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The subject of economic modeling is mathematical models of real economic objects. The object of study of modeling economics as a discipline is economics and its divisions.
The purpose of the manual is to give the reader the opportunity to look at economics through the eyes of a researcher, learn to apply the methodology, techniques and tools of economic and mathematical modeling in theoretical research and use the acquired knowledge in practice.

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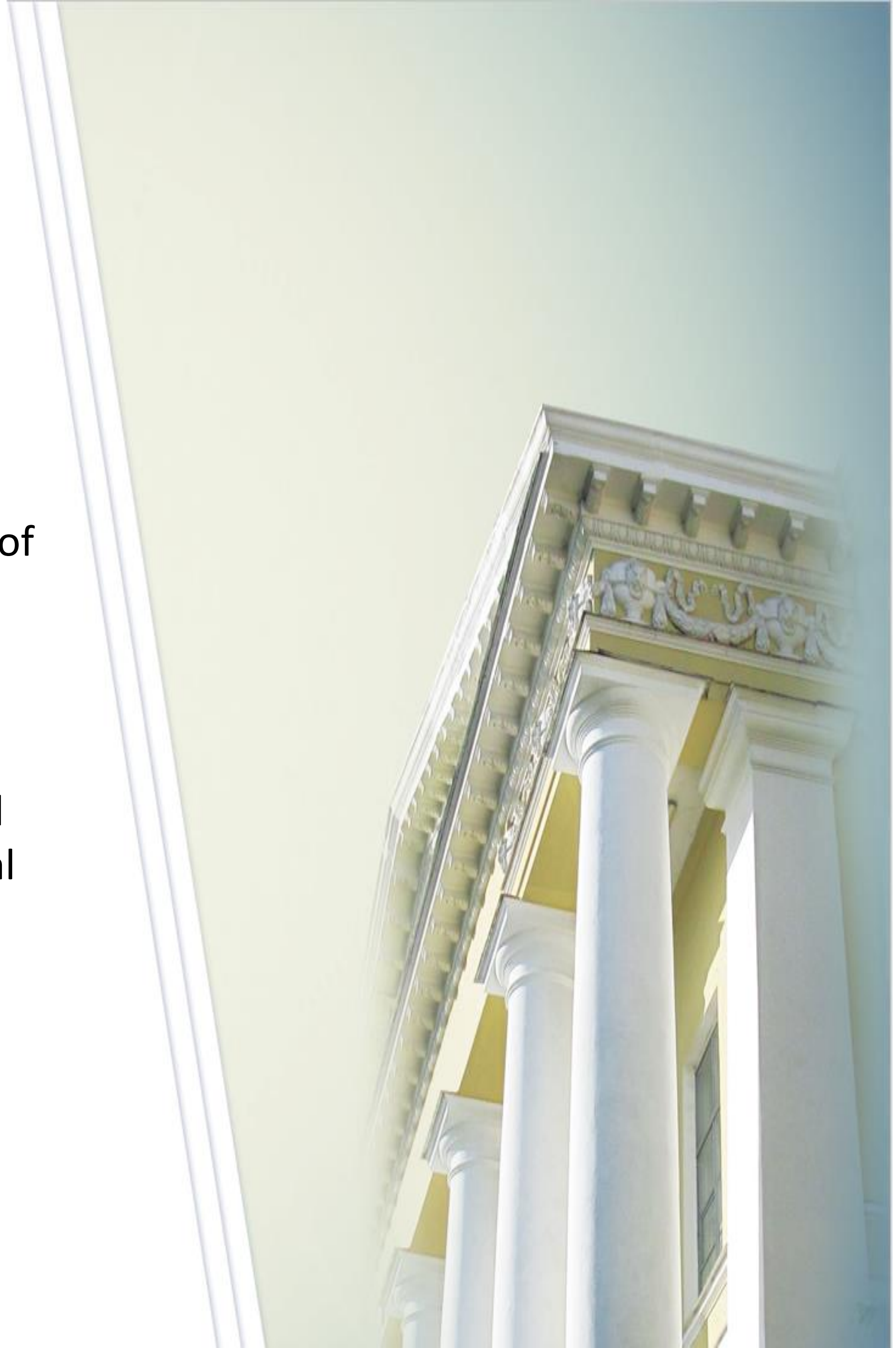

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PREFACE

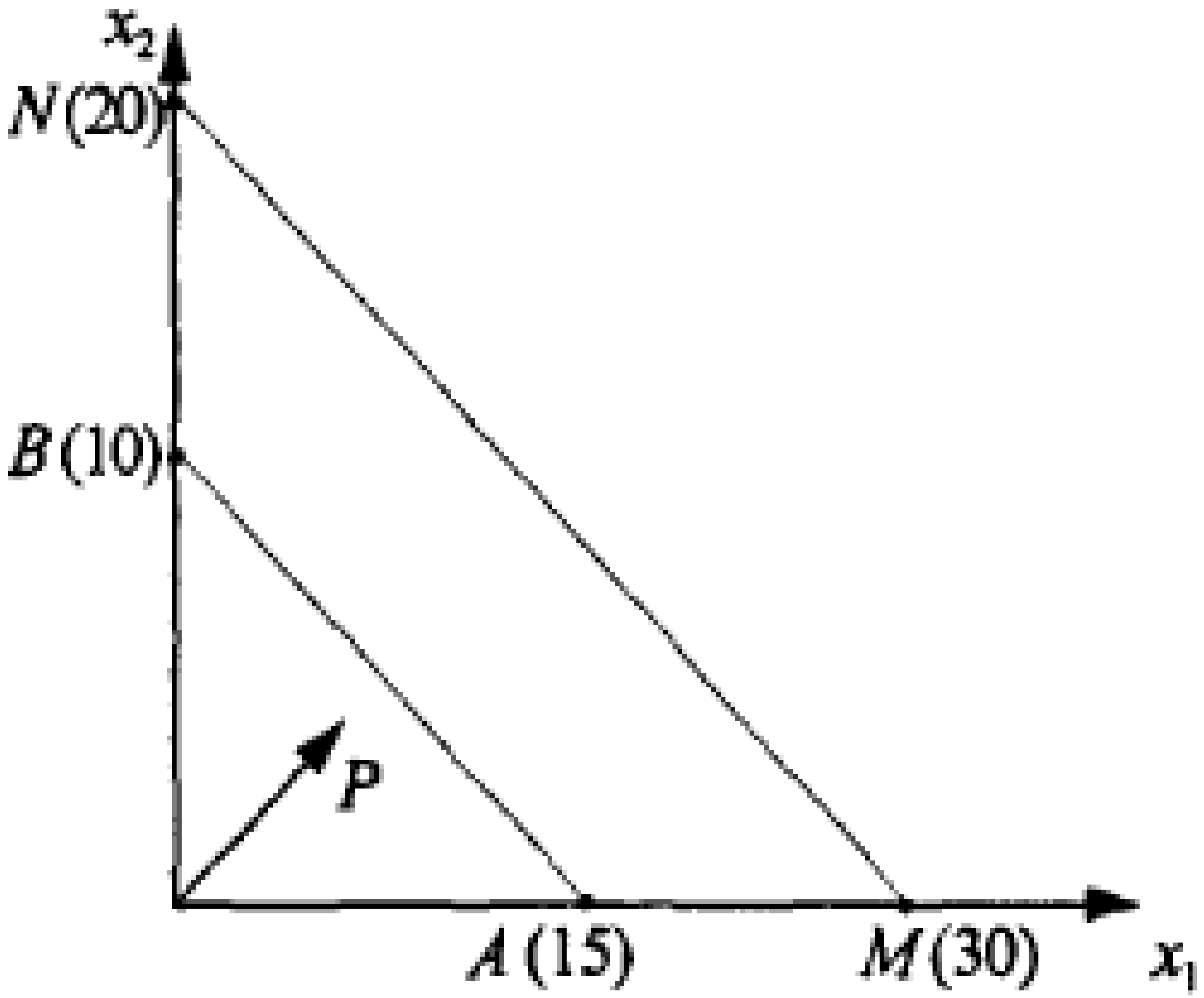
Successful mastering of the material presented in the manual requires knowledge of a number of fundamental and professionally oriented disciplines, the mastering of which is provided by the educational-professional program of higher education in the field of "Bachelor of Economics and Entrepreneurship".
These include political economy, macro- and microeconomics, finance, management, marketing, higher mathematics, probability theory and mathematical statistics, computer science and computer engineering,mathematical programming, econometrics, analysis, modeling and risk management, operations research.

