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The textbook presents the basic procedure for creating an investment project, a list of non-financial selection criteria, basic coefficients that characterize the investment project, securities theory and, based on these data, the main methods of finding optimal investment solutions by financial criteria.

Each section contains theory, examples of solutions and individual tasks to consolidate the acquired knowledge. The text provides recommendations for calculations on a computer using free software, including Calc, Open Office.

Designed for university students and can be useful for planners working on investment projects.

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## PREFACE

Based on disciplines:

INVESTMENT ACCOUNTING

ECONOMICS OF ENTERPRISES

INFORMATICS AND COMPUTER EQUIPMENT

MATHEMATICAL PROGRAMMING



# 1. INVESTMENT DESIGN OF ENTREPRENEURIAL ACTIVITY

- 1.1. Stages of project creation and implementation
- 1.2. A typical procedure for selecting investment projects
- 1.3. Financial ratios of the analysis of enterprise activity
  - 1.3.1. Financial indicators that are calculated in the interests of the state
  - 1.3.2. Coefficients that reflect the interests of owners
  - 1.3.3. Coefficients that reflect the interests of short-term creditors
  - 1.3.4. Coefficients that reflect the interests of long-term creditors
- 1.4. Investment project management
  - 1.4.1. Ways to manage funds in the enterprise
  - 1.4.2. Methods of optimizing the capital structure
  - 1.4.3. Selection and evaluation of staff
- 1.5. Determining the effectiveness of the investment project
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- 1.7. Assessment of the sustainability of the investment project**
  - 1.7.1. Assessment of the financial condition of the enterprise**
  - 1.7.2. Break-even point and level**
- 1.8. Optimal decisions when planning investments**
  - 1.8.1. The concept of optimal balance. Optimality criterion**
  - 1.8.3. The optimal choice of investment in capital diversification**
  - 1.8.5. Optimal planning of the moment of the beginning of investments**

## **2. INVESTMENT DESIGN OF FINANCIAL OPERATIONS**

- 2.1. Bond valuation models**
- 2.2. Valuation of shares**
- 2.3. Statistical characteristics of shares**
- 2.4. Securities portfolio Statistical calculations of the portfolio**
- 2.5. Basic principles of securities portfolio formation.**
- 2.6. Measures to assess the effectiveness of investments in securities.**
- 2.7. The optimal portfolio of securities**
  - 2.7.1. Markovitz and Tobin models**
  - 2.7.2. Risk-return model and development-based model Sharpe.**

