

Application of the Radial Basis Function interpolation method in selected reservoirs of the Croatian part of the Pannonian Basin System

Josip Ivšinić¹✉, Tomislav Malvić²✉

¹INA-Industry of Oil Plc., Zagreb, 10000, Croatia

²University of Zagreb, Zagreb, 10000, Croatia

*Corresponding author: e-mail josip.ivsinovic@ina.hr, tel. +38598311307

Abstract

Purpose. The use of interpolation methods of mapping Radial Basis Function (RBF) on reservoir data from one field in Croatian part of Pannonian Basin System (CPBS).

Methods. The RBF method (with five single basic mathematical functions) was applied to small datasets. Application of the Radial Basis Function (RBF) method and comparison with previous application of the Inverse Distance Weighting (IDW) method applied in the CPBS area. The IDW and RBF methods were compared by cross-validation value and visual inspection of interpolated maps.

Findings. The RBF method was tested on a small data sample. The RBF method can be used independently when using the Inverse Multiquadric Function (RBF-IM) mathematical function, while the remaining analyzed mathematical multilog function (RBF-M) and “multiquadric function” (RBF-M2) can be used as additional sources of information when mapping.

Originality. For the first time RBF is applied as a method in the CPBS area for small input data sets.

Practical implications. For small sample the RBF method cannot be applied independently. According to the cross-validation value and visual inspection of interpolated maps, the method that can be used with the IDW method when mapping a small sample is RBF-IM. It could be primary or additional method for a small sample, while for a large sample it offers additional information.

Keywords: Pannonian Basin System, Radial Basis Function (RBF), Inverse Distance Weighting (IDW), geostatistics, small dataset

1. Introduction

When analyzing reservoir parameters, it is very important to apply the appropriate interpolation method. The choice of the appropriate interpolation method is based on the size of the input dataset. A sample of less than 20 data is considered a small set of numbers [1]-[3]. In the paper, the radial basis function (RBF) method was applied to small number of samples (datasets).

An example of the mapping of a small dataset is reservoir “K” of field “B” located in the western part of the Sava Depression within Croatian part of the Pannonian Basin System (CPBS). The RBF method for a small data set is compared to inverse distance weighting (IDW) method, which is a common mapping interpolation method for a small sample in the CPBS [4]-[6].

The RBF method has been applied in various scientific fields when mapping: mesh deformation [7], antenna design [8], earthfill dam [9], porosity [10] etc. The RBF method, so far, has not been applied for subsurface geological mapping in the CPBS. The RBF method will be applied to map geological variable porosity on reservoirs “K” (for 19 data). In this paper this method has been tested and com-

pared with the IDW method that previously resulted in satisfactory mapping results of selected reservoir [4]-[7]. Such comparison is done for small data set and offered guideline how and when to use this mapping method. The results are evaluated by the cross-validation and visual inspection.

2. Geological settings of the study area

The Sava Depression (Fig. 1) is located in the southeastern part of the Pannonian Basin System, i.e. in the Croatian part of the Pannonian Basin System (CPBS).

The typical geological section of the Sava Depression sediments from Lower Pontian to Quaternary is shown in Figure 2. Deposition in depression started in Early Neogene (Ottungian), but figure shows sections where are analysed reservoir and younger deposits.

Hydrocarbon reservoirs in the filed “B” have been confirmed in the Klostar-Ivanic Formation (Fig. 2). The lithological characteristics of the Lower Pontian reservoirs of the Klostar Ivanic Formation are a well sorted arenites and pelites.