1. MODELS OF CONSUMER BEHAVIOR

A good is a good or service that goes on sale and has a price of R_i and a quantity of X_i. The set of prices and quantities of goods is the scalar product of these vectors

$$PX = p_1x_1 + p_2x_2 + \ldots + p_nx_n$$

and is called the price of the set X. The ratio of true value divides the space of goods into classes that do not intersect.Consider two classes worth 30 and 60 units for 2 products, if the price vector $P(2, 3)$



x_2
N (20)
B (10)
P
A (15)
M (30)
x_1

Budget set

Let the individual have an income *Q*. Then the set of goods, at prices *P* worth no more than *Q*, is called the budget set *B*.

The set of goods of value that is exactly equal to *Q* is called the limit of this budget set.

That is,

$B(P, Q) = \{(x_1,\dots x_n)\} \geq 0, p_1x_1 + p_2x_2 + \dots + p_nx_n \leq Q$

$G(P, Q) = \{(\mathrm{x1},\dots \mathrm{xn} \geq 0, p_1x_1 + p_2x_2 + \dots + p_nx_n = \mathrm{Q}$

Consumer benefits

If the consumer distinguishes between sets of goods *X* and *Y*, considering one of them better than the other, the following relations may occur:

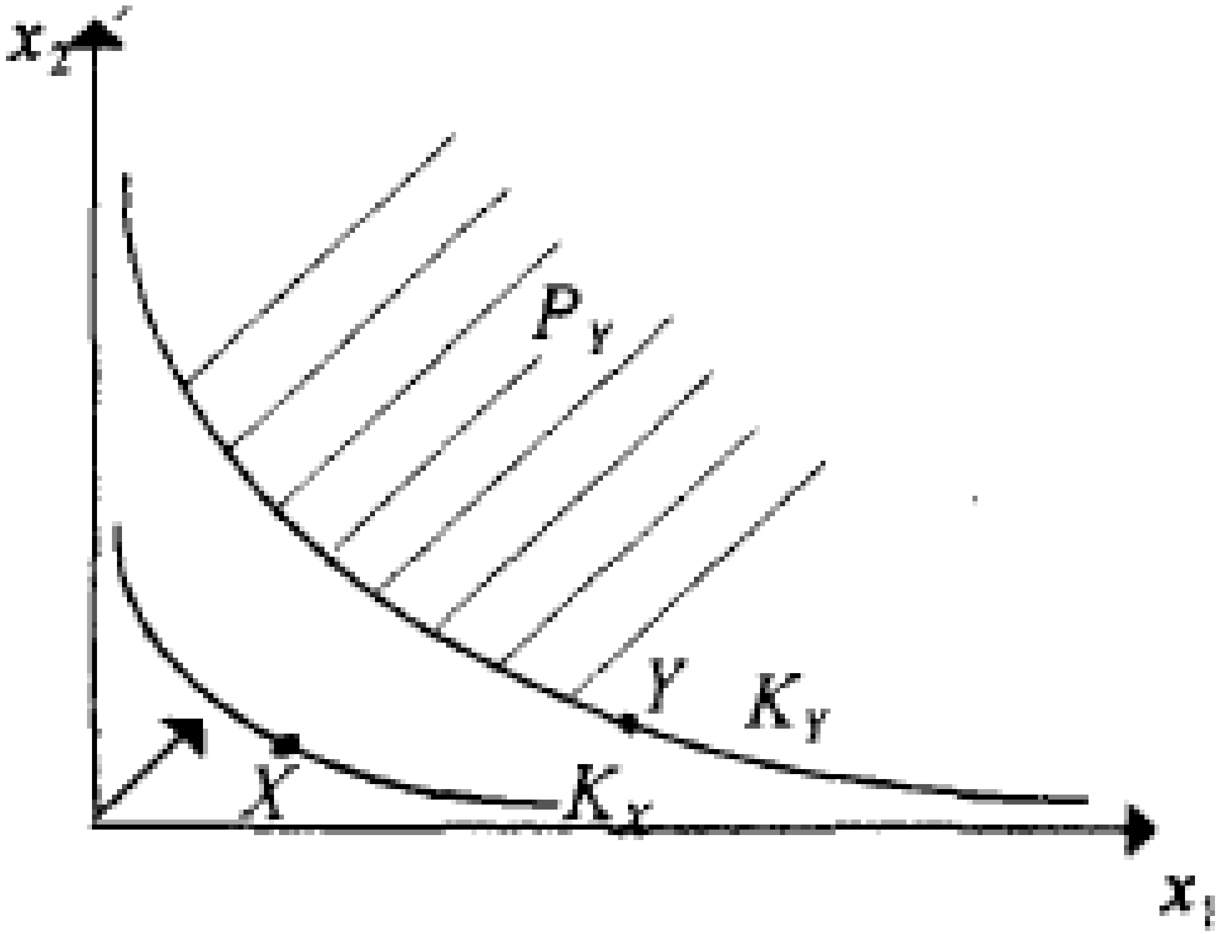
Strong advantage $X > Y$

Weak advantage $X \geq Y$

Indifference $X = Y$

For each system of preferences, you can determine the utility function of the species $U(p, x) = F(p, x)$.The greater the value of a function, the greater the advantage of one set of prices and goods over another

A typical picture for two types of goods Kx and Ky. The arrow is an indicator of equivalence, the shaded field is the set of advantages of Py prices over Px



In the theory of consumption hypotheses are assumed and it is considered that the utility function has such properties

1) $\frac{\partial u}{\partial x_i} > 0$ - with the growth of consumption of goods usefulness increases;

2) $\lim_{x_i \to 0} \frac{\partial u}{\partial x_i} = \infty$ - a small increase in the good in its initial absence dramatically increases the usefulness;

3) $\frac{\partial^2 u}{\partial x_i^2} < 0$ - with the growth of consumption of goods, the rate of growth of utility decreases (decreases);

4) $\lim_{x_i \to \infty} \frac{\partial u}{\partial x_i} = 0$ - when there is a very large amount of good, its further growth does not lead to an increase in utility.

The surface of indifference is called the hypersurface dimension (n - 1), on which the utility is constant

$$u(x) = c = \text{const},$$

ore have a next form

$$du = \sum_{i=1}^{n} \frac{\partial u}{\partial x_i} dx_i = 0$$

This condition means that the tangent to the surface of indifference is perpendicular to the gradient of utility.

This means (from the consumer's point of view) the possibility of replacing one product with a certain amount of another (equivalent) product.

Let $dx_i = 0$ for i = 1, 2, ... n, then this formula has the form

$$\frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 = 0,$$

whence

$$-\frac{dx_2}{dx_1} = \frac{\partial u / \partial x_1}{\partial u / \partial x_2},$$

That is, the marginal rate of substitution of the first commodity for the second is equal to the ratio of the marginal utility of the first and second commodities.

SLUTSKY EQUATION

$$\frac{\partial x^*}{\partial p_n} = \left(\frac{\partial x^*}{\partial p_n}\right)_{comp} - \frac{\partial x^*}{\partial M} x_n^* .$$

It combines the problems of changing demand in the event of a change in the price of the nth product, as well as changes in demand in the event of a change in income *M*.

* - the optimal solution.

The equation is composed in vector form

2. THE PROBLEM OF OPTIMAL (RATIONAL) CONSUMER CHOICE

$$\max_{x \in B \cap X} u(x) = \max_{px = M} u(x).$$

To find the extremum, we compose the Lagrange equation.

$$\boldsymbol{L(x, l) = u(x) - \lambda\,(px - M)}$$

Necessary conditions of local extremum

$$\sum_{j=1}^{n} p_j x_j^* = M,$$

$$\frac{\partial L}{\partial x_i} = \frac{\partial u}{\partial x_i}(x_i^*) - \lambda^* p_i = 0, \quad i = 1, \ \dots, n.$$

Example

Suppose that the utility function has the form.

$$L(x_1, x_2, \lambda) = 20\sqrt{x_1 x_2} - \lambda(p_1 x_1 + p_2 x_2 - 400)$$

Prerequisites for local extremum

$$\frac{\partial L}{\partial x_1} = 10\frac{\sqrt{x_2}}{\sqrt{x_1}} - \lambda p_1 = 0 \qquad \frac{\partial L}{\partial x_2} = 10\frac{\sqrt{x_1}}{\sqrt{x_2}} - \lambda p_2 = 0 \qquad \frac{\partial L}{\partial \lambda} = p_1 x_1 + p_2 x_2 - 400 = 0$$