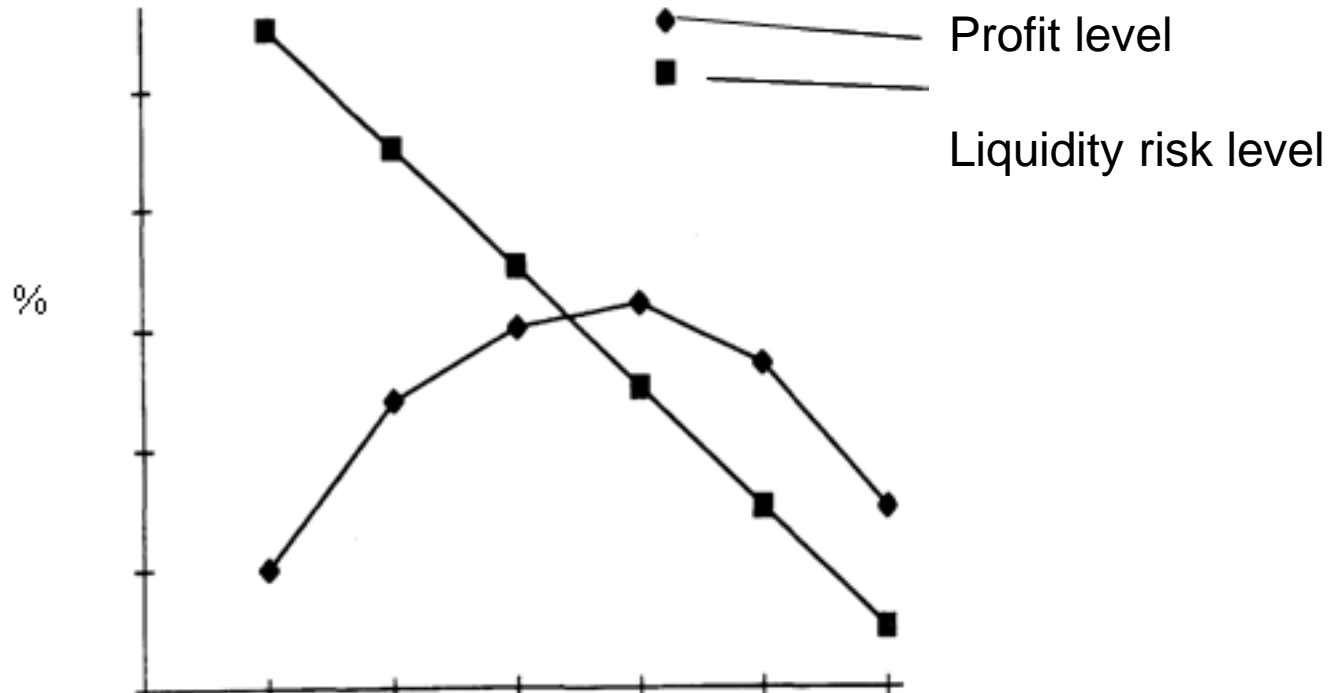


# Indicators that characterize the company in terms of interests...

- **State:** Profitability, Return on total assets, Return on net assets, Return on average total assets before interest and taxes, Return on average net assets before interest and taxes;
- **Owner's:** Profitability, Gross margin ratio, Profit margin ratio, Earnings before interest and taxes ratio, Operating expense analysis, Cost of goods sold ratio, Selling expense ratio, Administrative expense ratio, Interest expenses ratio;
- **Shareholders :** Return on average common equity), Earnings per share, Disposition of Earnings, Dividend yield ratio, Payout ratio, Dividends per share, Market indicators, Earnings multiple (price/earnings ratio), Market to book ratio;
- **Creditors:** Liquidity, Current ratio, Acid-test (quick) ratio, Financial Leverage, Debt to assets ratio, Debt to capitalization ratio, Debt to equity ratio, Debt Service ratio.



# Investment project management



The level of net working capital of the enterprise

]



Net cash flow is defined as the difference between income and expenses from the implementation of the investment project and includes as income - profit from production activities ( $P$ ) and depreciation deductions ( $Am$ ), as costs - investment in capital production, ie the reproduction of fixed assets that fall into this period ( $I$ ). Therefore, the deducted cash flow is determined by the formula

$$C_t = P + Am - I.$$

Net profit in general is defined as follows

$$P = [X \cdot C - (A + b \cdot X) + L] \cdot \left(1 - \frac{N_p}{100\%}\right),$$

where  $X$  - the volume of production,  $C$  - the price of production,  $A$  - conditionally fixed costs of production,  $b$  - conditionally variable costs per unit of output,  $L$  - the liquidation value of the object of production,  $N_p$  - the current rate of taxation of enterprise profits.





# Determining the effectiveness of the investment project

The values of inflows and outflows of finance determine the net present value (NPV) allows to obtain the most generalized characteristics of the investment result.

This parameter can be given as at the end of the investment project

$$NPV = \sum_{t=1}^T \frac{\Pi_t - B_t}{(1 + E_t)^t}$$

and at the beginning of the investment project

$$NFV = \sum_{t=1}^T (\Pi_t - B_t)(1 + E_t)^t$$

where  $E_t$  is the discount rate, which in general may be different for each step of the calculation.



Profitability Index ( $III$ ) - the sum of the reduced effects to the amount of capital investment:

$$III = \frac{NPV}{K}$$

where  $K$  is the total amount of investment in the project.

The internal rate of return (denoted as  $GNP$  or  $IRR$ ) is the discount rate ( $E$ ) at which the magnitude of the reduced effects is equal to the reduced investment.

$$\sum_{t=1}^T \frac{\Pi_t - B_t}{(1 + BIII)^t} = 0$$

Payback period of the investment project

$$T_{OK} = \frac{K}{\sum_{t=1}^T \frac{\Pi_t - B_t}{(1 + E_t)^t}}$$



## Calculation of the discount rate

$$E_t = BK + \Pi P + I_t,$$

where  $BK$  - the cost of capital involved in the investment project,

$\Pi P$  - risk premium,  $I_t$  - inflation index at time  $t$ .

The cost of capital is this

$$BK = BBK + B3K, \text{ or}$$

$$BK = \frac{\sum_{i=1}^n BK_i S_i}{\sum_{i=1}^n S_i}$$

where  $BBK$  - the cost of equity, which is a percentage of return on equity or the rate on deposits in the bank,

$B3K$  - the cost of borrowed capital, ie the rate on loans from the bank or the promised rate of return on shares issued for this project.