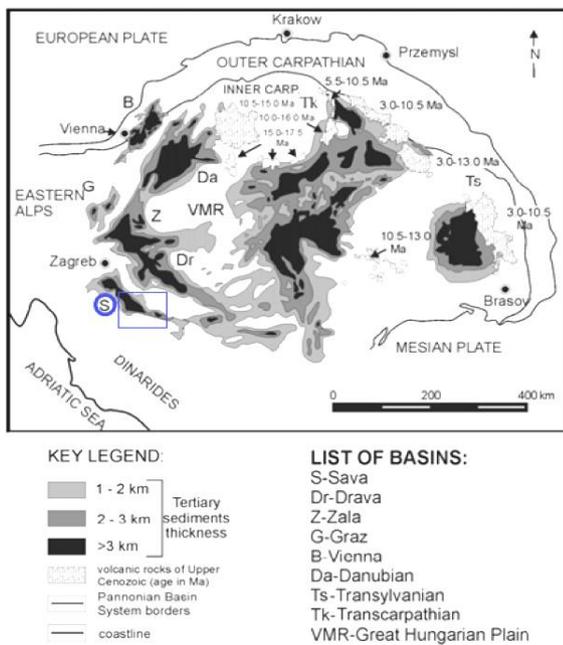


Figure 1. Sava Depression (blue) within the Pannonian Basin System [11]

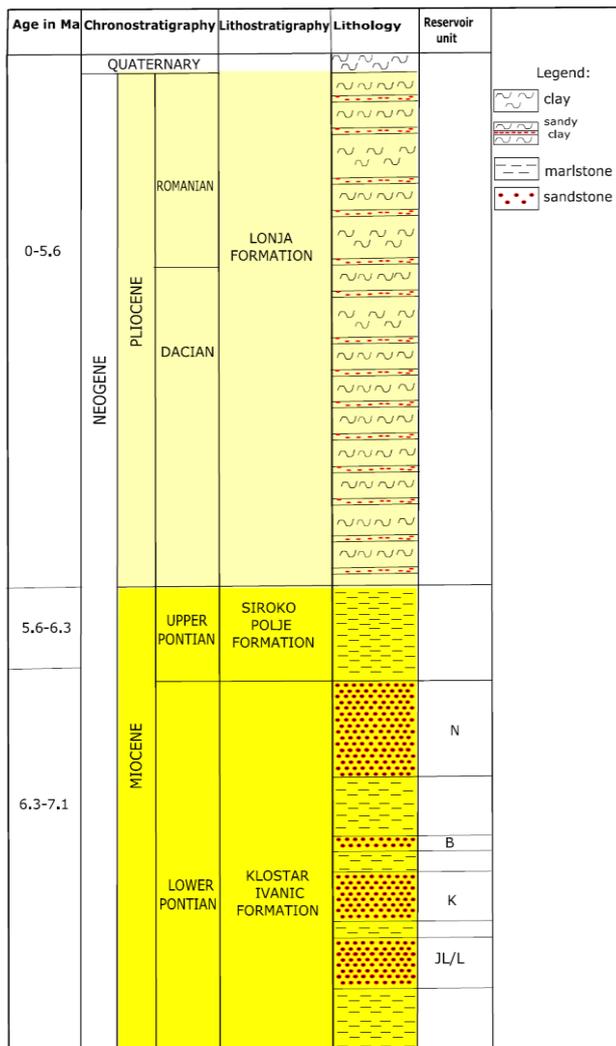


Figure 2. Typical chrono and lithostratigraphic section from Lower Pontian to Quaternary of the Sava Depression [12]

Reservoir rocks in the lower part there are hard sandstones, which towards the top of the formation, and especially in the Siroko Polje Formation (Upper Pontian), become poorly bounded, and even fine-grained, unbounded sands. The marls of this formation are gray to gray-brown, and of medium hardness. The marl intervals are isolator rocks for each sandstone reservoir. The marl thickness is 30-150 m, while the average sandstone thickness is 20-150 m. Reservoir "K" fields "B" hydrocarbon production began in 1970. The structural map of reservoir "K" is shown in Figure 3.

