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ESTIMATION OF PARAMETERS OF GAS STORAGE OPERATION IN INHOMOGENEOUS AQUIFERS

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ОЦІНКА ПАРАМЕТРІВ РОБОТИ СХОВИЩА ГАЗА У НЕОДНОРІДНОМУ ВОДОНОСНОМУ ПЛАСТІ

The purpose of the article is development and testing of a mathematical model of gas storage in a layered aquifer with a low permeability interlayer for a case of plane-parallel and axial-symmetric filtration.

Methodology. One of the most common models of anisotropy of rocks is a model of a layered seam, which is explained by geological conditions of sedimentation, which lead to stratification of layers with different collecting properties. In the practice of underground gas storage, consideration of such a model is of particular importance. This is due to possible significant difference in advancing a boundary of gas-water contact through the interlayers with different filtration characteristics caused by changes in a position of a gas zone. A comprehensive approach is applied, which includes collection, systematization and analysis of actual data on filtration and physical and mechanical properties of host rocks that affect the formation of natural and technogenic deposits, as well as analytical and numerical methods for solving equations of gas-water contact in different conditions.

Results. Gas-hydrodynamic model of underground gas storage in an inhomogeneous aquifer is justified for calculation of its cyclic operation in a three-layer seam considering cross-flows through a low permeability interlayer. The results can be used in evaluation calculations at a design stage of gas storage facilities in aquifers.

Scientific novelty. A mathematical model of gas storage in a layered aquifer with a low permeability interlayer for a case of plane-parallel and axial-symmetric filtration is developed and tested. A new method of linearization of a system of differential equations for determining pressures in a collecting seam is obtained in the article and it is a generalization of previously used methods, with an introduction of "boundary schemes".

Practical significance. Calculation results indicate a significant influence of characteristics of a layered porous medium on the advance of gas-water contact along individual layers. The results can be used in the evaluation calculations at a design stage of gas storage facilities in aquifers.

Keywords: *aquifer, gas storage, filtration, gas-water contact, inhomogeneity.*

Introduction. The urgency of creating new underground gas storages in aquifers of the southeastern part of Ukraine is justified in detail earlier in [1-5].This article is focused on estimating gas-hydrodynamic performance of their operation in these conditions. It should be noted that such an estimation, considering the real structure of aquifers, should be carried out at an early stage of gas storage design, when a large amount of computational work related to a project justification is performed. In this regard, the calculation methods used to calculate gas-hydrodynamic parameters of gas storage facilities must meet certain requirements: consider the main features of an object; have sufficient accuracy and simplicity in realization, be tested and be universal.

Methodology. Currently, most of the methods associated with calculation of parameters of underground gas storage facilities are based on an assumption of homogeneity and anisotropy of a collecting seam. This approach greatly simplifies the solution of problems and allows accurate modelling of a process of multiphase filtration in a porous medium in many cases [6–10]. However, the real structure of rocks is more consistent with a model of inhomogeneous seam (environment, in which all components of a permeability tensor are random or have a certain type of symmetry). In this case, one of the most common models of anisotropy of rocks is a model of a layered seam, which is explained by geological conditions of sedimentation, which lead to stratification of seams with different collecting properties. In the practice of underground gas storage, consideration of such a model is of particular importance, as due to changes in a position of a gas zone, the advance of a boundary of gas-water contact through strata with different filtration characteristics may differ significantly. In turn, neglecting this unevenness along seam thickness leads to significant losses of gas because of its leakage outside the storage boundaries, as well as to irrational use of pore space and reduction of active gas volume. In this regard, the purpose of this article is to develop and test a mathematical model of gas storage in a layered aquifer with low permeability interlayer for a case of plane-parallel and axial-symmetric filtration.

Results and discussion. Plane-parallel filtration. Consider a three-layer horizontal seam consisting of two high permeability layers and one dense layer that separates them (Fig. 1). Roof and floor of the seam are impermeable, the seam is limited, constant pressure over time P_k on a contour of its inflow ($x = L$) is given. Parameters of high permeability layers are k_1, n_1, m_1 and k_2, n_2, m_2 are respectively, permeability, porosity and thickness, and the parameters of anointer layer k_0, n_0, m_0 are also considered given.