# • Recommended risk premium

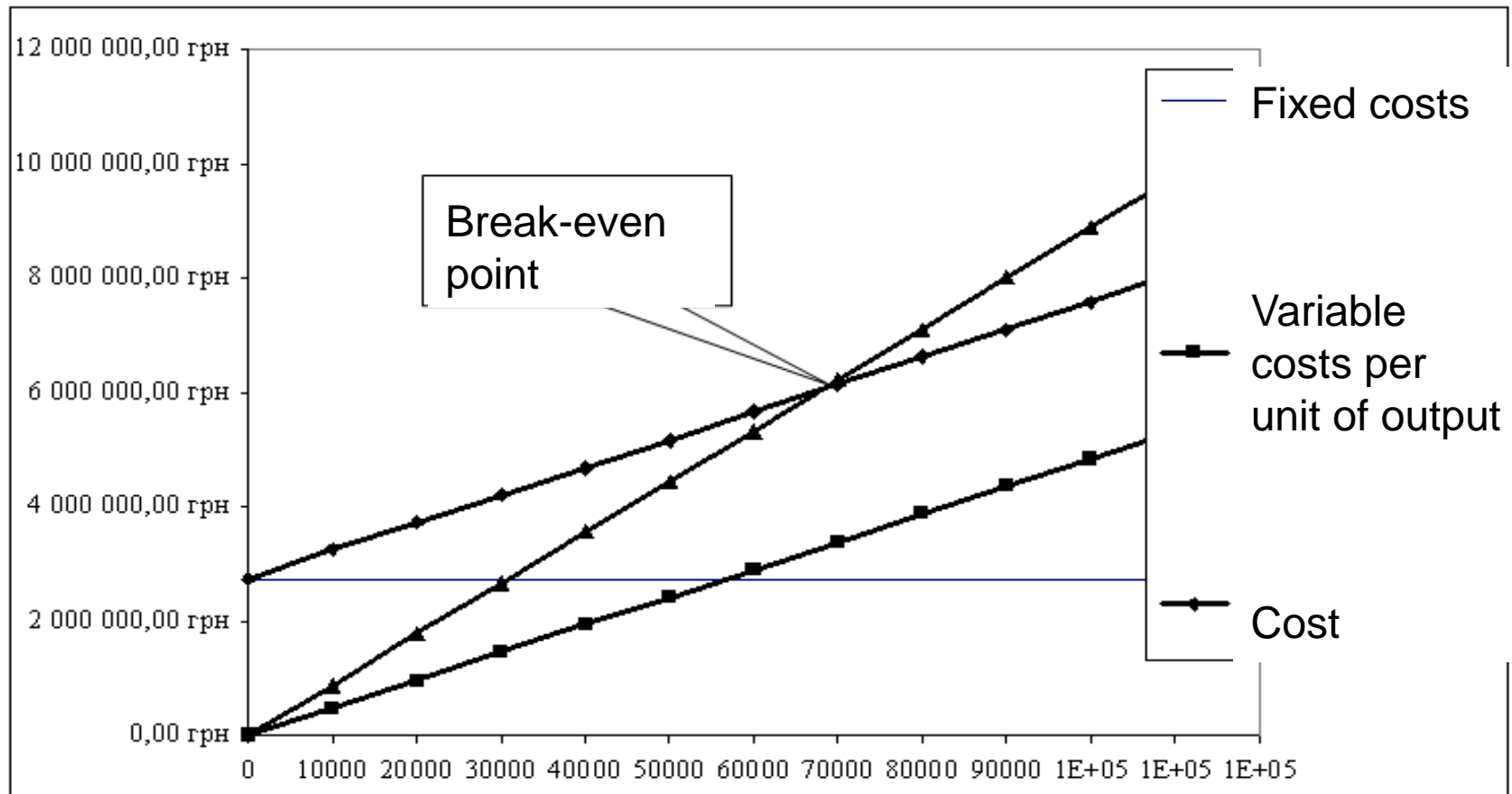
| Investment group  | Risk premium (IIP) |
|---|--------------------|
| Replacement investments - category 1 (new machines and equipment, vehicles, etc., which will perform basically the same functions as the old equipment being replaced)  | 0                  |
| Replacement investments - category 2 (new machines and equipment that will replace the old equipment, but are technologically more advanced, require higher skills of workers, other production approaches, etc.) | 0,03               |
| Replacement investments - category 3 (new facilities replacing old facilities, new plants at the same place)  | 0,06               |
| New investments - category 1 (new facilities or similar to old equipment, which will be used to produce or sell those products that have already been produced)   | 0,05               |
| New investments - category 2 (new facilities or machines for the production or sale of production lines that are closely related to existing production lines)  | 0,08               |
| New investments - category 3 (new facilities or machines or acquisitions (acquisitions) of other forms for the production or sale of production lines not related to the original activities of the company)      | 0,15               |
| Investments in R&D - category 1 (applied R&D aimed at specific goals)   | 0,10               |
| Investment in R&D - category 2 (basic research, the purpose of which may not yet be precisely defined, and the result is not known)   | 0,20               |



# Break-even point and level

For single-product production, ie one that produces only one type of product, break-even point

$$T_B = \frac{B_{\Pi}}{Ц - B_V}$$



A safety margin can also be found for all initial investment project data. To do this, we formulate the problem of optimizing the form of

$$NPV(PD) \rightarrow 0 \quad PD \geq 0$$

where NPV (PD) is the functional dependence of net present income on a specific point of PD - initial data.