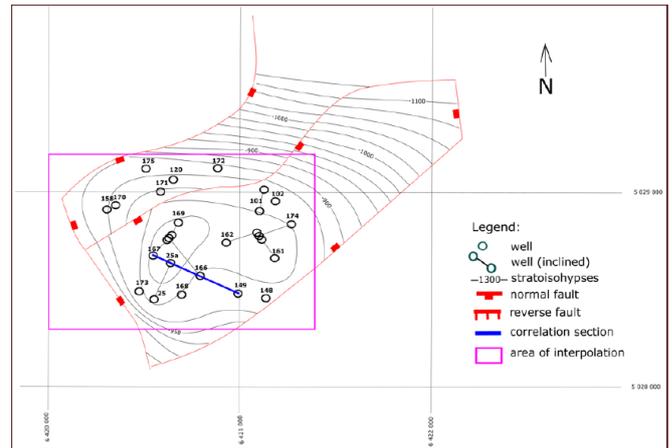


Figure 3. Structural map of reservoir "K" [14]

In selected part of the reservoir (purple box), secondary hydrocarbon production method (formation water injection) is applied. Reservoir "K" characteristics are: porosity 0.27-0.32 units and permeability $29.6-121.2 \cdot 10^{-3} \mu\text{m}^2$. The histogram of porosity of "K" reservoir data used for interpolation is shown in Table 1 and Figure 4.

Table 1. Raw data of porosity interpolated in the "K" reservoir

Well	Surface X	Surface Y	Porosity (part of units)
J-101	6421096	5028877	0.217
J-120	6420658	5029068	0.272
J-161	6420957	5028870	0.217
J-162	6421034	5028593	0.217
J-167	6420529	5028674	0.217
J-168	6420699	5028475	0.315
J-169	6420724	5028825	0.217
J-170	6420349	5028926	0.223
J-174	6421298	5028863	0.217
J-175	6420475	5029136	0.223
J-158	6420303	5028910	0.223
J-171	6420576	5028970	0.223
J-172	6420928	5029147	0.223
J-102	6421208	5028926	0.217
J-148	6421126	5028437	0.217
J-149	6420959	5028501	0.217
J-166	6420771	5028650	0.217
J-25	6420546	5028460	0.315
J-173	6420539	5028382	0.217

Secondary methods of hydrocarbon production (injection of formation water [15]) were applied at the reservoir "K" and modified gravel pack [16] is applied in case of sand occurrence in flow during hydrocarbon production.

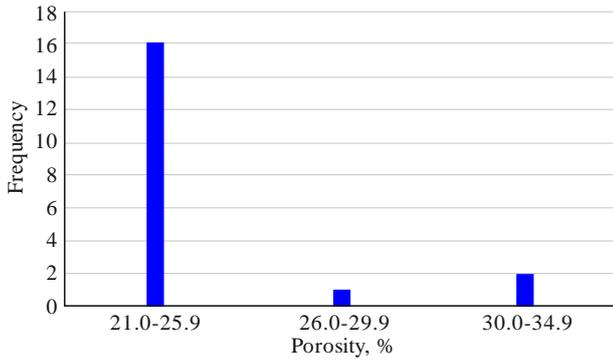


Figure 4. Histogram of porosity of reservoir "K"

3. Mathematical settings of applied methods

The IDW interpolation method is one of the most common method when mapping different variables in geology [4], [5]. It is widely used, especially for smaller input datasets [3]. In this method, the estimation of a single value of a variable depends on the inverse distance between the measured and estimated data and the size of the radius of coverage of the other values around the original data. The mathematical expression (1) for IDW estimation, e.g., [17]-[19], is as follows:

$$z_{IU} = \frac{\frac{z_1}{d_1^p} + \frac{z_2}{d_2^p} + \dots + \frac{z_n}{d_n^p}}{\frac{1}{d_1^p} + \frac{1}{d_2^p} + \dots + \frac{1}{d_n^p}}, \quad (1)$$

where:

- z_{IU} – interpolated (unknown) value;
- d_n – distance of the "i-th" location;
- p – power of distance;
- z_i – measured (known) value at "i-th" location.

The interpolation result depends on the distance exponent. Most commonly used is amount 2, which has been empirically proven to be the most appropriate value for acceptable deep geological mapping of the CPBS. Higher values of power (p) favor a larger influence of measured points closer to the interpolated point, and eventually ended up in a zonal interpolation. Oppositely, the value $p = 1$ emphasis larger influence of more distant measured points and larger regional smoothing of the entire map.

The RBF is an interpolation method consisting of one of the several allowed basic mathematical functions. Generally, that is a real-valued function f whose value depends only on the distance between the input and some fixed point, so that $f(x) = f(|x|)$, or some other fixed point c called a *center*, so that $f(x) = f(|x-c|)$. Any function f that satisfies the such properties is a radial function.