At the initial moment of time ($t = 0$), an ideal gas is pumped into a seam, which is completely filled with water, through a mine gallery located on the line ($x = 0$). Assume that permeability of the upper layer is higher than permeability of the lower one ($k_2 > k_1$), permeability of the interlayer$k_0 \ll k_1$.Gas displacement is considered to be piston-like, pressure in the entire gas area for any moment of time is constant and equal to the gallery pressure $\overline{P}(t)$. The filtration area is conditionally divided into several zones. Zone $D_{i,1}$ ($0 \leq x \leq l_1$), zone $D_{i,2}$ ($l_1 \leq x \leq l_2$) and zone $D_{i,3}$ ($l_2 \leq x \leq L$), where i indicates the interlayer number ($i = 0, 1, 2$). Zones $D_{1,1}$, $D_{1,2}$ and $D_{2,2}$, $D_{2,3}$ are separated from each other by movable separation boundaries B_1 and B_2, which are considered vertical within high permeability layers. The equation of separation boundaries in a low permeability interlayer is written in a form:

$$y_0 = m_1 + y\,(x, t) \tag{1}$$

Pressure in each of the zones $D_{i,j}$ is denoted by $P_{i,j}(x, y, t)$ ($i = 1, j = 2, 3$; $i = 2$, $j = 3$). The problem is to find pressure distribution $P_{i,j}(x, y, t)$, as well as a law of motion for the boundaries B_1, B_2 and B_0. The liquid is assumed to be incompressible, and pressure in the water area satisfies the Laplace's equation.

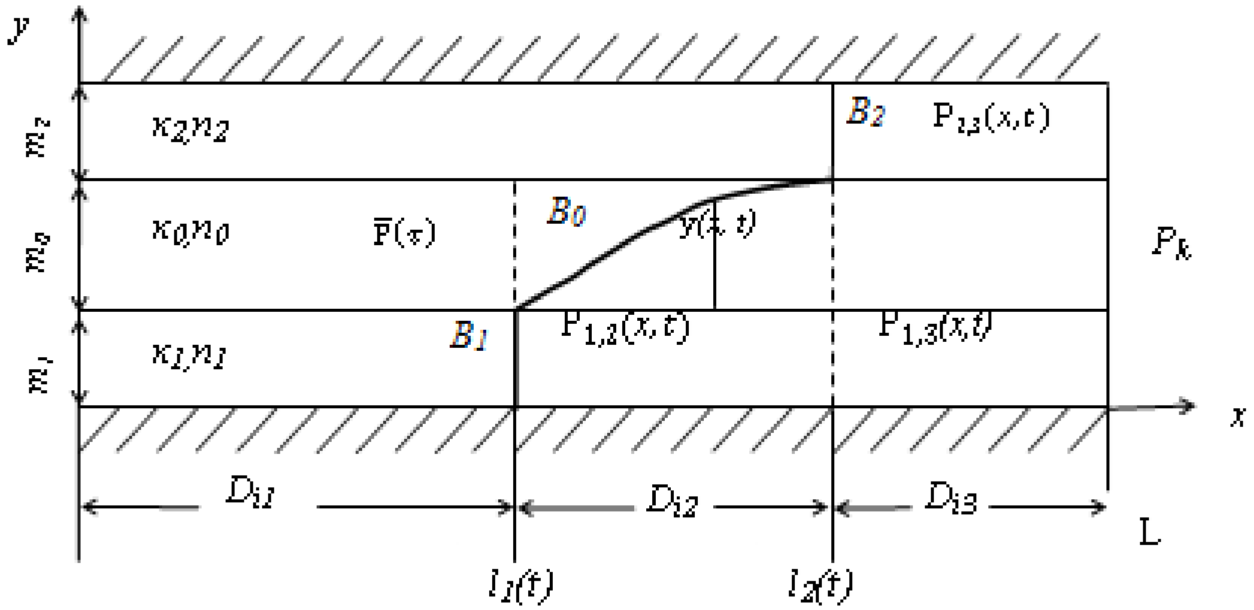

Fig. 1. Model of a three-layer seam

Using the condition $k_0 \ll \ll k_1$, a horizontal component of filtration rate in anointer layer can be neglected according to the Myatiev-Girinsky scheme [11–15]. Applying the averaging method by seam thickness and satisfying the conditions of flow continuity on contact surfaces of layers with different permeability values, as well as using the conditions of impermeability of the seam roof and floor, differential equations are obtained to determine the average pressure $P_{i,j}$ in respective zones $D_{i,j}$:

$$\frac{\partial^2 P_{1,2}}{\partial \xi} - \bar{\lambda}^2 \left(P_{1,2} - \bar{P}(\tau)\right) = 0, \tag{2}$$

$$\frac{\partial P_{1,3}}{\partial \xi} - \lambda_i^2 (-1)^{i+1} \left(P_{1,3} - P_{2,3}\right) = 0, \tag{3}$$

$$\bar{\lambda}^2 = \frac{k_0 L^2}{m_1 \left(3k_1 y(\xi,\tau) + k_0 m_1\right)}, \tag{4}$$

$$\lambda_i^2 = \frac{3k_0 k_1 k_2 L^2}{m_i k_i \left(3m_0 k_1 k_2 + m_1 k_0 k_2 + m_2 k_0 k_1\right)}, \tag{5}$$

$$\xi = \frac{x}{L}; \; r_i = \frac{l_i}{L}; \; P_{i,j} = \frac{P_{i,j}}{P_{atm}}; \; \tau = \frac{t}{T}; \; h_i = \frac{m_i}{m_0}; \; \bar{P} = \frac{\bar{P}}{P_{atm}};$$

where P_{atm} is atmospheric pressure; T is characteristic time of the process (period of gas storage operation).

Boundary conditions are as follows:

$$P_{1,2} = P_{1,3}; \; \frac{\partial P_{1,2}}{\partial \xi} = \frac{\partial P_{1,3}}{\partial \xi};$$

$$P_{2,3} = \bar{P}(\tau) \text{ for } \xi = r_2(\tau); \tag{6}$$

$$P_{1,2} = \bar{P}(\tau) \text{ for } \xi = r_1(\tau);$$
$$P_{1,2} = P_{1,3} = P_k \text{ for } \xi = 1.$$

In addition, the following kinematic relations are true at the moving boundaries $r_1(\tau)$ and $r_2(\tau)$:

$$\frac{\partial r_i(\tau)}{\partial \tau} = -\frac{\alpha_1 \partial P_{i,j}}{\partial \xi}\Bigg|_{\substack{(i=1, j=2; i=2, j=3) \\ \xi = r_i(\tau)}}, \quad \alpha_i = \frac{k_i T P_{atm}}{\sigma \mu n_i L^2}, \tag{7}$$

where μ is water viscosity; σ is gas saturation at the displacement front.

Additionally, the equation of material balance is used for solution control

$$P(\tau)\sigma(n_1 h_1 r_1 + (n_2 h_2 + n_1 h_1) r_2 - n_0 \int_{r_1}^{r_2} y(\xi,\tau) d\xi) = \frac{T}{LP_{atm} m_0}(V_0 + \int_0^{\tau} G(\tau) d\tau), \tag{8}$$

where V_0 is initial gas volume in a seam; $G(\tau)$ is gas consumption in a seam.

Due to uncertainty of a boundary location in the interlayer y (ξ, τ), the equations (2) – (8) cannot be solved analytically. An approach based on the introduction of "boundary" schemes [16-20] is used for a numerical solution of the problem. In accordance with the first "boundary" scheme, it is assumed that the low permeability interlayer in zone D_2 is completely filled with liquid, thus

$$y(\xi,\tau) \equiv m_0, \; r_1 \le \xi \le r_2 .$$

Otherwise (in the second "boundary" scheme) it is assumed that the interlayer in zone D_2 is filled with gas

$$y(\xi,\tau) \equiv 0, r_1 \le \xi \le r_2 .$$

Real solution can be estimated from below and above, which allows determining the boundaries within which the exact solution is located.