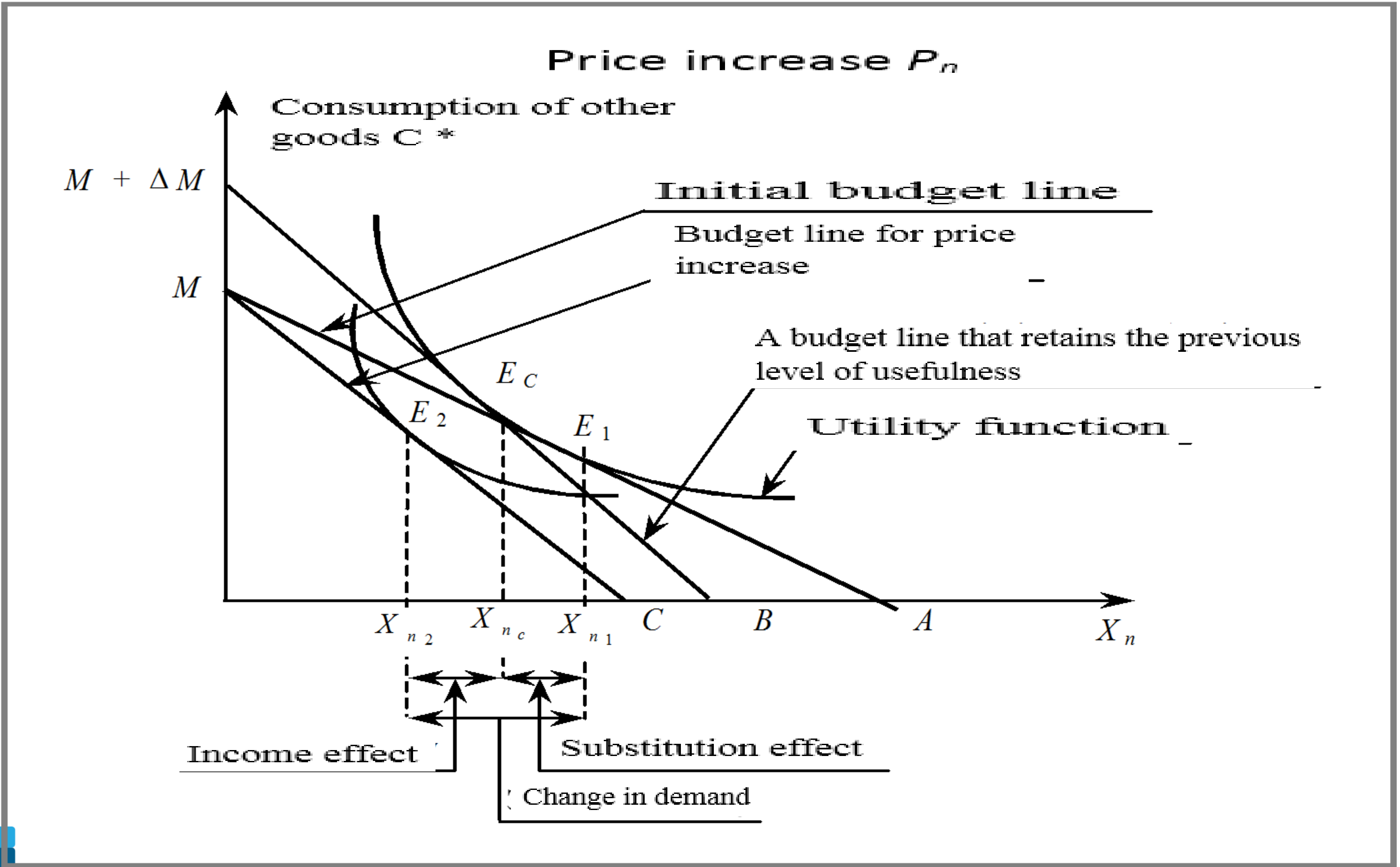
Solving the system of equations

$$\begin{cases} 10\dfrac{\sqrt{x_2}}{p_1\sqrt{x_1}} = \lambda \\ 10\dfrac{\sqrt{x_1}}{p_2\sqrt{x_2}} = \lambda \\ p_1 x_1 + p_2 x_2 = 400 \end{cases}$$

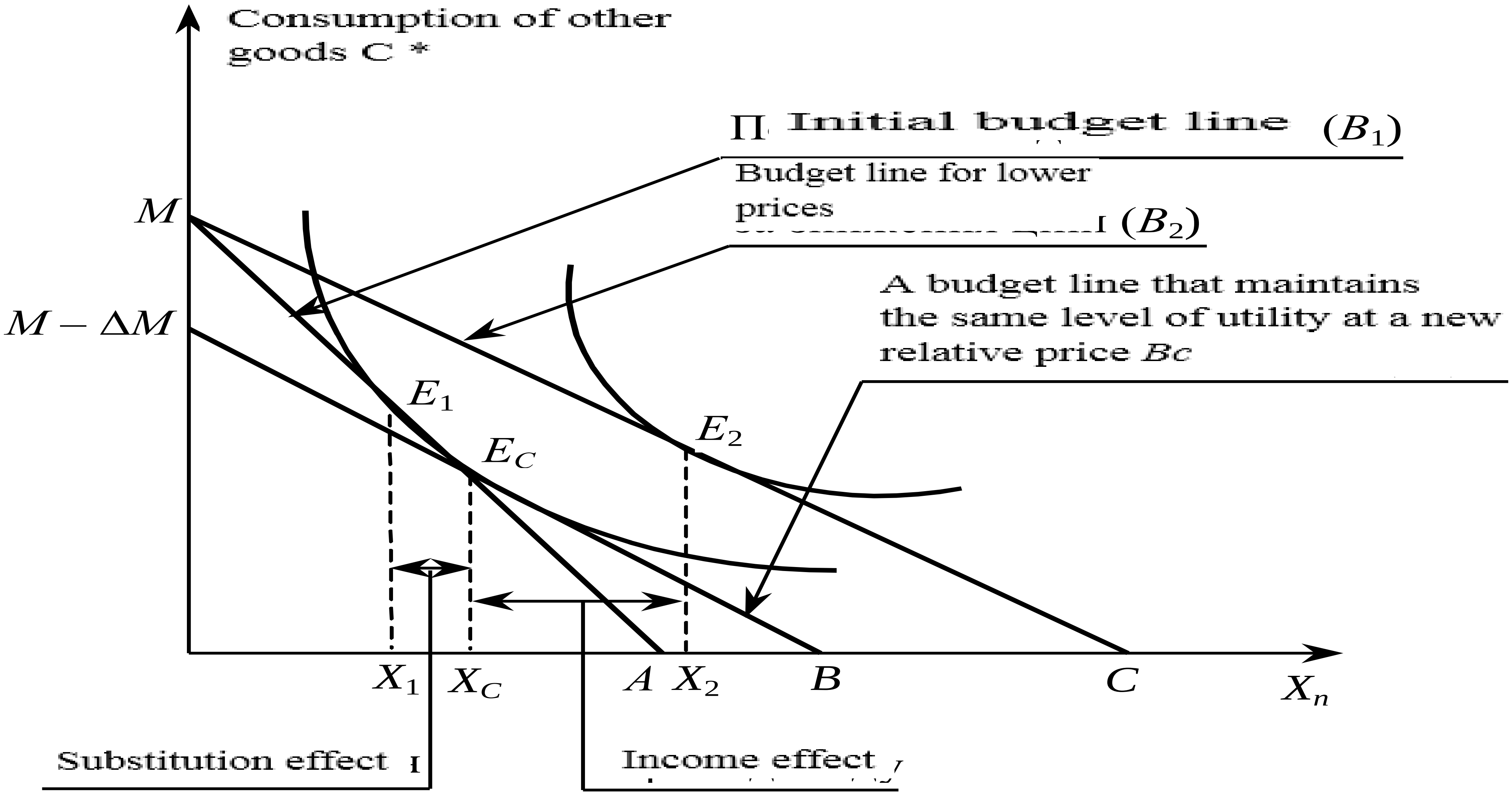
we obtain the function of consumer demand.

Then

$$\begin{cases} x_1^* = \dfrac{200}{p_1} \\ x_2^* = \dfrac{200}{p_2} \end{cases}$$


Price increase P_n
Consumption of other goods C *
$M + \Delta M$
M
Initial budget line
Budget line for price increase
A budget line that retains the previous level of usefulness
Utility function
E_C
E_2
E_1
X_{n2}
X_{nc}
X_{n1}
C
B
A
X_n
Income effect
Substitution effect
Change in demand

Price reduction Pn



3. MODELS OF BEHAVIOR OF PRODUCERS

PRODUCTION FUNCTION (PF)

PF is an economic and statistical model of the production process in this economic system and expresses a stable regular quantitative relationship between the volume of resources and output.

In the theory of production functions, the production process is analyzed in terms of the transformation of resources into a product (products). Inputs are flows of resources of various kinds, fully or partially used in production, output - ready-to-sell products. Resources (factors), technology and conditions of production organization functioning in the system determine potential possibilities and state of the process (system).

