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INTEGRATED MODEL OF INVESMENT PORTFOLIO OPTIMISATION

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An improved synthetic mathematical model of optimal investment portfolio is proposed. Modeling of the optimal securities portfolio of the US energy companies was conducted in order to compare the proposed model with those previously known. Criterion of relative riskiness was developed to combine ratios of risk and profit in order to compare effectiveness of investment portfolio models

***Keywords**:* investment portfolio, stock market, profitability, investment risk, Markowitz model, Sharpe model.

Statement of problem. Investment portfolio optimization was an issue of scientific and practical interest since stock markets establishment.

However, since the global economy in general and Ukraine economy in particular are unstable, the challenge of financial resources efficient allocation becomes more urgent.

Traditionally, financial tools and mathematical-economical models that reduce risk and provide high-profitable investment are of particular interest in the periods of economic uncertainty.

Analysis of recent papers. Many foreign and domestic researchers and practitioners traders investigate the problem of optimal portfolios formation. Application of the theory of random matrices in the analysis of investment portfolios is discussed in the works [1, 2]. This method is an effective tool for filtering incoming data on securities volatility. As for research works relevant for CIS countries, a significant contribution to the theory of optimal investment portfolio is made by A. N. Burenin [3] and A. S. Shapkin [4].

Despite the existing variety of scientific and practical approaches to formation of investment strategy and risk management, classical Markowitz [5] and Sharpe models [6] are widely applied for direct distribution of funds among the assets.

Major drawback of mentioned above models is that the expected return is based on the results of previous periods. On the top of that, usage of each of the models involves the selection of one of the mutually opposing objective functions: profit maximization or risk minimization. There is a need for the development of a mathematical model that would allow aligning conflicting goals of investment portfolio formation.

The aim of this study is to improve the investment portfolio optimization model by combining existing Markowitz and Sharpe models.

Aim of the paper is to consider the most common model of investment portfolio optimization.

Materials and methods. Markowitz Model. According to Markowitz, investment portfolio is considered optimal under conditions that

1) for the given level of profitability there is no any-other investment opportunity with lower risk;

2) there is no other more profitable investment portfolio, that is characterized by the same level of risk. This refers to the so-called non-systematic risks.

Markowitz optimal portfolio for maximum profitability criteria (1). Markowitz optimal portfolio for the minimum risk criterion is (2). Sharpe model. Unlike Markowitz model, which considers the relationship of considered securities rate of return, Sharpe model examines the relationship of rate of return of given securities with an average market rate of return (3).

$$\begin{cases} m_p = \sum_{i=1}^{N} x_i d_i \to \max \\ \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j v_{ij} = r_p \\ \sum_{i=1}^{N} x_i = 1 \end{cases} \quad (1)$$

$$\begin{cases} \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j v_{ij} \to \min \\ \sum_{i=1}^{N} x_i d_i = m_p \\ \sum_{i=1}^{N} x_i = 1, \end{cases} \quad (2)$$

where x_i – is the share of capital spent on the purchase of securities *i*, d_i – is expected rate of return on asset *i*, m_p – is expected portfolio return, v_{ij}– is covariance of asset *i* and *j*, r_p – is given covariance of portfolio return (risk), *N* – is number of assets.

$$\begin{cases} R_f + \sum_{j=1}^{N} \alpha_j x_j + (R_m - R_f)\sum_{j=1}^{N} \beta_j x_j \to \max \\ \sqrt{p_m^2 (\sum_{j=1}^{N} \beta_j x_j)^2 + \sum_{j=1}^{N} p_j^2 x_j^2} \le p_{req} \\ \sum_{j=1}^{N} x_j = 1 \\ x_j \ge 0,\ j = 1,..,N, \end{cases} \quad (3)$$

where x_j – is share of *j* equity in diversified portfolio, α_j – is excess of expected rate of return on asset *j*, β_t– is estimated risk of asset *j* in the portfolio, R_f – is rate of return on the risk-free asset, R_m – is average asset rate of return, ρ_{req} – is risk frontier set by the investor, *N* – is number of assets.

The major shortcomings of the Sharp model are: neglecting of fluctuations of rate of return of the risk-free asset and necessity of returns rate prediction for stock market and risk-free securities (assets). Sharpe model is used to analyze a large number of securities that compose the most part of relatively stable stock market.