| Initial data  | Break-even point | Initial data | Stability margin |
|---|------------------|--------------|------------------|
| <b>Total investment</b>                                     | 85               | 45           | -40              |
| <b>The share of investment in fixed assets</b>              | 85%              | 80%          | -5%              |
| <b>Residual value of fixed assets</b>                       | 11%              | 12%          | 1%               |
| <b>The share of equity in the structure of finance</b>      | 30%              | 45%          | 15%              |
| <b>Cost of equity</b>                                       | 27%              | 30%          | 3%               |
| <b>The cost of borrowed capital</b>                         | 21%              | 20%          | -1%              |
| <b>Revenue (income) of the enterprise in the first year</b> | 220,37           | 142,86       | -77,51           |



## BREAK-FREE POINT OF MULTIPRODUCT PRODUCTION

We hypothesize that the fixed costs (ПВ) for each type of product are proportional to the income for each type of product to each type of product. In addition, it is assumed that the prices and variable costs per unit of each product type are known. Then the break-even point for each type of product can be found as

$$T_{\text{ТОВ.багат.}i} = \frac{D_i}{D_{\text{заг.}}} * \text{ПВ}$$

where,

$T_{\text{мов.багат}i}$  – break-even point of the  $i$ -th type of product;  $D_i$  - income of the  $i$ -th type of product;

$D_{\text{заг.}}$  - total income from sales;

$\text{Ц}_i$   $3B_i$  - respectively Price / Change in costs of the  $i$ -th type of product;  $i$  - product type number (  $1 \geq i \geq n$ );  $n$  is the number of products.

Then the total break-even point of multi-product production will be found as the sum of the break-even points

$$T_{\text{заг.}} = \sum_{i=1}^n T_{\text{мов.багат}i} = \frac{\text{ПВ}}{D_{\text{заг.}}} \sum_{i=1}^n \frac{D_i}{\text{Ц}_i - 3B_i}.$$



# The concept of optimal balance

Suppose there are some balance sheet items of the enterprise, which include items of the statement of losses and gains of  $CB_j$  ( $1 \leq i \leq N$ ), where  $N$  is the number of such balance sheet items), which are related to each other by correspondent relations of the form:

$$CB_i = Fl(CB_j) \quad (1 \leq i, j \leq N, i \neq j, 1 \leq l \leq K),$$

where  $K$  is the number of correspondent links for this balance,  $F_l$  is the function of correspondent (for balance) or settlement links. Suppose there is also a set of financial ratios ( $\Phi K$ ), which are derived from the balance sheet items by forming from them certain complexes of the form

$$\Phi K_i = \prod_{j=1}^{Z_i} CB_j^{S_j}$$

where  $1 \leq i \leq M$ ,  $M$  is the number of financial ratios,  $Z_i$  is the number of balance sheet items included in the  $i$ -th ratio,  $S_j$  is equal to “1” or “-1”. Based on research, it is known that for each of these coefficients there is a certain limit of their values, more or less of which the balance becomes inefficient, ie

$$\Phi K_i \leq [100\% \cdot Y - (2Y - 1) \cdot OB_i],$$





where  $OB_i$  is the value of these constraints for the  $i$ -th coefficient.  $Y = 0$ , if the coefficient is required to be less than the constraint;  $Y = 1$ , if greater.

In the process of capital diversification, let several investment projects be proposed, the implementation of which should lead to a change in certain balance sheet items in the form of

$$CB_{hi} = Cbi + III_i,$$

where  $CB_{hi}$  is the new meaning of the balance sheet item after the introduction of the next proposal of the investment project of  $III_i$ . This implies that all possible variants of investment projects are substituted into the model at once.

If the  $III_i$  is not related to other  $III_i$ , its value should be limited to

$$0 \leq III_i \leq III_{imax},$$

where  $III_{imax}$  is the largest possible value of  $III_i$ . If there is a certain group of  $III_{imax}$ , they are related by the dependence of appearance

$$\sum III_i = const,$$

Let us now choose as an objective function some item of the balance sheet of  $CB_0$  (for example, equity), financial indicator (for example, profit before tax) or the financial ratio of  $\Phi K_0$  (for example, the highest efficiency of invested capital), and get the optimization problem

$$CB_0 \rightarrow \min \text{ or } \max \text{ або } \Phi K_0 \rightarrow \min \text{ or } \max.$$



# The optimal choice of investment in capital diversification

Suppose we have  $N$  different investment projects, each of which has its own term  $T_i$ , the amount needed to start it  $S_i$  and the percentage increase in invested capital at the end of the project  $P_i$  ( $1 \leq i \leq N$ ).

Then, the calculation horizon  $T = \max(T_i)$ , calculation step  $t = \min(T_i)$ .