Function with fixed proportions of factors (Leontief function).

$$y = \min\left(\frac{x_1}{a_1}, \frac{x_2}{a_2}\right)$$

Cobb-Douglas function

$$y = a_0 x_1^{a_1} x_2^{a_2}$$

$$X = AK^{\alpha} \cdot L^{1-\alpha}, \quad \alpha_1 = \alpha, \quad \alpha_2 = 1 - \alpha.$$

Parameter *A* is mostly interpreted as a parameter of neutral technical progress: for the same values of α_1 and $_{\alpha 2}$, the output at point (*K, L*) will be greater the greater *A*.α_1 is the coefficient of elasticity of output on fixed assets, and α_2 is the coefficient of elasticity of output on labor. The coefficient of elasticity of the factor shows how many percent the output will increase if the factor increases by 1%.

Linear function

$$y = a_1 x_1 + a_2 x_2$$

Allen function

$$y = a_0 x_1 x_2 - a_1 x_1^2 - a_2 x_2^2$$

Function of constant elasticity of substitution of factors (CES function)

$$y = \left(a_1 x_1^{a_3} + a_2 x_2^{a_3}\right)^{a_4}.$$

Solow function

$$y = \left(a_1 x_1^{a_3} + a_2 x_2^{a_4}\right)^{a_5}$$

As resources (factors of production) at the macro level are mostly considered accumulated (materialized) labor in the form of production assets (capital) *K* and current (living) labor *L*. And as a result - gross output *X* (or gross domestic product *Y*, or national income *N*) .
In all cases, the result is generally called the issue and denoted by *X*.

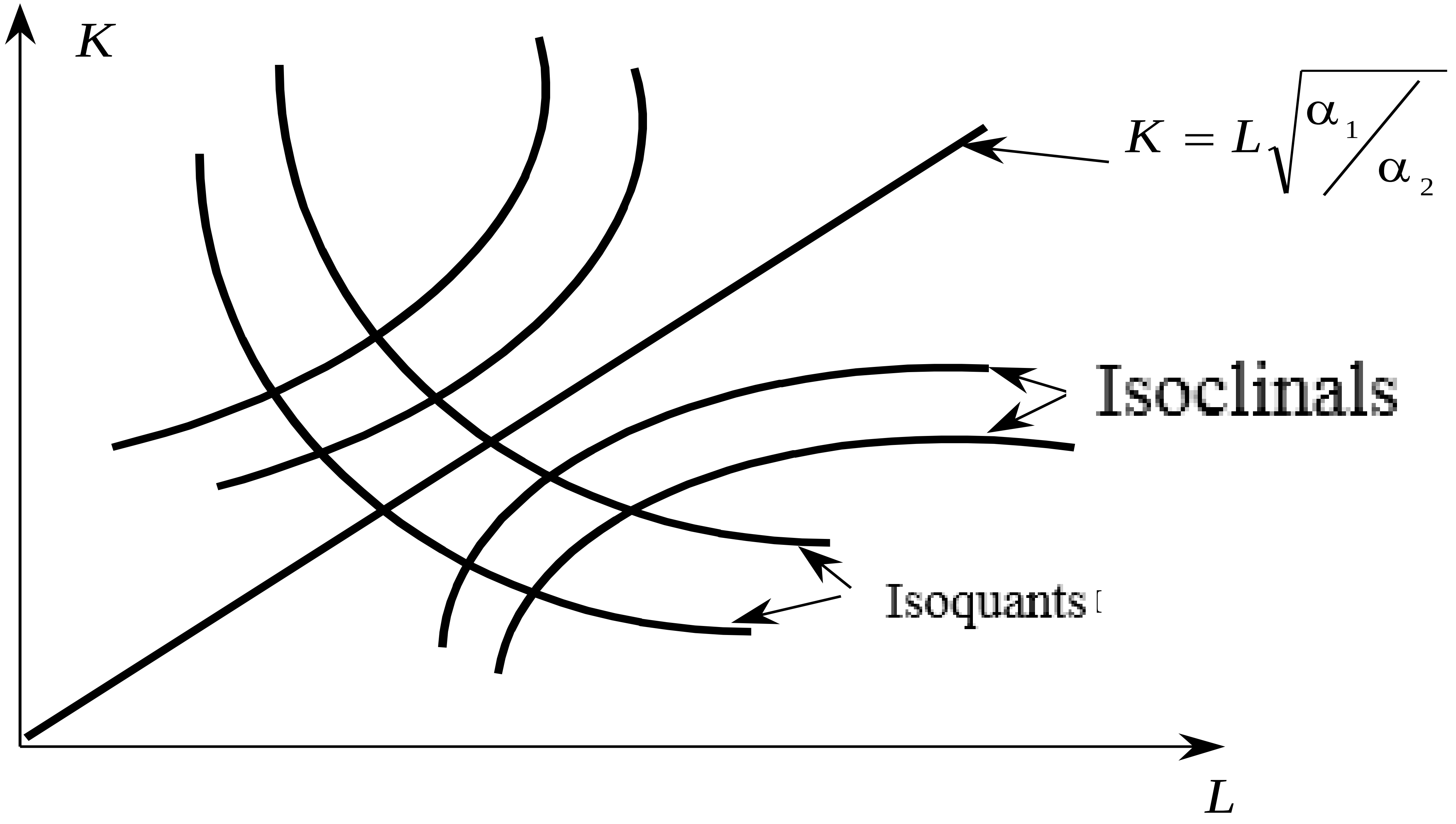
The level line on the plane K, L, the isoquant, is called the set of those points of the plane for which

$$F(K, L) = X_0 = \text{const.}$$

Isoclinals are the lines of the fastest growth of PF. Isoclines are orthogonal to zero-growth lines, ie, orthogonal to isoquants.

$$K = \sqrt{\frac{\alpha_1}{\alpha_2} L^2 + a}\,, \quad a = \text{const},$$

$$a = K_0^2 - \frac{\alpha_1}{\alpha_2} L_0^2\,,$$



K
$K = L\sqrt{\alpha_1 / \alpha_2}$
Isoclinals
Isoquants
L

4. MODELS OF OPTIMAL (RATIONAL) CHOICE OF THE MANUFACTURER

$$\Pi(x) = pF(x) - wx$$

$$\frac{\partial \Pi}{\partial x_j} = p\frac{\partial F}{\partial x_j} - w_j \le 0, \qquad \frac{\partial \Pi}{\partial x_j} x_j = \left(p\frac{\partial F}{\partial x_j} - w_j\right) x_j = 0, \; x_j \ge 0, \; j = 1, \ldots, n$$

$$p\frac{\partial F(x^*)}{\partial x_j} = w_j, \; j = 1, \ldots, n,$$

Let's determine the maximum issue, if 150 money is allocated for rent and wages. units, the cost of renting a unit of funds Wk = 5 money units, salary rate WL = 10 money units / people

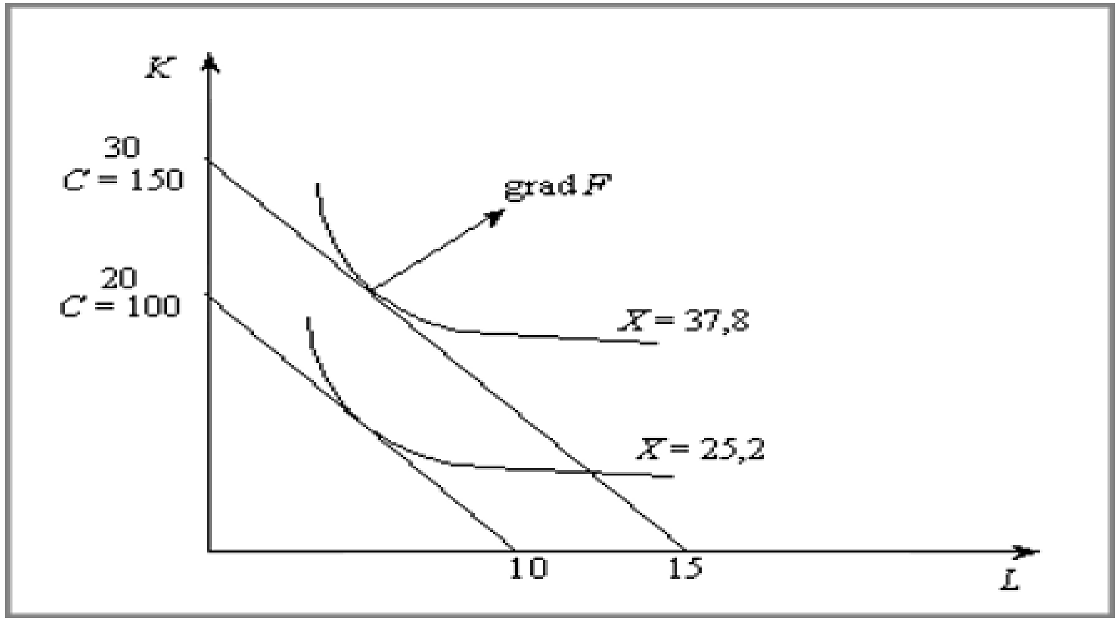
EXAMPLE

$$X = F(K, L) = 3K^{\frac{2}{3}} L^{\frac{1}{3}} .$$

$$\frac{\partial F}{\partial K} = \lambda w_K , \quad \frac{\partial F}{\partial L} = \lambda w_L$$

$$\frac{2}{3} \frac{F(K^*, L^*)}{K^*} = \lambda w_K , \quad \frac{1}{3} \frac{F(K^*, L^*)}{L^*} = \lambda w_L .$$

$$L^* = \frac{w_K}{2w_L} K^* \qquad w_K K^* + w_L L^* = 150, \qquad 5K + 10L = const ,$$



$$-\frac{dK}{dL} = S_K = \frac{\partial F / \partial L}{\partial F / \partial K} = \frac{1/3}{2/3} \frac{K^*}{L^*} = \frac{1}{2} \cdot \frac{20}{5} = 2,$$