Basic mathematical functions in SURFER 15 program are: Inverse Multiquadric (RBF-IM), Multilog (RBF-M), Multiquadric (RBF-M2), Natural Cubic Spline (RBF-NCS) and Thin Plate Spline (RBF-TPS). When interpolating a map, one of the basic equation must be chosen. The mathematical expressions for such functions are in SURFER 15 program are:

$$IM(h) = \frac{1}{\sqrt{h^2 + R^2}}; \quad (2)$$

$$M(h) = \log(h^2 + R^2); \quad (3)$$

$$M2(h) = \sqrt{h^2 + R^2}; \quad (4)$$

$$M2NCS(h) = (h^2 + R^2)^{3/2}, \quad (5)$$

$$TPS(h) = (h^2 + R^2) \log(h^2 + R^2), \quad (6)$$

where:

- $IM(h)$ – Inverse Multiquadric function;
- $M(h)$ – Multilog function;
- $M2(h)$ – Multiquadric function;
- $NCS(h)$ – Natural Cubic Spline function;
- $TPS(h)$ – Thin Plate Spline function;
- h – relative distance between measured and estimated point;
- R^2 – shaping factor.

Cross-validation (MSE) is a numerical estimation method for checking the performance of an interpolation method applied by calculating the value of the mean square error of the estimation value. The expression (7) for the mean square error of estimation is, e.g., [20], [21]:

$$MSE = \frac{1}{n} \sum_{i=1}^n (measured - estimated)^2, \quad (7)$$

where:

- MSE – mean square error;
- $measured$ – value measured d at location "i";
- $estimated$ – value estimated at location "i";
- n – number of locations.

When comparing multiple results of the same estimation made with different algorithms, one of the selection criteria is the most appropriate choice of the solution with the lowest cross-validation value [22], [23].

4. Results and discussion

The analyzed methods (RBF & IDW) are evaluated as follows:

- visual inspection of the interpolated maps (as soft condition);
- cross-validation values (as hard condition).

4.1. Visual inspection of the interpolated maps

IDW and RBF interpolation methods were applied to 19 data (Table 1) for the reservoir porosity variable. Porosity data were obtained from mutually wells cores analysis and logging measurements. According to the size ($n = 19$) the data set by the authors [3] belongs to a small sample (< 20). Data interpolation was done with the IDW and RBF (IM, M, M2) methods. The results of the interpolations for "K" reservoir porosity are shown in Figure 5.

Due to size and distribution of the input dataset, interpolated maps had pronounced local "bull eyes" effect (e.g. around wells Jam-25, Jam-168 and J-120). The map interpolated by the RBF-IM method (Fig. 5b) resulted in a similar map obtained by IDW (Fig. 5a), which is in line with expectations since the both algorithms have an inverse distance weight (1) & (2). Comparing the map obtained by IDW (Fig. 5a) and the other two RBF-M and RBF-M2 methods (Fig. 5c and 5d), differences can be observed due to the different mathematical functions used. Both sub-methods stronger favor the larger closed areas in low sampled parts, e.g., see area closed with isoporosity line 0.22.

