= e^{-\frac{(15.95-33.03)^2}{2 \cdot 14.18^2}} = 0.484; \end{aligned} \quad (19)$$

$$\begin{aligned} \mu_{CT_S}(14) &= f(14; 7.413; 5.043) = \\ &= e^{-\frac{(14-5.043)^2}{2 \cdot 7.413^2}} = 0.482; \end{aligned} \quad (20)$$

$$\begin{aligned} \mu_{CT_L}(14) &= f(14; 10.32; 26.35) = \\ &= e^{-\frac{(14-26.35)^2}{2 \cdot 10.32^2}} = 0.489. \end{aligned} \quad (21)$$

In the second layer firing strength measuring the degree to which rule matches the input are calculated for each rule via multiplication of calculated membership degrees.

For Rule 1 if D is L and ATR is L and CD is L and CT is S the firing strength can be calculated as follow:

$$\begin{aligned} w_1 &= \mu_{D_S}(71.5) \cdot \mu_{ATR_L}(6.454) \times \\ &\times \mu_{CD_L}(15.95) \cdot \mu_{CT_S}(14) = 0.105. \end{aligned} \quad (22)$$

For Rule 2 if D is L and ATR is L and CD is L and CT is H the firing strength can be calculated as follow:

$$\begin{aligned} w_2 &= \mu_{D_S}(71.5) \cdot \mu_{ATR_L}(6.454) \times \\ &\times \mu_{CD_L}(15.95) \cdot \mu_{CT_H}(14) = 0.106. \end{aligned} \quad (23)$$

Similarly for each rule firing strength are calculated. The calculated firing strength and corresponding rules are as shown in Table 7.

$$\ddot{w}_1 = \frac{w_1}{\sum_{i=1}^{16} w_i} = \frac{0.105}{0.828} = 0.126; \quad (24)$$

$$\ddot{w}_2 = \frac{w_2}{\sum_{i=1}^{16} w_i} = \frac{0.106}{0.828} = 0.128. \quad (25)$$

Table 7. Calculation steps for ANFIS example

No.	Rule	Firing strength, $w_i$	Normalized firing strength, $\bar{w}_i$	Constant, $f_i$	$\bar{w}_i \cdot f_i$
1	If D is L and ATR is L and CD is L and CT is S	0.105	0.126	0.240	0.030
2	If D is L and ATR is L and CD is L and CT is L	0.106	0.128	-0.166	-0.021
3	If D is L and ATR is L and CD is H and CT is S	0.062	0.075	-0.377	-0.028
4	If D is L and ATR is L and CD is H and CT is L	0.063	0.076	0.898	0.068
5	If D is L and ATR is H and CD is L and CT is S	0.032	0.039	4.483	0.175
6	If D is L and ATR is H and CD is L and CT is L	0.033	0.040	-2.735	-0.108
7	If D is L and ATR is H and CD is H and CT is S	0.019	0.023	-17.320	-0.399
8	If D is L and ATR is H and CD is H and CT is L	0.019	0.023	24.510	0.572
9	If D is H and ATR is L and CD is L and CT is S	0.088	0.106	-0.260	-0.028
10	If D is H and ATR is L and CD is L and CT is L	0.089	0.107	-0.298	-0.032
11	If D is H and ATR is L and CD is H and CT is S	0.052	0.062	2.835	0.177
12	If D is H and ATR is L and CD is H and CT is L	0.052	0.063	7.324	0.464
13	If D is H and ATR is H and CD is L and CT is S	0.027	0.033	-2.065	-0.068
14	If D is H and ATR is H and CD is L and CT is L	0.027	0.033	-1.615	-0.054
15	If D is H and ATR is H and CD is H and CT is S	0.027	0.033	9.883	0.323
16	If D is H and ATR is H and CD is H and CT is L	0.027	0.033	7.431	0.246
Sum:		0.828			1.319

The calculated firing strengths are normalized in the third layer. Example calculations for the first two rules are calculated as follows. The calculated normalized firing strengths for each rule are shown in Table 7.

Fifth layer is defuzzification layer in which constant functions  $f$  specified in Table 5 are multiplied by normalized firing strength for each rule. For the first two rules the calculations are shown below, all the calculated values are shown in Table 7.

For Rule 1 and 2:

$$\ddot{w}_1 \cdot f_1 = 0.126 \cdot 0.240 = 0.030; \quad (26)$$

$$\ddot{w}_2 \cdot f_2 = 0.128 \cdot (-0.166) = -0.021. \quad (27)$$

The last layer is the output layer. In this layer the output value is calculated by summing up the output of previous layer:

$$\begin{aligned} UCS &= \sum_{i=1}^{16} \ddot{w}_i \cdot f_i = \ddot{w}_1 \cdot f_1 + \dots + \ddot{w}_{16} \cdot f_{16} = \\ &= 0.030 + \dots + 0.246 = 1.319. \end{aligned} \quad (28)$$

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## СИСТЕМА АДАПТИВНОГО НЕЙРОНЕЧІТКОГО ЛОГІЧНОГО ВИВЕДЕННЯ ДЛЯ МОДЕЛЮВАННЯ МЕЖИ МІЦНОСТІ ПРИ ОДНООСЬОВОМУ СТИСКАННІ ЦЕМЕНТНОЇ ГІДРАВЛІЧНОЇ ЗАКЛАДКИ

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**Мета.** Побудова моделей для прогнозування межі міцності при одноосьовому стисканні цементної гідралічної закладки для заповнення вироблених просторів шахт.