5. ADVERTISING CAMPAIGN MODEL

$$\frac{dN}{dt} = [\alpha_1(t) + \alpha_2(t)N(t)](N_p - N(t)).$$

$$\ddddot{N}(t) = \frac{\ddddot{N}_p}{\left[1 + \left(\ddddot{N}_p \frac{a_2(t)}{a_1(t)} - 1\right) e^{-\ddddot{N}_p a_2(t) t}\right]}$$

$$\ddddot{N}_p = \frac{a_1(t)}{a_2(t)} + N_p \qquad \ddddot{N}(t) = \frac{a_1(t)}{a_2(t)} + N(t) \qquad a_1(t) = Ae^{-Bt}$$

$$a_2(t) = C\left(1 - e^{-Dt}\right) \qquad P = pN(t)\int_0^t \alpha_1(t)dt, \qquad S = s\int_0^t \alpha_1(t)dt.$$

$$P = pN(t)(1 - \ell^{-Bt}) \qquad S = sAD(1 - \ell^{-Bt})$$

6. COBWEB MODELI

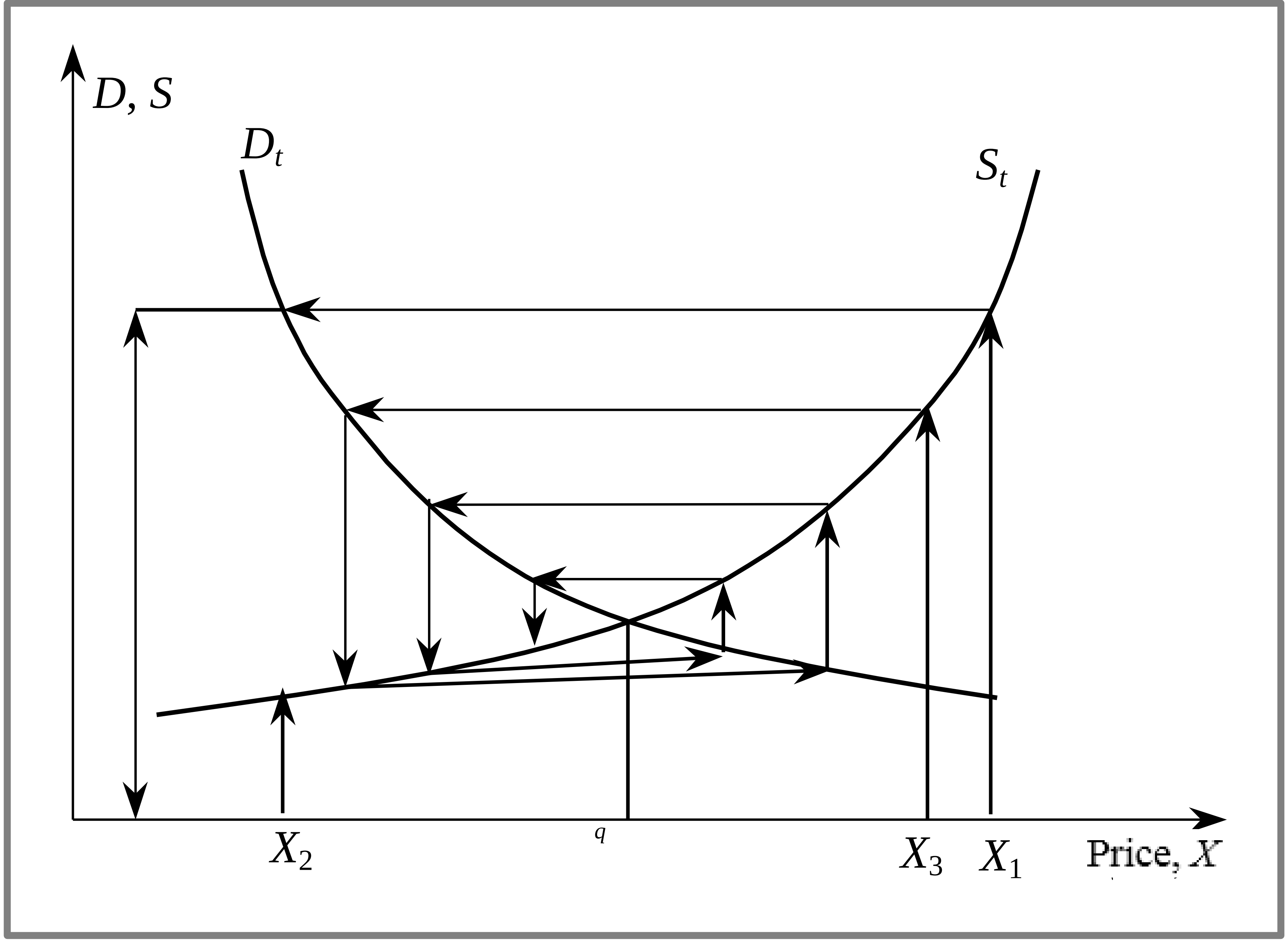
is a matter of formalizing the economic law of supply and demand, according to which:- the quantity of goods that can be sold on the market (ie demand) changes in the opposite direction to the change in the price of goods;- the quantity of goods produced and delivered to the market (ie supply) varies in the same direction as the price;- at the same time, the real market price is formed at a level at which supply and demand are approximately equal to each other (approximately coincide with some given accuracy), ie are in equilibrium.

Let X_t the price of goods at a time *t* and D_t and S_t - the quantity of goods bought and offered, respectively, on the market at the same time *t*. Then, taking into account one time interval required for manufacturers-sellers to respond to the price *X*, we can mathematically formulate the following patterns:

$$S_t = f(X_{t-1}),\ D_t = g(X_t),$$

$$\lim_{t\to\infty} f(X_{t-1}) = \lim_{t\to\infty} g(X_t);\ \lim_{t\to\infty} X_t = \overline{X}_q,$$

where *f(X)* is some monotonically increasing and *g(X)* is a monotonically decreasing function of the argument *X*, ie from the price, $\overline{X}_q$ is the equilibrium price


D, S
D_t
S_t
X_2
q
X_3
X_1
Price, X

7. MODELS OF INTERACTION OF CONSUMERS AND MANUFACTURERS
7.1. Evans model

$$d = d(t) = \Phi[p(t)],\ s = s(t) = \Psi[p(t)]$$

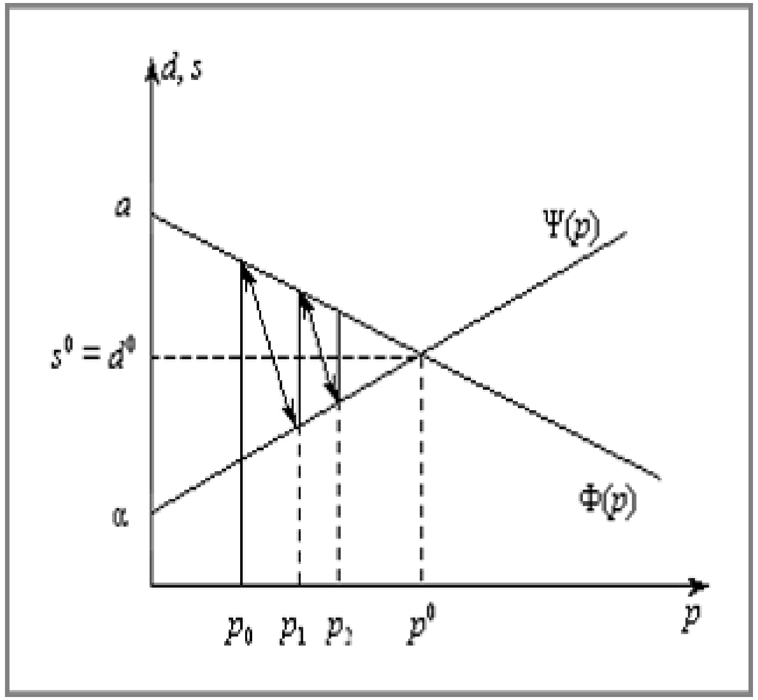
$\Phi(p) = \alpha - \beta p,\ \alpha > 0,\ \beta > 0$ (demand decreases with price increase)