For investment projects whose validity is within the limits, the number of times when this project can be applied is determined  $K_i = \{T / T_i\}$ , curly braces here denote an integer division. For each step of the calculation of  $t$  is the balance of cash flows of the form

$$\sum_{j=1}^t S_{t-k} (1 + P_i) = \sum_{i=1}^N Z_t S_i$$

where  $k$  is the number of the previous calculation step on which the sum of  $S_i$  was invested,  $Z_t$  is the multiplier. Is equal to 1, if in step  $t$  the investment  $S_i$  will be made, 0 - if not. Here, the left part of the equation is the receipt of funds after the end of the first investment project, and the right - the amount invested in investment projects. Obviously, such equations must be  $T-1$ . For the last step of the calculation, an equation is formed only for the receipt of sums, which will be the objective function of the optimization problem



$$\sum_{j=1}^t S_{t-k} (1 + P_i) \rightarrow \max$$

However, the previous equations will be the limitations of this problem. The variable parameters here will be the value of  $Z_t$ .

## Optimal planning of the moment of the beginning of investments

Suppose we have  $N$  different investment projects, each of which has its own term  $T_i$ , the amount needed to start it  $S_i$  and the percentage increase in invested capital at the end of the project  $P_i$  ( $1 \leq i \leq N$ ). Then, the calculation step and cash flow balances. Suppose there is a major investment project, for the implementation of which the amount is allocated  $K$ . The calculation horizon of this major project  $T$ . Other investment projects in this statement of the problem are considered as ancillary. It is known that the amount of  $K$  does not need to be invested simultaneously, but only at some stages of the project. Then, the task is to reduce the amount of  $K$  by investing temporarily free funds in ancillary investment projects.

The objective function in this case will look like  $K \rightarrow \min.$

$$K = \sum_{i=1}^N Z_t S_i$$

And the cash flow equation



## 2. BOND VALUATION MODELS

Basis Bond Valuation Model

$$P = \sum_{t=1}^T \frac{C}{(1+d)^t} + \frac{SH}{(1+d)^T}$$

where  $P$  is the real value of the bond with periodic interest payments;  $SH$  - no-term of the bond, which is subject to redemption at the end of its term;  $d$  - the expected rate of gross investment return (yield) on the bond, expressed as a decimal fraction;  $T$  - the number of periods remaining until the maturity of the bond,  $C$  - the amount of regular payments.

Model of valuation of the bond with payment of the full amount of interest upon its redemption:

$$B_{OB} = \frac{S_H + P}{(1+d)^T}$$

A model for estimating the value of a bond sold at a discount without interest

$$B_{OD} = \frac{S_H}{(1+d)^T}$$



Estimation of the current level of gross investment income on bonds using the coefficient of its current yield

$$K_{\text{IIIIO}} = \frac{S_H d}{P}$$

When calculating the future value of the annuity, carried out on the terms of subsequent payments (postnumerando), the following formula is used

$$\textit{Macaulay duration} = C \sum_{t=1}^T \frac{t}{(1+d)^t} + \frac{TS_H}{(1+d)^T}$$

$$\textit{Modified duration} = \textit{Macaulay duration} / (1 + d).$$



# Valuation of shares

The assessment of the nature of the stock's turnover on the stock market is primarily related to the indicators of its market quotation and liquidity. Among these indicators, the most important role is played by the following:

a) The level of dividend payment. This indicator characterizes the ratio of the amount of dividend and share price.

$$PД_A = \frac{ДB * 100}{Ц_A}$$

where  $PД_A$  - the level of dividend return on the share, %;  $ДB$  - the amount of dividend paid per share in a certain period;  $Ц_A$  - the share price at the beginning of the period.

b) The ratio of price and profitability. This indicator characterizes the relationship between the share price and earnings on it. The lower this ratio, the more attractive the stock for investment.

$$K_{Ц/Д} = \frac{Ц_A}{Д}$$

where  $K_{Ц/Д}$  - the ratio of price and earnings per share;  $Ц_A$  - share price at the beginning of the period;  $Д$  - total earnings per share in the period under review.

c) The liquidity ratio of the stock on the stock exchange. It characterizes the possibilities of rapid liquidity of the stock in case of need for its implementation.

$$K_{Л} = \frac{O_{ПР}}{O_{ПРОП}}$$

where  $K_{Л}$  is the liquidity ratio of the stock on the stock exchange;  $O_{ПР}$  - the total sales of the shares in question at this auction (or the amount of this indicator for all auctions for a certain period);  $O_{ПРОП}$  - the total volume of the offer of the considered actions at the given auctions (or the sum of this indicator on all auctions for the certain period)



d) The ratio of supply prices and demand for shares

$$K_{\text{ПРОП/П}} = \frac{\text{Ц}_{\text{ПРОП}}}{\text{Ц}_{\text{П}}}$$

where  $K_{\text{ПРОП/П}}$  - the ratio of supply prices and demand for shares;

$\text{Ц}_{\text{ПРОП}}$  - the average price level of the offer of the action at the auction;  $\text{Ц}_{\text{П}}$  - the average level of demand for shares at auction.

e) The turnover ratio of shares. It shows the volume of turnover of issued shares and is an indirect indicator of their liquidity. In foreign practice, this indicator is calculated based on sales results on both the stock exchange and the over-the-counter stock market. In our practice, accounting for sales of specific common shares on the OTC market is not organized, so the calculation of this indicator is possible only on the stock market

$$KO_A = \frac{O_{\text{ПР}}}{A_3 * \text{Ц}_{\text{ПР}}}$$

where  $KO_A$  - the turnover ratio of shares in a given period;  $O_{\text{ПР}}$  - the total sales of the shares in the auction for a certain period;  $A_3$  - the total number of shares of the company  $\text{Ц}_{\text{ПР}}$  - the average sale price of one share in the period under review.