This approach is equivalent to linearization of the equations (2) – (8), and in this case, the system (2) – (3) with boundary conditions (6) has an analytical solution. By substituting the determined solution for pressure into equation (7), a system of differential equations of motion for separation boundaries in high permeability layers is obtained:

$$\begin{cases} \dfrac{\partial r_1}{\partial \tau} = \alpha_1 \dfrac{\lambda_N sh(\lambda(1-r_2))(\bar{\mathrm{P}}(\tau) - \mathrm{P}_k)}{sh(\lambda_1(r_2 - r_1)) F(r_1, r_2)} \\ \dfrac{\partial r_2}{\partial \tau} = \alpha_2 \dfrac{(\lambda_N cth(\lambda_N (r_2 - r_1)) sh(\lambda(1-r_2)) + \lambda ch(\lambda(1-r_2)))}{F(r_1, r_2)} \end{cases} \tag{9}$$

where $F(r_1, r_2) = \beta sh(\lambda(1-r_2)) + \lambda(1-\beta)(1-r_1) ch(\lambda(1-r_2)) +$
$+\lambda_N (1-r_2) cth(\lambda_N (r_2 - r_1)) sh(\lambda(1-r_2));$

$$\lambda = \sqrt{\lambda_1^2 + \lambda_2^2}; \beta = \frac{k_1 m_1}{k_1 m_1 + k_2 m_2};$$

$$\lambda_N^2 = \frac{3k_0 L^2}{m_1\left(3k_1 y_N + k_0 m_1\right)}.$$

sh, ch,cth are hyperbolic sine, cosine and cotangent, respectively.

System (9) is integrated numerically with the initial conditions

$r_1(0) = r_2(0) = 0, V(0) = 0.$ (10)

Coefficients λ_N are calculated according to the accepted assumptions for two "boundary" schemes:

N = I is the first "boundary" scheme

$$y(\xi,\tau) = y_I = m_0; \lambda_I = \frac{3k_0 L^2}{m_1\left(3k_1 m_0 + k_0 m_1\right)}, \tag{11}$$

N = II is the second "boundary" scheme

$$y(\xi,\tau) = y_{II} = 0; \ \lambda_{II} = \frac{3L^2}{m_1^2}.$$

The solution of equations (1) – (11) is carried out by numerical integration using Mathcad software for the following geological conditions: $m_1 = m_2 = 30$ m; $m_0 = 10$ m; $k_1 = 0{,}5\cdot10^{-12}$ m^2; $k_2 = 2\cdot10^{-12}$ m^2; $k_0 = 5\cdot10^{-18}$ m^2; $n_1 = n_2 = 0{,}2$; $\bar{\sigma} = 0{,}3$; $P_k = 5$ MPa; $L = 10^4$ m; $T = 3{,}15\cdot10^7$ s. Based on the performed calculations, Fig. 2 indicates a dependency of change of gas volume pumped into the seam and reduced to normal conditions on time. Changes in pressure and location of displacement fronts for different points of time are given for the two "boundary" schemes N = I ($r_{1,I}$, $r_{2,I}$, $\bar{P}_I(\tau)$) and N = II ($r_{1,II}$, $r_{2,II}$, $\bar{P}_{II}(\tau)$). As can be seen from the graphs, with a given ratio of permeabilities $k_2/k_1 = 4$, there is a significant difference between the I and II "boundary" schemes. Moreover, as calculations have shown, this difference also depends on the ratios of m_0, m_1 and m_2 to some extent. However, as an estimate, the use of "boundary" schemes may be justified.

It should also be noted that if the interlayer in the area D_2 is filled with water (the first "boundary" scheme), location of fronts $r_1(\tau)$ and $r_2(\tau)$ differs by a factor of 2-3, this indicates the need to consider layered seam heterogeneity and significant influence of this factor on gas storage operation. It should be noted that the calculated time change in a gas volume entering the seam is characteristic of cyclic operation of an aquifer gas storage.