$\Psi(p) = a + bp,\ a > 0,\ b > 0$ (supply increases with price)

$$\frac{dp}{dt} = \gamma\{-(b+\beta)p + a - \alpha\},\ p(0) = p_0.$$

$$p^0 = \frac{a-\alpha}{b+\beta} > 0.$$

$$p(t) = p_0 e^{-\gamma(b+\beta)t} + \frac{a-\alpha}{b+\beta}\left[1 - e^{-\gamma(b+\beta)t}\right]$$



7.2. Walras model

$$K_i(p) = pb_i + l_i(p).$$

$$\psi_k(p) = \left\{ y_k^* : py_k^* = \max_{y_k \in Y_k} py_k \right\}.$$

$$X(p) = \{x : x \in X,\ px \le K(p)\}$$

$$\Phi(p) = \left\{ x^* : u(x^*) = \max_{x \in X(p)} u(x) \right\},$$

$$Y = \left\{ y : y = \sum_{k=1}^{m} y_k,\ y_k \in Y_k,\ k = 1, \ldots, m \right\}.$$

$$\begin{cases} x_i^* \in \Phi_i(p^*), & i = 1, \ldots, l, \\ y_k^* \in \psi_k(p^*), & k = 1, \ldots, m, \\ \sum_{k=1}^{m} y_k^* + b \ge \sum_{i=1}^{l} x_i^*, & \\ p^*\left(\sum_{k=1}^{m} y_k^* + b \right) = p^* \sum_{i=1}^{l} x_i^*. & \end{cases}$$

$$\sum_{k=1}^{m} p_{Ck}^* y_k + \sum_{i=1}^{l} p_{Ti}^* (b_i - x_i) \to 0$$

$$\sum_{k=1}^{m} y_k + \sum_{i=1}^{l} b_i \ge \sum_{i=1}^{l} x_i$$

$$x_i = \Phi_i(p_{Ti}), \quad i = 1, \ldots, l,$$

$$y_k = \Psi_k(p_{Ck}), \quad k = 1, \ldots, m.$$

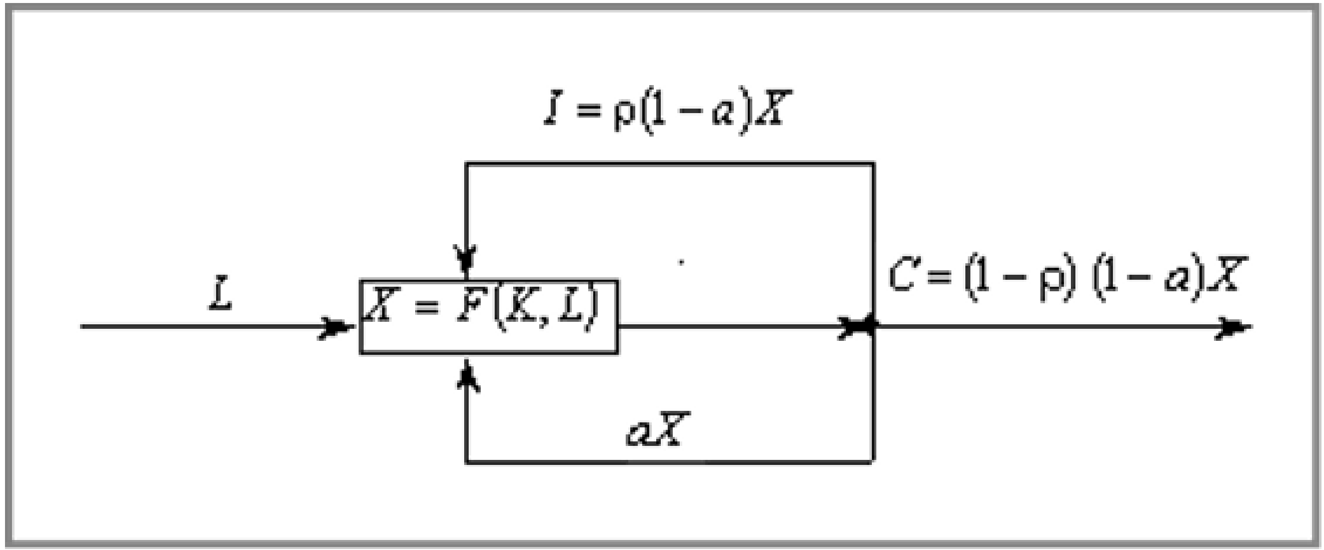
$$p_{\grave{\alpha}i},\ p_{Ck},\ y_k,\ x_i \ge 0.$$

7.3. One-sector model of economic development (SOLO MODEL)

$$-1 < \nu < 1,$$
$$0 < \mu < 1,$$
$$0 < a < 1,$$
$$0 < \rho < 1.$$

$$L = L_0 e^{\nu t}; \quad \frac{dK}{dt} = -\mu K + \rho(1-a)X, \quad K(0) = K_0;$$

$$X = F(K, L), \quad I = \rho(1-a)X; \quad C = (1-\rho)\,(1-a)X.$$



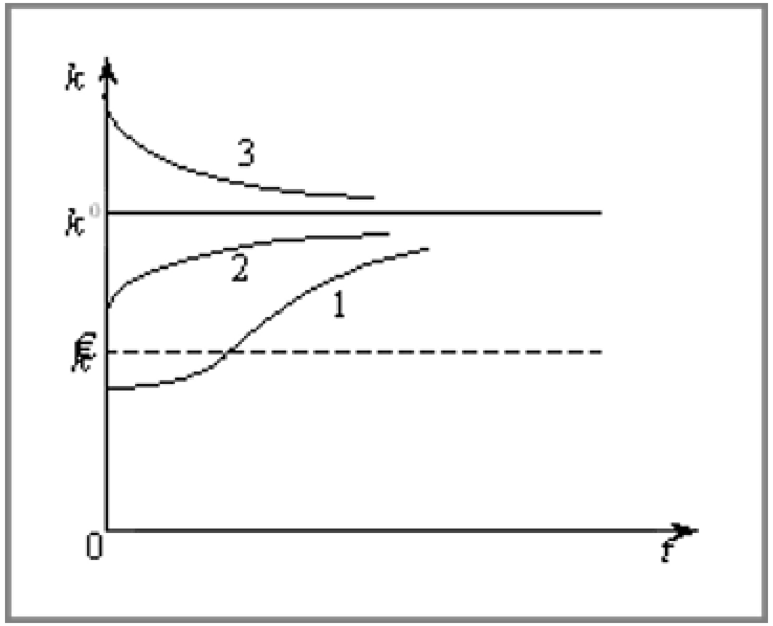
$$\frac{dk}{dt} = -\lambda k + \rho(1-a)f(k), \lambda = \mu + \nu, k(0) = k_0 = \frac{K_0}{L_0}$$

Transition mode in the Solow model

$$\frac{dk}{dt} = -\lambda k + \rho(1-a)f(k), \quad k(0) = k_0$$

$$\frac{d^2k}{dt^2} = \frac{dk}{dt}\left[\rho(1-a)f'(k) - \lambda\right]$$

$$f(k) = Ak^{\alpha}, \quad \hat{k} = \left[\frac{\alpha\rho(1-a)A}{\lambda}\right]^{\frac{1}{1-\alpha}}, \quad k^0 = \left[\frac{\rho(1-a)A}{\lambda}\right]^{\frac{1}{1-\alpha}},$$



$$u(t) = \left[\left(k^0\right)^{1-\alpha} e^{(1-\alpha)\lambda t} + k_0^{1-\alpha} - \left(k^0\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}}.$$

$$k(t) = \left[\left(k^0\right)^{1-\alpha} + e^{-(1-\alpha)\lambda t}\left(k_0^{1-\alpha} - \left(k^0\right)^{1-\alpha}\right)\right]^{\frac{1}{1-\alpha}},$$

8. EQUATION OF PUBLIC DEBT DYNAMICS

$$\dot{M} + \dot{B} = P(G - T) + RB,$$

$$\dot{M} \equiv \frac{d}{dt} M(t)$$

$$S = \frac{\dot{M}}{P}$$

$$\dot{b} = rb - S + (G - T),$$

$$\dot{B} \equiv \frac{d}{dt} B(t)$$

P - inflation, *P*(*G* - *T*) - the state budget deficit in nominal terms; *G* - budget expenditures in real terms; T - real taxes that do not change the volume of output; *RB* - the volume of public debt service at the rate of nominal interest *R* > 0.