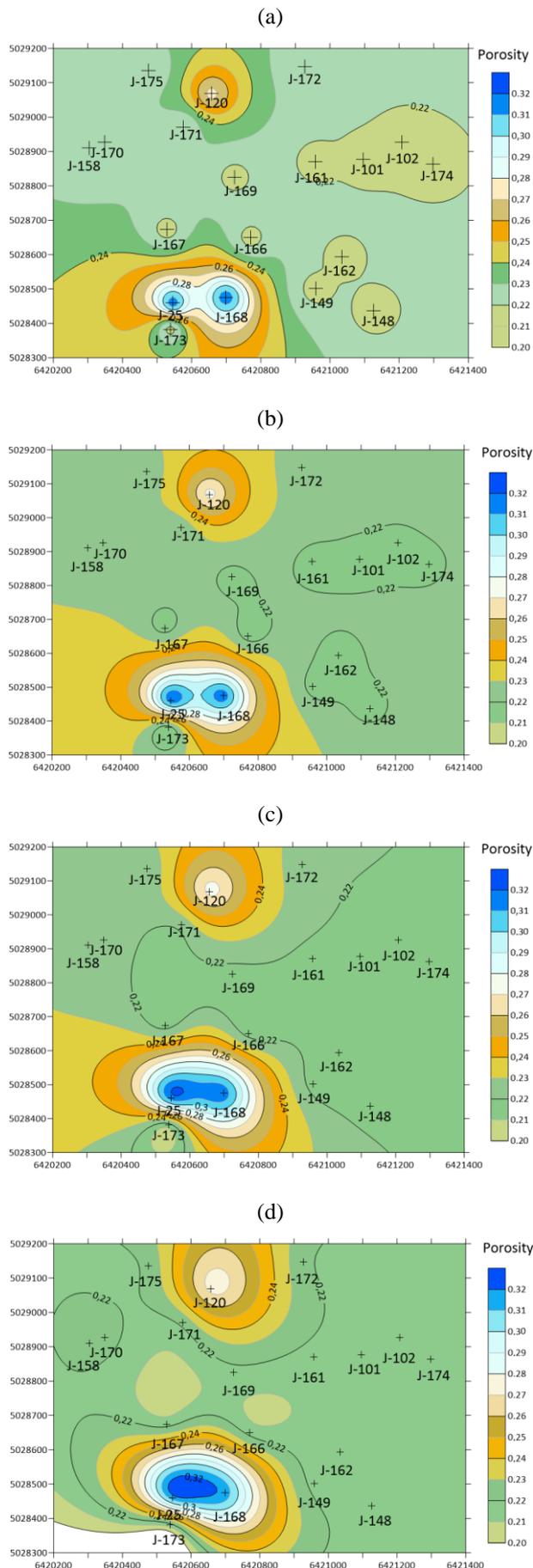


Figure 5. Porosity of reservoir “K” obtained by interpolation methods: (a) IDW; (b) RBF-IM (inverse multiquadric f.); (c) RBF-M (multilog f.); (d) RBF-M2 (multiquadric f.)

Additionally, those algorithms tend to close areas of the same values belonging to neighboring wells, e.g., like area between wells J-25 and J-168 (Fig. 5c and 5d). Also, a larger interpolation surface (isoporosity line 0.24) can be observed around the J-120 well in the RBF-M and RBF-M2 methods than in the case of the IDW and RBF-IM interpolation method.

Based on the visual inspection, it can be observed that the RBF-IM and IDW methods resulted in more applicable maps. RBF-M and RBF methods tends the create larger closed areas, favors in such ways mapping of structure/depositional volumes that could be too large regarding available data in total and locally. Bit of these sub-method could be applying as source of corrective solutions for maps obtained by the IDW or RBF-IM (sub + methods).

4.2. Cross-validation values

Cross-validation (7) is a mathematical procedure, based on (for the MSE type) the calculated squared difference between measured and estimated values in the same location. It is used for estimation of the acceptability of the mapping method. The calculated cross-validation values for the methods applied on the reservoir “K” are shown in Table 2.

Table 2. Cross-validation values for the reservoir “K” (19 data)

Variable	Method	MSE
Porosity	IDW	0.0011
Porosity	RBF-IM	0.0011
Porosity	RBF-M	0.0014
Porosity	RBF-M2	0.0016
Porosity	RBF-TPS	0.0025
Porosity	RBF-NCS	0.0035

Table 2 shows that cross-validation values for the analysed dataset with the RBF-IM and IDW (sub)methods (both are 0.0011). The cross-validation value of the RBF-M method (0.0014) is 27% higher than the previous value, while for RBF-M2 method (0.0016) is 45% higher. The cross-validation value of IDW and RBF-IM confirmed the visual analysis described previously. The same cross-validation value for the IDW and RBF-IM showed the similarity of algorithms, because both include some variant of inverse distance pondering. The higher values for RBF-M and RBF-M2 confirm the conclusion of the visual inspection in previous subsection, i.e. that forcing closures of larger areas with the same contour line, without regarding to scarce data, is not well-designed approach for this variable mapped in described geological environment.