Axial-symmetric filtration. Consider the solution of a problem of gas-water contact motion in a layered seam, which has radial symmetry. Introduce the following notation: L is radius of seam in flow contour; $R_1(t)$ and $R_2(t)$ are radii of a front of gas-water contact in the first and second layers, respectively.

Continuity equation has a form

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial P_{i,j}}{\partial R}\right) + \frac{\partial^2 P_{i,j}}{\partial Z^2} = 0 \tag{12}$$

The meaning of indices *i* and *j* is the same as for a case of plane-parallel motion. Using the averaging method and assumptions for a vertical character of filtration in a low permeability interlayer, equation (12) can be written as follows

$$\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P_{1,2}}{\delta r}\right) - \lambda^2\left(y(r,\tau)\right)\left(P_{1,2} - \overline{P}(\tau)\right) = 0; \\ \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P_{i,3}}{\partial r}\right) - (-1)^{i+1}\lambda_i^2\left(P_{1,3} - P_{2,3}\right) = 0; \end{cases} \tag{13}$$

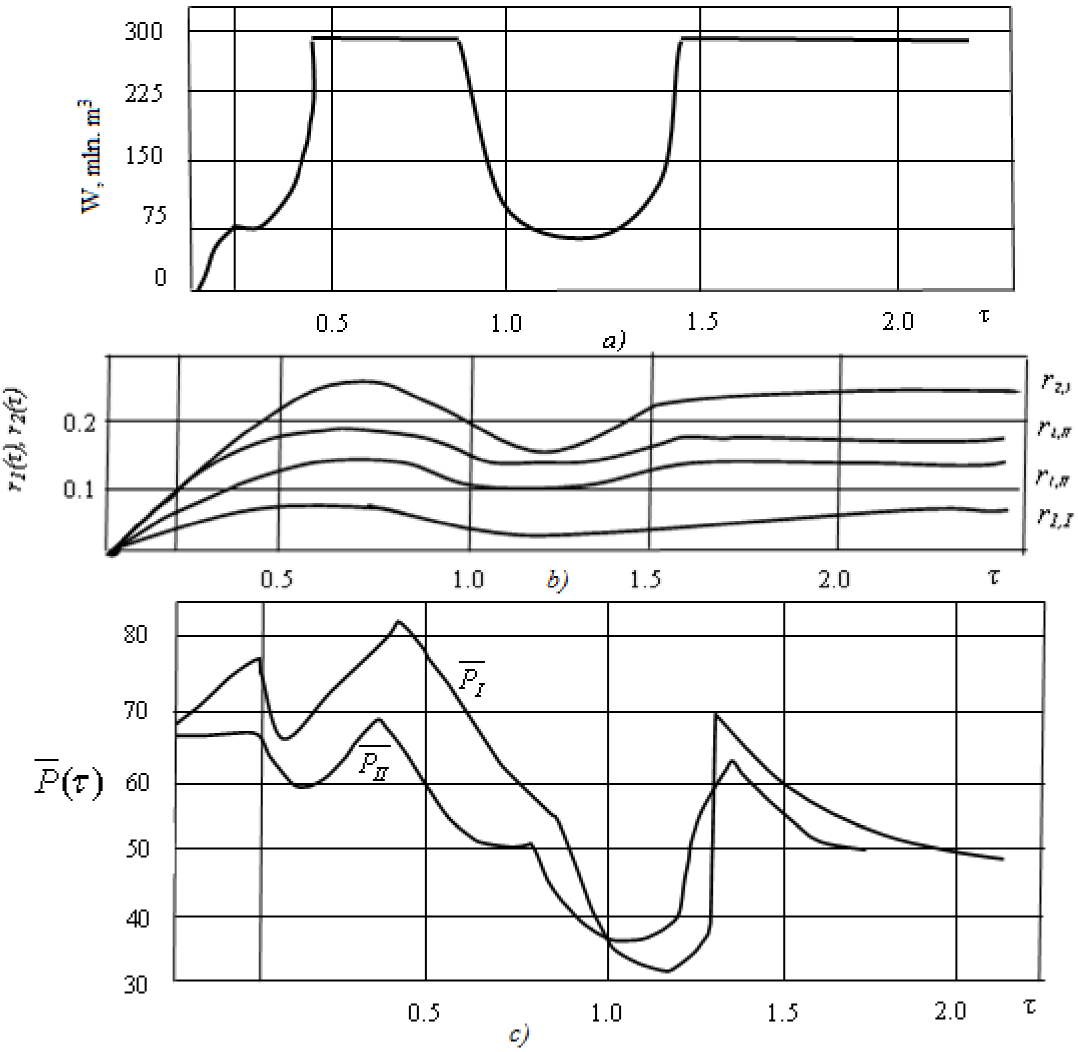

Fig. 2. Calculation results according to the I and II boundary schemes for a case of a linear collecting seam: a - c are changes of gas volume in time, location of a displacement front and a seam pressure, respectively

System (13) is written in dimensionless quantities

$$r = \frac{R}{L},\ \tau = \frac{t}{T},\ p_{i,j} = \frac{P_{i,j}}{P_{aT}},\ h = \frac{m_i}{m_0},$$

$$r_i(\tau) = \frac{R_i(\tau)}{L}, \overline{y}(r,\tau) = \frac{y(r,\tau)}{m_0}$$

Boundary and kinematic conditions on a moving boundary are written as follows