Pistunov-Sitnikov risk-revenue model. The model was created as a solution to multicriteria optimization task ensuring risk minimum and profit maximum [7]. As a result, convolution of two Markowitz criteria was formed. Whereby the criterion of "minimum" was placed in the numerator and the criterion of «maximum» - the denominator. Also, the numerator was added to the weighted average variance portfolio. The main idea of this model was to unite the Markowitz model with maximum rate of return and minimum risk.

Its advantage over the previously described models is that there is no need to determine the acceptable level of risk and income. This model requires the same statistical calculations as Markowitz model. This model works great with small number of assets and with relatively volatile stock market (4).

$$\begin{cases} \dfrac{\sqrt{\sum_i x_i^2 v_i^2} + \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j v_{ij}}{\sum_i^N x_i d_i} \to \min \\ \sum_{j=1}^{N} x_j = 1 \\ x_j \ge 0,\ j = 1,..,N \end{cases} \quad (4)$$

where d_i – is average rate of return of asset *i*, v_i– is variation (standard deviation) rate of return on asset *i*, v_{ij} – is covariance of profitability of assets *j* and *i*, $x_{i,j}$ – is the share of capital spent on the purchase of securities *i* and *j*.

Integrated Pistunov-Sitnikov-Sharpe model. We use the Pistunov-Sitnikov approach, that allows assembling two Markowitz models into one by putting the rate of return into numerator and the risk into the denominator.

Considering the Sharpe model we have R_f – rate of return on the risk-free asset. Elimination of this parameter provides us with portfolio level of profitability i.e. maximum aiming function.

There is a rate of return index in the denominator of Pistunov optimal portfolio model.

Two options exist to combine two criteria. One way is to merge them by summing, but then their weighting has to develop. Another, simpler option is to multiply two criteria, in this case the denominator will look like (5):

$$(\sum_{j=1}^{N}\alpha_j x_j + R_m\sum_{j=1}^{N}\beta_j x_j)\cdot\sum_{j=1}^{N}d_j x_j \qquad (5)$$

Let us consider the main constrain of the Sharpe model (6):

$$\sqrt{p_m^2(\sum_{j=1}^{N}\beta_j x_j)^2 + \sum_{j=1}^{N}p_j^2 x_j^2} \le p_{req} \qquad (6)$$

This inequality ensures the implementation of the premise that the risk of the portfolio should not exceed pre-specified risk frontier. The counterpart of this constrain is also present in the Markowitz model, as well as set beforehand expected portfolio return. But the Pistunov-Sitnikov model allows to omit the determination of such values as predefined profit and risk. On the top of that, the index that characterizes risk (β_i) is present in the denominator. Therefore, the Sharpe model constrain is discarded completely. So a simplified version of the denominator is multiplied by the denominator of Pisunov model and the numerator remains unchanged.

Thus, Integrated Pistunov-Sitnikov-Sharpe model is (7):

$$\begin{cases} \dfrac{\sqrt{\sum_i x_i^2 v_i^2} + \sum_{i=1}^{N}\sum_{j=1}^{N}x_i x_j v_{ij}}{(\sum_{j=1}^{N}\alpha_j x_j + R_m\sum_{j=1}^{N}\beta_j x_j)\cdot\sum_{j=1}^{N}d_j x_j} \to \min \\ \sum_{j=1}^{N}x_j = 1 \\ x_j \ge 0,\ j = 1,..,N \end{cases} \qquad (7)$$

Comparison of the models. To compare the effectiveness of the models data of NYSE trades are used. Calculation and comparison was conducted for the results obtained by the models of Sharpe, Pistunov-Sitnikov and Markowitz with a given income and a given risk. Especially for this comparison criterion of relative riskiness was developed, that is calculated according the formula (8):

$$Vr = R/M, \qquad (8)$$

Where R – is risk, a M – is rate of return of asset.

The criterion of the relative riskiness shows the effectiveness of the model, since it determines the ratio of risk to the income portfolio. The smaller *Vr*, the more effectively investment is distributed.

Calculations were made on the energy sector, as this stock market segment is unstable vibrations of this segment are typical. The portfolio is formed of six companies of the energy sector:

1. Ameren Corporation – holding company engaged in the transmission and distribution of electricity and natural gas. Deviations from the average share price (variation) – 0,104.

2. American Electric Power - an American company, the main electricity business owner with subsidiary enterprises in the different parts of the United States. Deviations from the average share price (variation) - 0.106.

3. DTE Energy Company - a diversified energy company in the United States. The company generates, buys, transmits, distributes and sells electricity in southeastern Michigan. Deviations from the average share price (variation) - 0.173.