5. Conclusions

The following conclusions can be made from previous analysis:

- all results are valid for entire Lower Pontian of the Sava Depression, i.e. for any petrophysical dataset collected in the sandstones of that area;
- when visually inspecting maps, RBF-IM method can be applied to a small data set (less than 20 points), while the RBF-M and RB-M2 methods cannot be applied independently but as additional information on the spatial distribution of mapped variable, i.e. about zone of strong localized effect;
- all obtained interpolation maps have a pronounced “bull-eyes” (localized) effects. This forms clearly outlined two larger areas closed with isoporosity lines 0.24 (south) and 0.23 (north);

– according to the cross-validation value, the most acceptable methods for analysed dataset is IDW (0.0011) and RBF-IM (0.0011);

– it is additionally confirmed by application of the same mapping algorithms on permeability values (as additional variable). In such case, the cross-validation value for the permeability of the reservoir “K” is for IDW method 1667 and 1648 for RBF-IM method.

The RBF-IM submethod can be applied to a small set ($n < 20$ data) independently in the case of using the inverse multiquadric function (RBF-IM). It is especially useful when selection of power exponent (p) value in the IDW is not straightforward process.

The remaining two analyzed mathematical functions “multilog function” (RBF-M) and “multiquadric function” (RBF-M2) can be used as an additional source of information.

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References

[1] Balić, D., Velić, J., & Malvić, T. (2008). Selection of the most appropriate interpolation method for sandstone reservoirs in the Kloštar oil and gas field. *Geologia Croatica*, 61(1), 27-35.

[2] Ivšinić, J. (2018). Deep mapping of hydrocarbon reservoirs in the case of a small number of data on the example of the Lower Pontian reservoirs of the western part of Sava Depression. In *Proceedings of the 2nd Croatian congress on geomathematics and geological terminology*, 59-65.

[3] Malvić, T., Ivšinić, J., Velić, J., & Rajić, R. (2019). Interpolation of small datasets in the sandstone hydrocarbon reservoirs, case study of the Sava Depression, Croatia. *Geosciences*, 9(5), 201. <https://doi.org/10.3390/geosciences9010036>

[4] Balić, D., Velić, J., & Malvić, T. (2008). Selection of the most appropriate interpolation method for sandstone reservoirs in the Kloštar oil and gas field. *Geologia Croatica*, 61(1), 27-35.

[5] Ivšinić, J. (2018). The relationship between sandstone depositional environment and water injection system, a case study from the Upper Miocene hydrocarbon reservoir in northern Croatia. In *Proceedings of the 2nd Croatian Scientific Congress from Geomathematics and Terminology in Geology*, 65-75.

[6] Husanović, E., & Malvić, T. (2014). Review of deterministic geostatistical mapping methods in Croatian hydrocarbon reservoirs and advantages of such approach. *Nafta*, (65), 57-63.

[7] De Boer, A., van der Schoot, M.S., & Bijl, H. (2007). Mesh deformation based on radial basis function interpolation. *Computers & Structures*, 85(11-14), 784-795. <https://doi.org/10.1016/j.compstruc.2007.01.013>

[8] Jakobsson, S., Andersson, B., & Edelvik, F. (2009). Rational radial basis function interpolation with applications to antenna design. *Journal of Computational and Applied Mathematics*, 233(4), 889-904. <https://doi.org/10.1016/j.cam.2009.08.058>

[9] Nourani, V., & Babakhani, A. (2013). Integration of artificial neural networks with radial basis function interpolation in earthfill dam seepage modeling. *Journal of Computing in Civil Engineering*, 27(2), 183-195. [https://doi.org/10.1061/\(asce\)jcp.1943-5487.0000200](https://doi.org/10.1061/(asce)jcp.1943-5487.0000200)

[10] Zou, Y.-L., Hu, F.-L., Zhou, C.-C., Li, C.-L., & Dunn, K.-J. (2013). Analysis of radial basis function interpolation approach. *Applied Geophysics*, 10(4), 397-410. <https://doi.org/10.1007/s11770-013-0407-z>

[11] Royden, L.H. (1988). Late Cenozoic tectonics of the Pannonian Basin System. *American Association of Petroleum Geologists Memoir*, (45), 27-48.