$$\begin{cases} P_{1,3} = P_{2,3} = P_k, \left(r = 1\right); \\ P_{1,2} = \bar{P}\left(\tau\right), \left(r = r_1\left(\tau\right)\right); \\ P_{1,2} = P_{1,3}, \dfrac{\partial P_{1,2}}{\partial r} = \dfrac{\partial P_{1,3}}{\partial r}, P_{2,3} = \bar{P}\left(\tau\right), \left(r = r_2\left(\tau\right)\right); \end{cases} \tag{14}$$

$$n_i \sigma \frac{dr_i}{d\tau} = -\frac{k_i T}{\mu L^2} \frac{\partial P_{i,j}}{\partial r}, \ (i = 2, j = 3; \ i = 1, j = 2) \tag{15}$$

In order to close the system (12) – (15) write down an equation of material balance

$$\bar{P}\sigma\pi(n_1 r_1^2\left(\tau\right) h_1 + r_2^2\left(\tau\right)\left(n_2 h_2 + n_0 h_0\right) - 2n_0 \int_{r_1}^{r_2} ry\left(r,\tau\right) dr) = V_0 + \int_0^{\tau} q\left(\tau\right) d\tau, \tag{16}$$

where $q\left(\tau\right) = \dfrac{Q\left(\tau\right)T}{m_0 L^2 P_{atm}}, V_0 = \dfrac{W_0}{m_0 L^2 P_{atm}}; Q\left(\tau\right)$ is consumption of gas pumped into a seam and reduced to normal conditions; W_0 is amount of gas in a seam at a moment of starting pumping in or pumping out.

By linearizing the system (13) and by introducing two "boundary" schemes and determining the expression for the coefficients λ_i and $\hat{\lambda}$, obtain a solution to the problem with respect to unknown functions $\bar{P}\left(\tau\right)$, $r_i\left(\tau\right)$ and $r_2\left(\tau\right)$, and also determine pressure distribution in the area filled with water $P_{1,2}\left(r,\tau\right), P_{1,3}\left(r,\tau\right)$ and $P_{2,3}\left(r,\tau\right)$.

Consider another approach to solving this problem, based on linearization of the initial system of equations not in the entire area $D_{1,2}$, but in subareas $D_j^{'}$, into which the initial area is divided (Fig. 3). The zone $D_{1,2}$ is divided by a finite number of zones $D_j^{'}$, and a continuity equation can be written for each of them