$$S_N \equiv S - (G - T) \leq 0,$$

$$b - \frac{r}{2} b^2 = S_N t \qquad \text{Roots of equation: } b_1 = 1 \qquad b_2 = -\frac{2 S_N t}{r}$$

$$S_N \equiv S - (G - T) > 0,$$

$$b(t) = e^{rt} \int^{\infty} e^{-r\tau} \left[S(\tau) + T(\tau) - G(\tau) \right] d\tau$$

Government loans and accumulated debt

$$dB = -rB\,dt,$$

$$dV = \alpha V dt,$$

$$B(t) = F \exp\,[-rt]$$

$$\hat{V}(t) = F \exp(\alpha t)$$

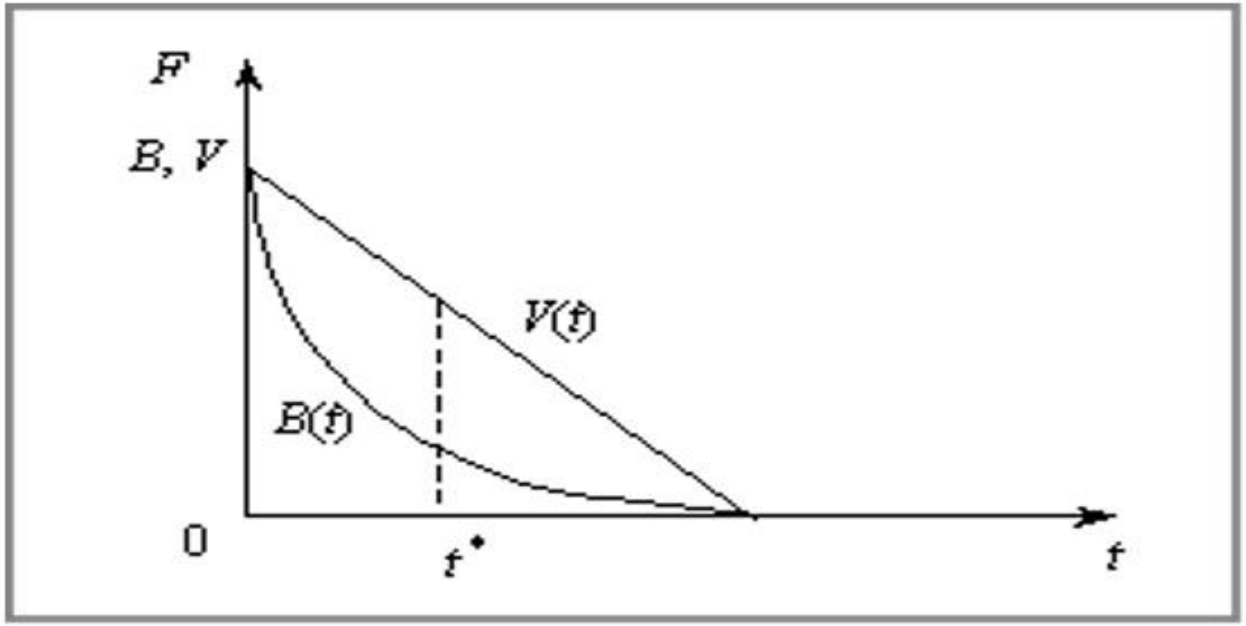
$$V(t) = [F \exp(\alpha t)] \exp[-rt] = F \exp[-(r-\alpha)\,t\,].$$

$$\max\{V(t^*) - B(t^*)\} = \max\{F|\exp(\alpha t^*) - 1|\exp(-rt^*)\}$$

$$f(t) = [V(t) - B(t)].$$

$$t^* = \frac{1}{\alpha}\ln\frac{r}{r-\alpha}$$

$$f(V) = \max\{V - B, 0\}.$$



9. LEONTIEV'S MODEL OF DIVERSIFIED ECONOMY

Denote by X_i the volume of gross output of the i-th industry for the planned period, and through Y_i the volume of final products of the i-th industry, intended for non-productive consumption (i = 1, 2, ... n), x_{ij} - part of the products of the i-th industry consumed in the j-th to ensure the release of its products in the amount of X_j. Values are called direct cost ratios.

Values $a_{ij} = \frac{x_{ij}}{X_j}, \ (i, j = 1, 2, \ldots, n)$ are called direct cost ratios.

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix},$$

These coefficients form a square matrix of direct cost coefficients.

The matrix $A = \|a_{ij}\|$ contains information about the structure of intersectoral relations and the technology of production of this economic and production system.

We obtain a system of equations of the form

$$\begin{cases} X_1 = x_{11} + x_{12} + \ldots + x_{1n} + Y_1, \\ X_2 = x_{21} + x_{22} + \ldots + x_{2n} + Y_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ X_i = x_{i1} + x_{i2} + \ldots + x_{in} + Y_i, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ X_n = x_{n1} + x_{n2} + \ldots + x_{nn} + Y_n. \end{cases}$$

In matrix form, the balance ratios can be written as

$$\mathbf{(E\text{-}A)\ X = Y,}$$

where **E** is a unit matrix.

The main task of the industry balance is to find the wind of gross output **X**, which, at a given matrix of direct costs provides a given vector of the final product **Y**. Therefore, if the matrix **(E - A)** is not degenerate, we have

$$\mathbf{X = (E - A)^{-1}\,Y}.$$

The matrix $\mathbf{S = (E - A)^{-1}}$ is called the full cost matrix.

Example

Branch.	Product consumption		Gross output
	Energy	Engineering	
Energy	100	160	500
Engineering	275	40	400

Calculate the required volume of gross output of each industry, which provides the vector of output of final products

$$Y = \begin{pmatrix} 200 \\ 100 \end{pmatrix}.$$

$$A = \begin{pmatrix} 0{,}2 & 0{,}4 \\ 0{,}55 & 0{,}1 \end{pmatrix},$$

$$S = (E - A)^{-1} = \frac{1}{0{,}5}\begin{pmatrix} 0{,}9 & 0{,}4 \\ 0{,}55 & 0{,}8 \end{pmatrix} = \begin{pmatrix} 1{,}8 & 0{,}8 \\ 1{,}1 & 1{,}6 \end{pmatrix}$$

$$X = \begin{pmatrix} 1{,}8 & 0{,}8 \\ 1{,}1 & 1{,}6 \end{pmatrix}\begin{pmatrix} 200 \\ 100 \end{pmatrix} = \begin{pmatrix} 440 \\ 380 \end{pmatrix}.$$

Solving the Leontief's problem by MatLab

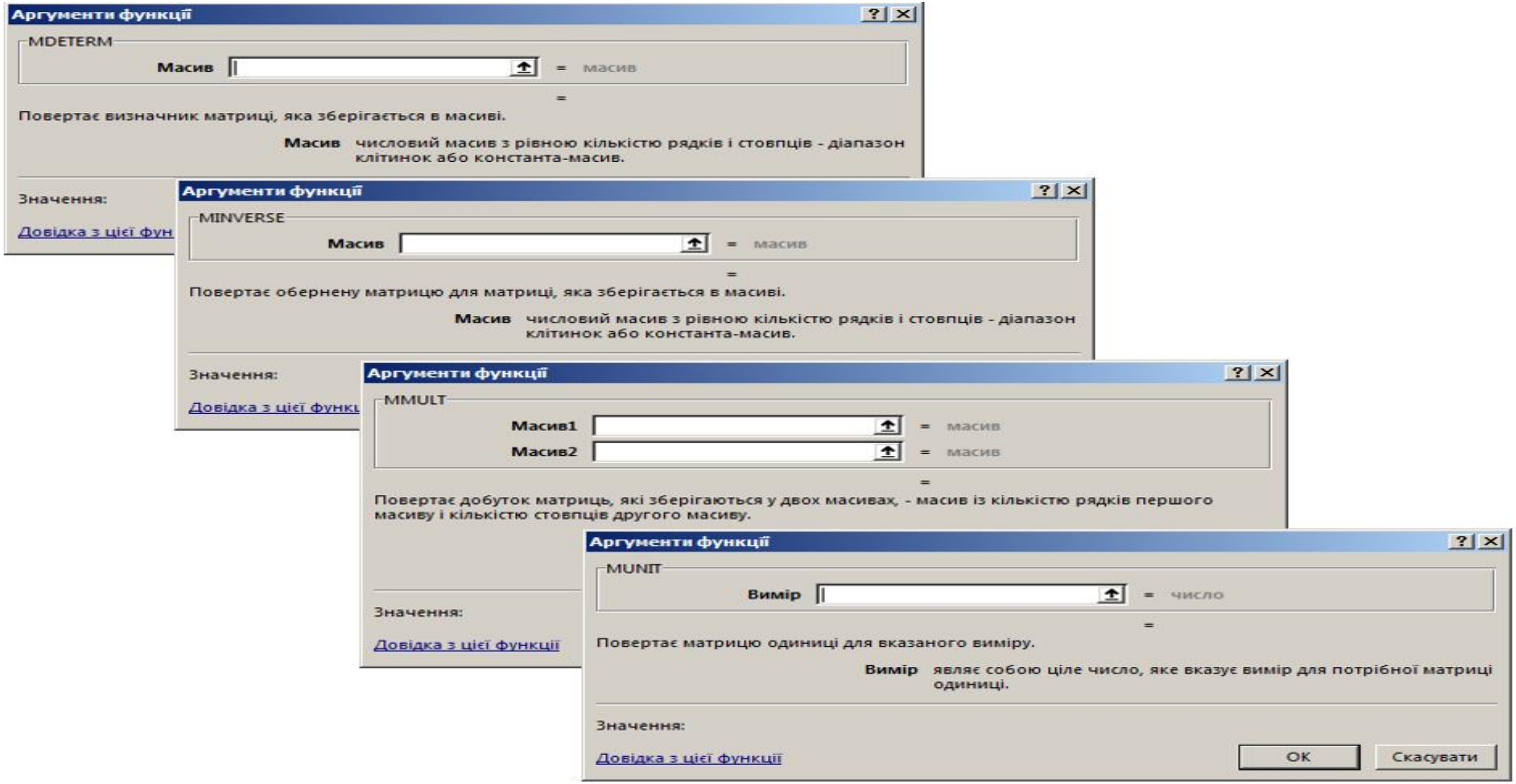
```
>>>E=eye(3)
>>> A=[0,2, 0,4; 0,55, 0,4]
>>> Y=[200; 100]
```