[12] Ivšinić, J. (2019). *Selection and geomathematical calculation of variables for sets with less than 50 data regarding the creation of an improved subsurface model, case study from the western part of the Sava Depression*. Ph.D. Zagreb, Croatia: University of Zagreb.

[13] Velić, J. (2007). *Geology of oil and gas*. Zagreb, Croatia: University of Zagreb.

[14] Malvić, T., Ivšinić, J., Velić, J., Sremac, J., & Barudžija, U. (2020). Application of the modified Shepard's method (MSM): a case study with the interpolation of Neogene reservoir variables in Northern Croatia. *Stats*, (3), 68-83. <https://doi.org/10.3390/stats3010007>

[15] Ivšinić, J. (2018). The cost analysis of the separation of produced formation water from the hydrocarbon reservoir using the example of the upper Miocene sandstone deposits of the Sava depression. *Rudarsko-Geološko-Naftni Zbornik*, 33(1), 35-43. <https://doi.org/10.17794/rgn.2018.1.5>

[16] Ivšinić, J., Pleteš, V., & Marinić, M. (2018). Production of hydrocarbons from weakly consolidated sandstone reservoirs in the croatian part of the Pannonian basin system. *Mining of Mineral Deposits*, 12(2), 116-121. <https://doi.org/10.15407/mining12.02.116>

[17] Setianto, A., & Triandini, T. (2013). Comparison of kriging and inverse distance weighted (IDW) interpolation methods in lineament extraction and analysis. *Journal of Applied Geology*, 5(1), 21-29. <https://doi.org/10.22146/jag.7204>

[18] Li, L., Losser, T., Yorke, C., & Piltner, R. (2014). Fast inverse distance weighting-based spatiotemporal interpolation: a web-based application of interpolating daily fine particulate matter PM_{2.5} in the contiguous U.S. using parallel programming and k-d tree. *International Journal of Environmental Research and Public Health*, (11), 9101-9141. <https://doi.org/10.3390/ijerph110909101>

[19] Stachelek, J., & Madden, C.J. (2015). Application of inverse path distance weighting for high-density spatial mapping of coastal water quality patterns. *International Journal of Geographical Information Science*, 29(7), 1240-1250. <https://doi.org/10.1080/13658816.2015.1018833>

[20] Rodriguez, J.D., Perez, A., & Lozano, J.A. (2010). Sensitivity analysis of k-fold cross validation in prediction error estimation. *IEEE transactions on pattern analysis and machine intelligence*, 3(32), 569-575. <https://doi.org/10.1109/TPAMI.2009.187>

[21] Arlot, S., & Lerasle, M. (2016). Choice of V for V-fold cross-validation in least-squares density estimation. *Journal of Machine Learning Research*, (17), 1-50.

[22] Malvić, T., Ivšinić, J., Velić, J., & Rajić, R. (2019). Kriging with a small number of data points supported by jack-knifing, a case study in the Sava Depression (Northern Croatia). *Geosciences*, 9(36). <https://doi.org/10.3390/geosciences9010036>

[23] Malvić, T., Ivšinić, J., Velić, J., Sremac, J., & Barudžija, U. (2020). Increasing efficiency of field water re-injection during water-flooding in mature hydrocarbon reservoirs: a case study from the Sava Depression, Northern Croatia. *Sustainability*, 12(786). <https://doi.org/10.20944/preprints201912.0350.v1>

Використання методу інтерполяції радіальної базисної функції у застосуванні до деяких родовищ хорватської частини Паннонського басейну

Й. Івшинич, Т. Мальвич

Мета. Використання методу інтерполяції при картуванні радіальної базисної функції (РБФ) стосовно даних одного родовища в хорватській частині Паннонського басейну.