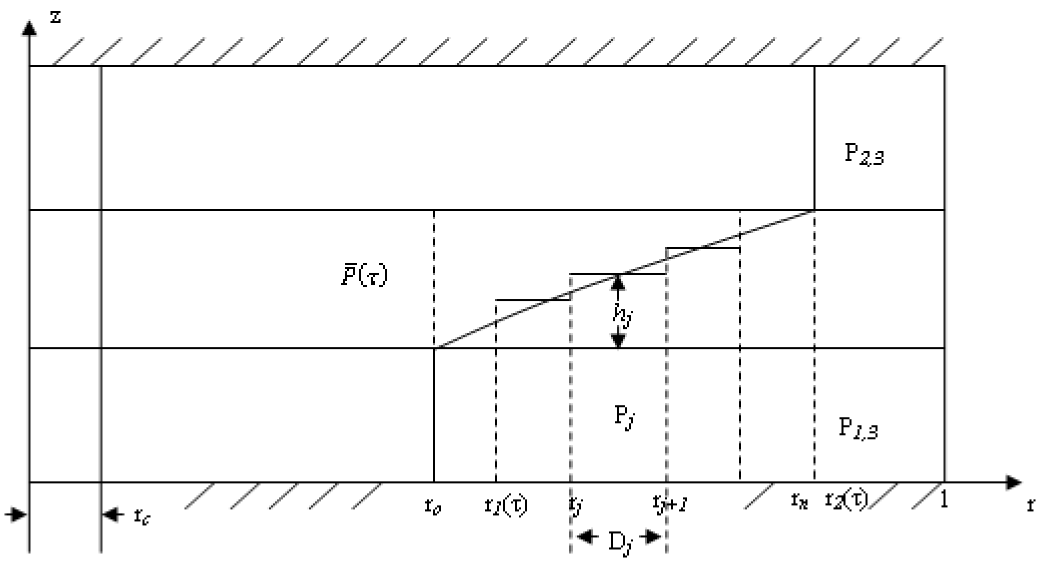

Fig. 3. Approximation scheme of unknown boundary in a low permeability interlayer

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P_j}{\partial r}\right)+\frac{\partial^2 \mathrm{P}_j}{\partial z}=0 \tag{17}$$

By averaging equation (17) with respect to thickness, obtain the following:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \mathrm{P}_j^{'}}{\partial r}\right)-\lambda_j^2\left(\bar{P}(\tau)-p_j^{'}\right)=0,\ \left(j=1,\ 2,\ \ldots,\ n\right), \tag{18}$$

where $P_j^{'}$ ispressure in area $D_j^{'}$.

Boundary y (r, τ) in the area $D_{0,2}$ is approximated by a piecewise-constant line, which allows determining coefficients λ_j^2 on each of the segments $\left(r_{j-1}^{'}\le r\le r_j^{'}\right)$

$$\lambda_j^2=\frac{3k_0k_1L^2}{m_1\left(3m_0h_jk_1+m_1k_0\right)} \tag{19}$$

Boundary conditions are written as follows

$$\begin{cases} P_1^{'}=\bar{P}(\tau),\ \text{ when } r=r_0^{'}=r_1(\tau); \\ P_j^{'}=P_{j+1}^{'},\ \dfrac{\partial p_j^{'}}{\partial r}=\dfrac{\partial p_{j+1}^{'}}{\partial \tau},\ \text{ when } r=r_j^{'}; \\ P_n^{'}=P_{1,3},\quad \text{when } r=r_n^{'}=r_2(\tau). \end{cases} \tag{20}$$

In order to determine the unknown function y (r, τ) write down the law of motion of a separation boundary B_0 in the interlayer. Using the fact that a horizontal component of filtration velocity is zero, the following is obtained

$$\frac{dh_j}{dt}=-\frac{k_0}{n_0\mu\sigma}\frac{\delta P_j^{'}}{\delta z}\bigg|\, z=h_j,\left(j=1,2,\ldots,n\right) \tag{21}$$

Note that a system of differential equations (21) must be solved together with the system (15).

The suggested linearization method of the system (13) is a generalization of previously discussed approach based on introduction of "boundary" schemes, and both "boundary" schemes can be obtained as separate cases.