4. Edison international - a holding company that manages the Southern Calofornia Edison Company (California utility company) and Edison Missing Group Inc. Deviations from the average share price (variation) - 0,331.

5. Enbridge - Canadian energy sector enterprises. The main activity of the enterprice is the construction of pipelines for crude oil and petroleum products, natural gas pipelines and natural gas distribution. Deviations from the average share price (variation) - 0.418.

6. Public Service Enterprise Group Inc. - US energy company that owns a diversified portfolio of energy assets. The company has significant generating capacity in the electricity in the northeastern and mid-Atlantic regions of the United States. Deviations from the average share price (variation) - 0,142.

Estimation on described optimization models defines different structure of optimal portfolios (shown in Table 1.). The developed integrated model shows the lowest relative riskiness *Vr* = 0.000702 and the highest income *Mp* = 64.37. The second performance is proved to be model-Pistunov Sitnikova with estimated relative riskiness *Vr* = 0,00077, and rate of return *Mp* = 62,5.

Return on the portfolio for the integrated model exceeds the return for Sharpe and Markowitz models by 8.7% and 7% respectively.

Thus, the proposed model is perfectly suited for the current unstable conditions of stock market. Worst compared results were provided by Markowitz profitable model, although it provides a high income.

Table 1

Comparison of calculations on optimization models

Model	A	AE	DTEE	EI	En	PSEG	*Mp*	*rp*	*Vr*
Developed integrated model	22.8%	33.8%	22.6%	6.6%	1.7%	12.5%	64.3	0.0452	0.000702
Pistunov-Sitnikov model	22.7%	42.6%	24.7%	3.1%	0.0%	6.9%	62.5	0.04814	0.0008
Sharpe model	0.0%	0.0%	22.7%	52.0%	25.3%	0.0%	59.3	2.8	0.0472
Markowitz (risk minimisation)	91.6%	8.5%	0.0%	0.0%	0.0%	0.0%	48.6	4.999	0.102801
Markowitz (profit maxomisation)	20.6%	79.4%	0.0%	0.0%	0.0%	0.0%	59.9	6.2144	0.1036

Legend for corporations: A – Ameren, AE – American electric, DTEE – DTE Energy, EI – Edison international, En – Enbridge, PSEG – Public Service Enterprice Group,

Conclusion. Analysis of existing approaches to optimal investment portfolio formation allowed us to develop the model of optimal investment portfolio structure. The model was tested on real data of equities of the energy sector, traded on the New York Stock Exchange. The effectiveness of the model was proved by the comparison with the results of calculations on existing optimal portfolio models: Markowitz, Sharpe and Pistunov-Sitnikov. The optimal investment portfolio was formed of equities of six companies: Ameren, Amirican Electric, DTE Energy, Edison International, Enbridge, Public Service Enterprice Group.

The obtained results indicate that the developed integrated model is the most effective among models considered on the rate return criteria. This conclusion is supported with the criterion of relative riskiness that was developed specially for this study. The relative riskiness (0.000702) is minimal for the portfolio formed on integrated model.

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ІНТЕГРОВАНА МОДЕЛЬ ОПТИМІЗАЦІЇ ПОРТФЕЛЮ ЦІННИХ ПАПЕРІВ

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У статті запропоновано удосконалену синтетичну економіко-математичну модель оптимального портфелю цінних паперів. Здійснено моделювання оптимального портфелю цінних паперів енергетичних компаній США. З метою порівняння ефективності моделей

інвестиційного портфеля розроблено критерій відносної ризикованості, що об'єднує показники ризику і прибутку.

***Ключові слова**:* інвестиційний портфель, фондовий ринок, прибутковість, інвестиційний ризик, модель Марковіца, модель Шарпа.

ИНТЕГРИРОВАННАЯ МОДЕЛЬ ОПТИМИЗАЦИИ ПОРТФЕЛЯ ЦЕННЫХ БУМАГ

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В статье предложена усовершенствованная синтетическая экономико-математическая модель оптимального портфеля ценных бумаг. Осуществлено моделирование оптимального портфеля ценных бумаг энергетических компаний США. С целью сравнения эффективности моделей инвестиционного портфеля разработан критерий относительной рискованности, объединяющий показатели риска и прибыли.

Ключевые слова: инвестиционный портфель, фондовый рынок, доходность, инвестиционный риск, модель Марковица, модель Шарпа.

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