Методика. Метод РБФ (з п'ятьма основними математичними функціями) застосовувався у порівнянні з використовуваним раніше методом ОВР (зворотних зважених відстаней) для хорватської частини Паннонського басейну. Методи РБФ і ОВР порівнювалися за значенням перехресної перевіркою і візуальним оглядом інтерпольованих карт. Метод РБФ застосовувався до невеликих масивів даних.

Результати. Встановлено, що при візуальному огляді карт метод RBF-IM може застосовуватися до невеликого набору даних (менше 20 точок), в той час як методи RBF-M і RB-M2 не можуть застосовуватися незалежно, але в якості додаткової інформації про просторовий розподіл відображається змінна, тобто про зону сильного локалізованого ефекту. Даний метод може застосовуватися автономно при використанні зворотної мультиквадратичної математичної функції (ОММФ), при цьому залишилися математи-

чні багатофункціональна функція (МФФ) і мультикватрична функція (МКФ) можуть бути використані як додаткові джерела інформації при картуванні. Порівнюючи карту, отриману IDW і двома іншими методами RBF-M і RBF-M2, можна спостерігати відмінності через різних використовуваних математичних функцій. Обидва допоміжних методи більш кращі для великих закритих областей в частинах з низькою вибіркою, наприклад, область, закриту лінією ізопористості 0.22.

Наукова новизна. Вперше використаний метод РБФ для аналізу невеликих масивів даних, що стосуються хорватської частини Паннонського басейну.

Практична значимість. Для невеликої вибірки недостатньо використовувати тільки метод РБФ. У відповідності зі значенням перехресної перевірки і візуальної перевірки інтерпольованих карт, метод, який може використовуватися з методом IDW при відображенні невеликої вибірки, є ОММФ. Він може застосовуватися як основний або додатковий метод у разі невеликої вибірки, і як метод отримання додаткової інформації в разі великої вибірки.

Ключові слова: *Паннонський басейн, радіальна базисна функція, метод ОВР (зворотних зв'язаних відстаней), геостатистика, невеликий масив даних*

Использование метода интерполяции радиальной базисной функции в применении к некоторым месторождениям хорватской части Паннонского бассейна

Й. Ившинич, Т. Мальвич

Цель. Использование метода интерполяции при картировании радиальной базисной функции (РБФ) применительно к данным одного месторождения в хорватской части Паннонского бассейна.

Методика. Метод РБФ (с пятью основными математическими функциями) применялся в сравнении с использовавшимся ранее методом ОВР (обратных взвешенных расстояний) для хорватской части Паннонского бассейна. Методы РБФ и ОВР сравнивались по значению перекрестной проверки и визуальному осмотру интерполированных карт. Метод РБФ применялся к небольшим массивам данных.

Результаты. Установлено, что при визуальном осмотре карт метод RBF-IM может применяться к небольшому набору данных (менее 20 точек), в то время как методы RBF-M и RB-M2 не могут применяться независимо, но в качестве дополнительной информации о пространственном распределении отображаемая переменная, т.е. о зоне сильного локализованного эффекта. Данный метод может применяться автономно при использовании обратной мультикватричной математической функции (ОММФ), при этом оставшиеся математические многофункциональная функция (МФФ) и мультикватричная функция (МКФ) могут быть использованы как дополнительные источники информации при картировании. Сравняя карту, полученную IDW и двумя другими методами RBF-M и RBF-M2, можно наблюдать различия из-за различных используемых математических функций. Оба вспомогательных метода более предпочтительны для больших закрытых областей в частях с низкой выборкой, например, область, закрытую линией изопористости 0.22.

Научная новизна. Впервые использован метод РБФ для анализа небольших массивов данных, касающихся хорватской части Паннонского бассейна.

Практическая значимость. Для небольшой выборки недостаточно использовать только метод РБФ. В соответствии со значением перекрестной проверки и визуальной проверки интерполированных карт, метод, который может использоваться с методом IDW при отображении небольшой выборки, является ОММФ. Он может применяться как основной или дополнительный метод в случае небольшой выборки, и как метод получения дополнительной информации в случае большой выборки.

Ключевые слова: *Паннонский бассейн, радиальная базисная функция, метод ОВР (обратных взвешенных расстояний), геостатистика, небольшой массив данных*

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