$$AA^{-1} = A^{-1}A = E$$

A^{-1}=E/A

```
>>> X=E/(E-A)*Y
```

Solving Leontief's problem by Excel



10. COEFFICIENT OF UNEVEN DISTRIBUTION OF INCOME TAX

Let *y* be part of the total income tax, which is proportional to part *x* of the total population. graph of the function $y = f(x)$. which describes the actual distribution of income tax, called the **Lorentz curve.**

The coefficient of uneven distribution of the Lorentz curve tax (Gini coefficient) is the ratio of the area of the figure bounded by the Lorentz curve and the line $y = x$ (in the figure it is shaded) to the area of the figure below the y = x (ie to the area of a right triangle) $0 \le x \le 1;\ 0 \le y \le 1;\ y = x.$

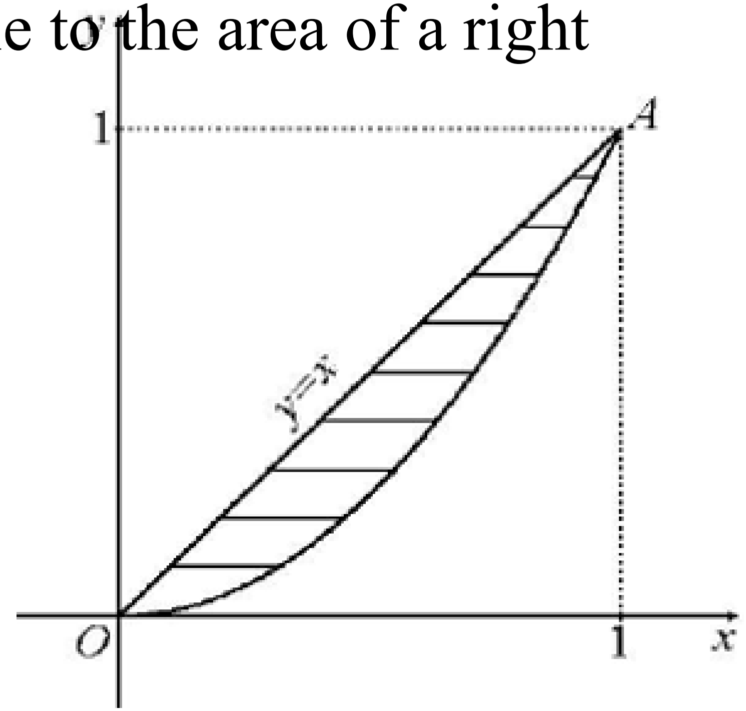
The area of the triangle

$$S_{OAB} = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}.$$

The area of the shaded figure I

s obtained through a definite integral

$$S_1 = \int_0^1 (x - f(x))dx.$$



Therefore, by definition, the Gini coefficient is calculated by the formula

$$L = \frac{S_1}{S_{OAB}}.$$

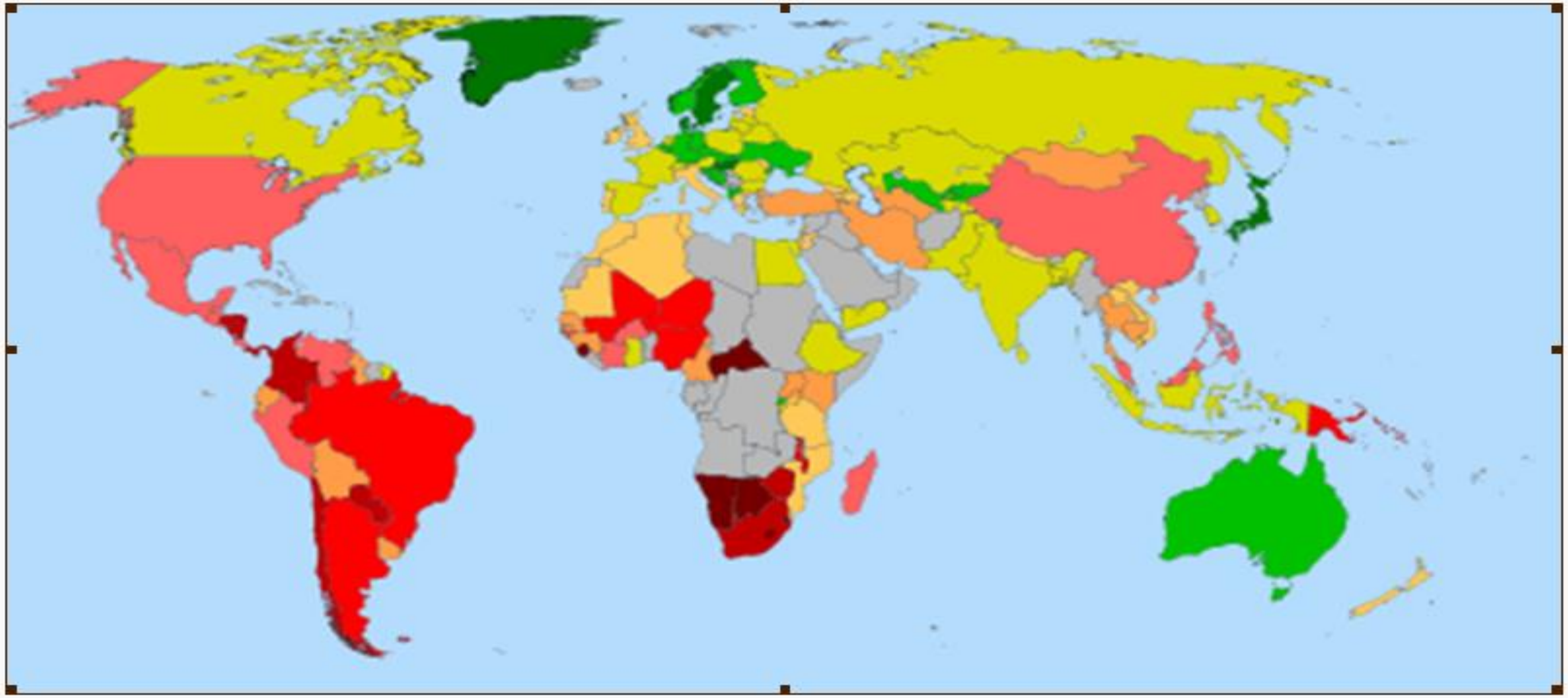
In real calculations, the graph is not built, but Brown's formula is used

$$G = \left|1 - \sum_{k=1}^{n} (X_k - X_{k-1})(Y_k + Y_{k-1})\right|,$$

or Ginny's formula

$$G = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} |y_i - y_j|}{2n^2 \bar{y}},$$

where G is the Gini coefficient, X_k is the cumulative share of the population (pre-ranked by increasing incomes), Y_k is the share of income that X_k receives in total, *yk* is the share of household income in total income, and $\bar{y}$ - the arithmetic mean share of household income.



Ginny coefficient (0 1), Ginny index (0 100 %)

< 0.25 | 0.25–0.29 | 0.30–0.34 | 0.35–0.39 | 0.40–0.44

0.45–0.49 | 0.50–0.54 | 0.55–0.59 | Γ 0.60 | N/A