Valuation of preferred stock

$$BA_{\text{П}} = \frac{D_{\text{П}}}{d}$$

where  $BA_{\text{П}}$  - the real value of the preferred share;  $D_{\text{П}}$  - the amount of dividends to be paid on a preferred share in the future;  $d$  is the expected rate of gross investment return (yield) on the preferred stock, expressed as a decimal fraction.

The valuation of a common stock when used for an indefinite period of time is as follows

$$BA_H = \sum_{t=1}^{\infty} \frac{D_o}{(1+d)^t}$$

where  $D_o$  - the amount of dividends expected to be received in each  $n$ -th period;



# Statistical characteristics of shares

As the profitability of stocks is constantly changing over time, there is a need to determine the degree of their risk. This characteristic is due to studies of changes in profitability over time, summarized in the table, which has the following form

| Date | Type of share 1 | Type of share 2 | Type of share $j$ | .... | Type of share $n$ |
|------|-----------------|-----------------|-------------------|------|-------------------|
| $t$  | $d_{t1}$        | $d_{t2}$        | $d_{tj}$          | .... | $d_{tn}$          |

For each type of shares there are:

- average yield;

$$M_j = \frac{1}{N} \sum_{t=1}^N d_{tj}$$

- variance of profitability;

$$D_j = \frac{N}{N-1} \sum_{t=1}^N d_{tj}^2 - M_j^2$$

- standard deviation of profitability or mathematical standard

$$\sigma_j = \sqrt{D_j}$$

The measure of the relative deviation of the values of profitability relative to the average is the variation and the coefficient of variation

$$\text{var}_j = \frac{D_j}{M_j}$$

$$K \text{ var}_j = \frac{\sigma_j}{M_j}$$





The latter serves as a measure of the riskiness of stocks. If  $K \text{ var}_j < 0.1$  - this type of shares is considered low risk, if  $0.1 \leq K \text{ var}_j < 0.25$  - medium risk, and when  $K \text{ var}_j > 0.25$  - high risk.

If we now find the average yield of all shares on the financial market for each observation date -  $M_t$  - it is possible to find two more important indicators -  $\alpha$  and  $\beta$ . They are found as coefficients of the linear equation of the dependence of the change in the yield of the  $j$ -th type of stock on the average yield of the financial market

$$d_{tj} = \alpha + \beta M_t$$

The level of financial risk of individual securities is determined on the basis of the following values of beta coefficients:

$\beta = 1$  - average level;

$\beta > 1$  - high level;

$\beta < 1$  - low level.