**Методика.** Для досягнення поставленої мети були використані різні методи моделювання: лінійна та нелінійна множинна регресія, а також порівняно недавно став популярним метод програмування – адаптивне нейронечітке логічне виведення (ANFIS). За їх допомогою було спрогнозовано зміну міцності на одноосьове стискання, що є ключовим показником для визначення складу закладної суміші. Для побудови моделей використана значна база даних, яка включає результати більш ніж 1600 випробувань на одноосьове стискання. Лабораторними дослідженнями також визначалися властивості закладних матеріалів і суміші.

**Результати.** Модель ANFIS дала найкращу продуктивність з урахуванням статистичних показників ефективності, таких як середня абсолютна процентна похибка і змінний обліковий запис. Статистичні показники про-

дуктивності, які використовуються для оцінки ефективності розроблених моделей, свідчать, що моделювання за допомогою ANFIS дозволяє отримати результати, які більше відповідають експертній оцінці та даним з сучасної літератури, ніж інформація, отримана за допомогою традиційного моделювання. Встановлено, що на відміну від регресивного моделювання, ANFIS не вимагає заздалегідь визначених математичних рівнянь для взаємозв'язку між вхідними та вихідними змінними і використовує наданий набір даних для ефективного визначення структури моделі.

**Наукова новизна.** Вперше для прогнозування міцності при одноосьовому стисканні були використані не лише традиційні способи моделювання, засновані на регресії, а й інноваційний метод програмування – адаптивне нейронечітке логічне виведення ANFIS. У статті наведені чисельні приклади впровадження нових побудованих моделей.

**Практична значимість.** Статистичні індикатори продуктивності показали, що розроблені моделі можуть бути використані для надійного прогнозування міцності при одноосьовому стисканні й оптимальної рецептури закладної суміші. Однак, щоб отримати більш точні результати, необхідно мати більш широку базу даних і створити більш досконалі моделі на основі алгоритму, запропонованому в даній статті.

**Ключові слова:** цементна гідралічна закладка, адаптивне нейронечітке логічне виведення, множинна регресійна модель, підземна розробка

## СИСТЕМА АДАПТИВНОГО НЕЙРОНЕЧЕТКОГО ЛОГИЧЕСКОГО ВЫВОДА ДЛЯ МОДЕЛИРОВАНИЯ ПРЕДЕЛА ПРОЧНОСТИ ПРИ ОДНООСНОМ СЖАТИИ ЦЕМЕНТНОЙ ГИДРАВЛИЧЕСКОЙ ЗАКЛАДКИ

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**Цель.** Построение моделей для прогнозирования предела прочности при одноосном сжатии цементной гидравлической закладки для заполнения выработанных пространств шахт.

**Методика.** Для достижения поставленной цели были использованы различные методы моделирования: линейная и нелинейная множественная регрессия, а также сравнительно недавно ставший популярным метод программирования – адаптивный нейронечеткий логический вывод (ANFIS). С их помощью было спрогнозировано изменение прочности на одноосное сжатие, что является ключевым показателем для определения состава закладочной смеси. Для построения моделей использована обширная база данных, которая включает результаты более чем 1600 испытаний на одноосное сжатие. Лабораторными исследованиями также определялись свойства закладочных материалов и смеси.

**Результаты.** Модель ANFIS дала наилучшую производительность с учетом статистических показателей эффективности, таких как средняя абсолютная процентная погрешность и переменная учетная запись. Статистические показатели производительности, используемые для оценки эффективности разработанных моделей, свидетельствуют, что моделирование с помощью ANFIS позволяет получить результаты, которые более соответствуют экспертной оценке и данным из современной литературы, чем информация, полученная при помощи традиционного моделирования. Установлено, что в отличие от регрессионного моделирования, ANFIS не требует заранее определенных математических уравнений для взаимосвязи между входными и выходными переменными и использует предоставленный набор данных для эффективного определения структуры модели.

**Научная новизна.** Впервые для прогнозирования прочности при одноосном сжатии были использованы не только традиционные способы моделирования, основанные на регрессии, но и инновационный метод программирования – адаптивный нейронечеткий логический вывод ANFIS. В статье приведены численные примеры внедрения новых построенных моделей.

**Практическая значимость.** Статистические индикаторы производительности показали, что разработанные модели могут быть использованы для надежного прогнозирования прочности при одноосном сжатии и оптимальной рецептуры закладочной смеси. Однако, чтобы получить более точные результаты, необходимо иметь более широкую базу данных и создать более совершенные модели на основе алгоритма, предложенного в данной статье.

**Ключевые слова:** цементная гидравлическая закладка, адаптивный нейронечеткий логический вывод, множественная регрессионная модель, подземная разработка

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