UDC 51–74, 517.968.21

Gorev V. N., Ph.D., Associate Professor of the Department of Information Security and Telecommunications
(*Dnipro University of Technology, Dnipro, Ukraine*)

ON THE ACCURACY OF SOME POLYNOMIAL APPROXIMATIONS FOR THE KOLMOGOROV–WIENER FILTER WEIGHT FUNCTION

The problem of telecommunication traffic prediction is important for telecommunications and cyber security, see [1]. In paper [2] telecommunication traffic is described as a continuous stationary random process with a power-law structure function. In the framework of this model in papers [3–6] we proposed to use the Kolmogorov–Wiener filter for telecommunication traffic prediction. An approximate solution of the corresponding integral equation for the unknown weight function was obtained on the basis of the truncated polynomial expansion method.

We used the following polynomial sets: the polynomials without weight, the Chebyshev polynomials of the first kind and of the second kind. It was shown that the results for the accuracy of approximations were almost identical for all the above-mentioned polynomial sets. A strange behavior of approximation accuracy was established. Some polynomial approximations were rather accurate, but it was mentioned that some approximations failed. For example, in [3–6] it was stressed that for the parameters (1)

$$T = 100,\ z = 3,\ H = 0.8,\ \sigma = 1.2,\ \alpha = 3 \cdot 10^{-3} \tag{1}$$

the approximations of 9–15 polynomials lead to huge errors; here T is the time interval for which the input data is given, z is the time interval for which the forecast is made, H is the Hurst parameter, σ is the traffic variance and α is the proportionality coefficient between the traffic structure function and the power-law term.

The calculation of the left-hand side of the Wiener–Hopf integral equation in [3–6] was made with the help of the NIntegrate function which is built in the Wolfram Mathematica package, the corresponding function is used for the numerical integration of the given function. In this paper all the calculations are made for the Chebyshev polynomials of the second kind. In this paper the corresponding left-hand side is calculated on the basis of the method of trapezoids rather than the NIntegrate function, see (2),

$$\mathrm{Left}(t) = \int_0^T d\tau h(\tau) R(t-\tau) \approx \frac{1}{2}\sum_{i=1}^{N}\Big(h(\tau_{i-1})R(t-\tau_{i-1}) + h(\tau_i)R(t-\tau_i)\Big)\cdot\frac{T}{N} \tag{2}$$

where $h(\tau)$ is the Kolmogorov–Wiener filter weight function, $R(t)$ is the traffic correlation function in the power–law structure function model and the value $N = 1000$ is taken. The function $R(t)$ is given by the expression (3):

$$R(t) = \sigma^2 - (\alpha/2) \cdot |t|^{2H}. \quad (3)$$

The mean average error (MAE) for the different polynomial approximations is calculated as follows, see (4):

$$\text{MAE} = \frac{1}{N+1} \sum_{j=0}^{N} \left| \text{Left}\left(\frac{jT}{N}\right) - \text{Right}\left(\frac{jT}{N}\right) \right|, \ \text{Right}(t) = R(t+z). \quad (4)$$

The corresponding results are given in Table 1.

Table 1

MAE for approximations of different numbers of polynomials for parameters (1)

Number of polynomials	MAE, NIntegrate function [6]	MAE, method of trapezoids
9	$1{,}7 \cdot 10^{2}$	$1{,}3 \cdot 10^{-3}$
10	$2{,}3 \cdot 10^{2}$	$1{,}2 \cdot 10^{-3}$
11	$6{,}4 \cdot 10^{3}$	$7{,}5 \cdot 10^{-4}$
12	$4{,}3 \cdot 10^{4}$	$6{,}7 \cdot 10^{-4}$
13	$2{,}1 \cdot 10^{5}$	$4{,}7 \cdot 10^{-4}$
14	$1{,}1 \cdot 10^{6}$	$4{,}2 \cdot 10^{-4}$
15	$5{,}9 \cdot 10^{7}$	$3{,}1 \cdot 10^{-4}$

So, as can be seen, the function NIntegrate built in Wolfram Mathematica does not work correctly for the considered polynomial approximations, and the corresponding approximations are rather accurate.

References

1. C. Katris and S. Daskalaki, Expert Systems with Applications, **42**, Issue 21, 8172 (2015), doi: 10.1016/j.eswa.2015.06.029
2. V. Kh. Bagmanov, A. M. Komissarov and A. Kh. Sultanov, Bulletin of Ufa State Aviation Technical University, **9**, No. 6 (24), 217 (2007), in Russian.
3. V.N. Gorev, A.Yu. Gusev and V.I. Korniienko, Radio Electronics, Computer Science, Control, No. 2, 44 (2019), doi: 10.15588/1607-3274-2019-2-5
4. V. Gorev, A. Gusev and V. Korniienko, Ceur Workshop Proceedings, **2353**, 596 (2019).
5. V. Gorev, A. Gusev and V. Korniienko, IAPGOS, No. 1, 58 (2020), doi: 10.35784/iapgos.912
6. V. Gorev, A. Gusev, V. Korniienko and M. Aleksieiev, in book "Current Trends in Communication and Information Technologies" edited by P. Vorobiyenko, M. Ilchenko and I. Strelkovska, Lecture Notes in Networks and Systems book series, volume 212, Springer, 2021, p. 111–129, doi: 10.1007/978-3-030-76343-5_7