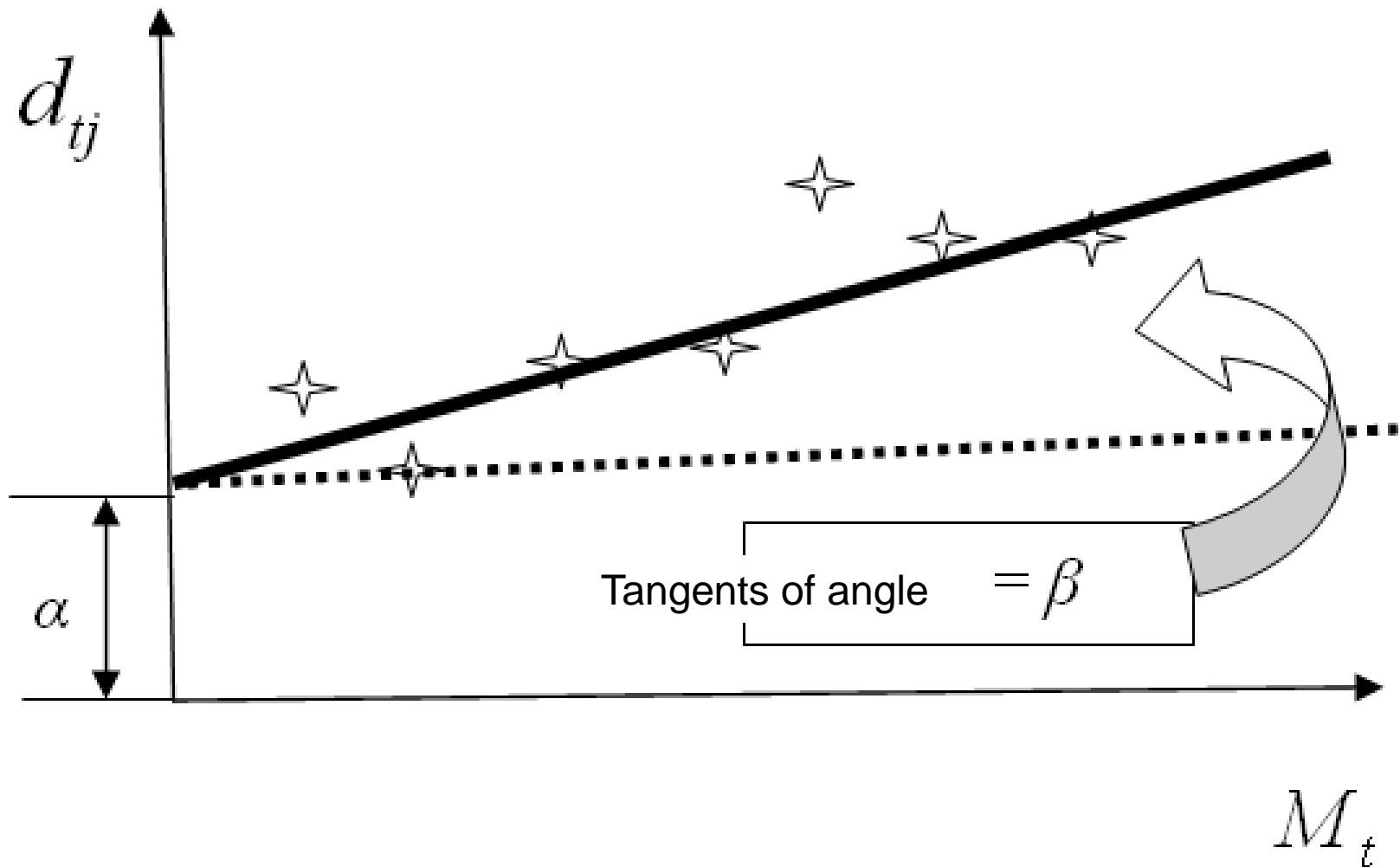
The quality of management of this type of shares is determined by the alpha factor:

$\alpha < 0$  - low level;

$\alpha = 0$  - average level;

$\alpha > 0$  - high level.





**Graphic content of coefficients  $\alpha$  and  $\beta$**



# Measures to assess the effectiveness of investments in securities

$$\begin{array}{l} \textit{Treynor Index} \\ \textit{Sharpe Index} \end{array} \quad I_T = \frac{d_{\Pi} - d_0}{\beta_{\Pi}} \quad I_{Sh} = \frac{d_{\Pi} - d_0}{\sigma_{\Pi}}$$

The degree of influence of the level of risk of a particular financial investment instrument on the formation of the risk level of the portfolio can be calculated by the following formula

$$L_j = \frac{\text{cov}_{j\Pi} - D_{\Pi}}{\sigma_{\Pi}}$$

where  $\text{cov}_{j\Pi}$  – co-variation of fluctuations in the profitability of the financial instrument and the profitability of the portfolio,  $D_{\Pi}$  - the variance of the profitability of the portfolio (the value of the set level, its risk);  $\sigma_{\Pi}$  is the standard deviation of the portfolio return.



# Markovits-Tobin models

The model of the optimal Markowitz portfolio, which provides minimal risk and a given return, has the form

$$\left. \begin{aligned} \sum_i \sum_j X_i X_j \text{cov}_{ij} &\rightarrow \min \\ \sum_i X_i M_i &= m_p \\ \sum_i X_i &= 1 \\ X_i &\geq 0 \end{aligned} \right\}$$

Markovitz's optimal portfolio of maximum profitability and given (acceptable) risk  $r_p$  can be represented as

$$\left. \begin{aligned} \sum_i X_i M_i &\rightarrow \max, \\ \sum_i \sum_j X_i X_j \text{cov}_{ij} &= r_p, \\ \sum_i x_i &= 1, \\ X_i &\geq 0. \end{aligned} \right\}$$

Tobin set the optimal task of forming a portfolio of securities based on the Markovic model. But to it was added the concept without risky securities, ie those whose profitability does not change over time. The following notation was introduced for them:  $d_0$  - return without risky security,  $X_0$  - share without risky security in the portfolio. Then, portfolios similar to Markovic's portfolios have the form

$$\left. \begin{aligned} \sum_i \sum_j X_i X_j \text{cov}_{ij} &\rightarrow \min \\ X_0 d_0 + \sum_i X_i M_i &= m_p \\ \sum_i X_i &= 1 \\ X_i &\geq 0 \end{aligned} \right\}$$

$$\left. \begin{aligned} X_0 d_0 + \sum_i X_i M_i &\rightarrow \max, \\ \sum_i \sum_j X_i X_j \text{cov}_{ij} &= r_p, \\ \sum_i x_i &= 1, \\ X_i &\geq 0. \end{aligned} \right\}$$



# Sharpe model of securities portfolio optimization

$$\left\{ \begin{array}{l} R_f + \sum_{i=1}^N (\alpha_i \cdot W_i) + (R_m - R_f) \cdot \sum_{i=1}^N (\beta_i \cdot W_i) \rightarrow \max; \\ \sqrt{\left( \sum_{i=1}^N (\beta_i \cdot W_i) \right)^2 \cdot \sigma_m^2 + \sum_{i=1}^N (\sigma_{\varepsilon i}^2 \cdot W_i^2)} \leq \sigma_{req}; \\ W_i \geq 0 \\ \sum W_i = 1. \end{array} \right.$$

Where  $W_j$  - share of the  $j$ -th set of shares in the diversified portfolio

$\alpha_j$  - Excess earnings of shares.

$\beta_i$  - risk of the  $i$ -th asset in the portfolio

$R_f$  - make a profit on risk-free transactions

$R_m$  - average return on shares.

$\rho_{re}$  - risk threshold set by the investor.

$N$  - number of securities in the portfolio.



## Risk-return model

$$\frac{\sum_{i=1}^M \sum_{j=1}^M X_i X_j \text{cov}_{ij} + \sqrt{\sum_{i=1}^M X_i^2 D_i}}{\sum_{i=1}^M X_i M_i} \Rightarrow \min$$

$$\sum_{i=1}^M X_i$$

$$X_i \geq 0$$

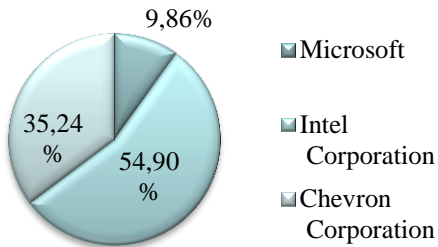


## Synthetic model.

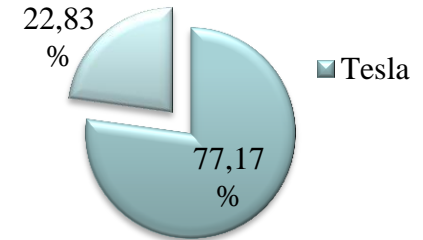
$$\left\{ \begin{array}{l} \sqrt{\sum_i x_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1}^N x_i x_j v_{ij}} \\ \hline (\sum_{j=1}^N \alpha_j x_j + R_m \sum_{j=1}^N \beta_j x_j) \cdot \sum_{j=1}^N d_j x_j \\ \sum_{j=1}^N x_j = 1 \\ x_j \geq 0, j = 1, \dots, N \end{array} \right. \rightarrow \min$$



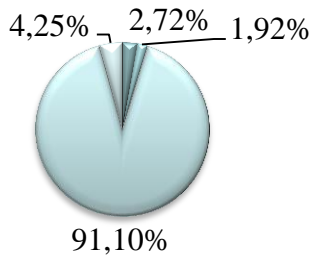
# Comparison of Pistunov's models with the Markovic and Sharpe models on the criterion of the ratio of portfolio risk to return.



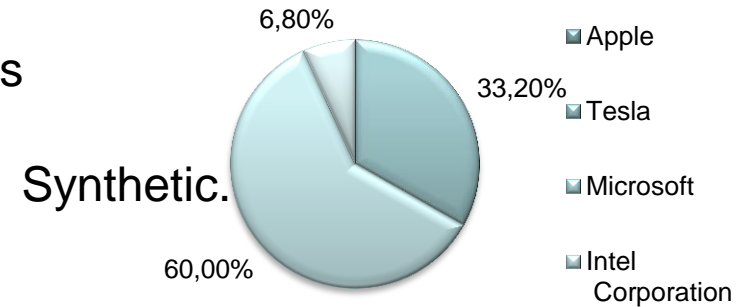
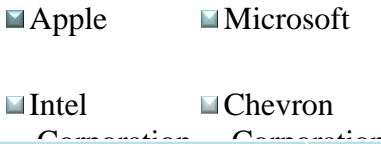
Markovic models  
minimal risk



maximum yield.



Pistunov's models  
Risk-profitable



Synthetic.

| Model                       | Risk (%) | Yield (%) |
|-----------------------------|----------|-----------|
| Minimal risk by Markovic    | 1,34     | 0,21      |
| Maximum yield by Markovic   | 5        | 0,65      |
| Risk-profitable by Pistunov | 5,64     | 0,7       |
| Synthetic model by Pistunov | 5,7      | 0,79      |

| Model                       | Sharp index |
|-----------------------------|-------------|
| Minimal risk by Markovic    | -0,184      |
| Maximum yield by Markovic   | 0,3         |
| Risk-profitable by Pistunov | 0,35        |
| Synthetic model by Pistunov | 0,41        |