General solution of equation (18) is written in a form

$$P_j(r,\tau)=\bar{P}(\tau)+\alpha_jK_0\left(\lambda_jr\right)+\beta_jI_0\left(\lambda_jr\right),\ \left(j=1,\ 2,\ \ldots,n\right). \tag{22}$$

Using conditions (20), recurrent relations for determining the coefficients α_j and β_j (j = 1, 2, …, n) are obtained:

$$\alpha_0=-\frac{\beta_0I_0(\lambda_0r_0)}{K_0(\lambda_0r_0)};$$

$$\beta_j=\frac{\left(F_{j-1}^{'}K_0\left(\lambda_jr_j\right)+\lambda_jF_{j-1}K_1\left(\lambda_jr_j\right)\right)}{\lambda_j(I_0\left(\lambda_jr_j)K_1(\lambda_jr_j\right)+I_1(\lambda_jr_j)K_0(\lambda_jr_j))}; \tag{23}$$

$$\alpha_j = \frac{\left(F_{j-1} - I_0(\lambda_j r_j)\beta_j\right)}{K_0(\lambda_j r_j)},$$

where $F_{j\text{-}1} = \alpha_{j\text{-}1}K_0(\lambda_{j\text{-}1}r_j) + \beta_{j\text{-}1}I_0(\lambda_{j\text{-}1}r_j)$;

$$F_{j-1}^{'} = -\lambda_{j-1}\left(\alpha_{j-1}K_1\left(\lambda_{j-1}r_j\right) - \beta_{j-1}I_1\left(\lambda_{j-1}r_j\right)\right). \quad (24)$$

where I_0, K_0, I_1, K_1 are modified Bessel and Hankel functions of zero and first order [15-17].

Pressure distribution in areas $D_j^{'}$ and $D_{i,3}$ has the following form

$$P_j^{'}(r,\tau) = \bar{P}(\tau) + \frac{(1-c)\Phi(\lambda_n r_n)F_j(\lambda_j r)}{S(r_0,r_n)}\left(\bar{P}(\tau) - P_k\right), \ \left(j = 1,\ 2,\ \ldots,\ n,\ r_{j-1} \le r \le r_j\right) \quad (25)$$

$$P_{i,3}(r,\tau) = \frac{\left(\bar{P}(\tau) - P_k\right)}{S(r_0,r_n)}\left(\left(F_n^{'}(\lambda_n r_n)\Phi(\lambda_n r_n)(1-c) - \Phi'(\lambda_n r_n)F_n(\lambda_n r_n)\right)r_n lnr + c^{i-1}F_n(\lambda_n r_n)\Phi(\lambda_n r)\right),$$

$$(i = 1,2,\ r_n \le r \le 1), \quad (26)$$

where $S(r_0,r_n) = F_n^{'}(\lambda_n r_n)\Phi(\lambda_n r_n)r_n lnr_n(1-c) + cF_n(\lambda_n r_n)\Phi(\lambda_n r_n) - \Phi'(\lambda_n r_n)F_n(\lambda_n r_n)r_n lnr$ (27)

$$\Phi(\lambda r) = -\frac{I_0(\lambda)K_0(\lambda r)}{K_0(\lambda)} + I_0(\lambda r); \quad (28)$$

$$\Phi'(\lambda r) = \lambda\left(\frac{I_0(\lambda)}{K_0(\lambda)}K_1(\lambda r) + I_1(\lambda r)\right). \quad (29)$$

The system of differential equations (15) considering expressions (27) – (29) obtains the form

$$\begin{cases} \dfrac{\partial r_0}{\partial \tau} = -\dfrac{(1-c)\Phi(\lambda_n r_n)F_0'(\lambda_0 r_0)}{S(r_0,r_n)}\left(\bar{P}(\tau) - P_k\right), \\ \dfrac{dr_n}{dt} = -\dfrac{(1-c)}{S(r_0,r_n)}F_n'(\lambda_n r_n)\Phi(\lambda_n r_n) - F_n(\lambda_n r_n)\Phi'(\lambda_n r_n)\cdot\left(\bar{P}(\tau) - P_k\right). \end{cases} \quad (30)$$

Initial conditions in a case of an unlimited seam

$$r_0(0) = r_n(0) = r_c, \quad (31)$$

where r_c is well radius.

System (30) is integrated numerically in Mathcad software using the fourth-order Runge-Kutta method. When determining h_j, based on relation (21) and considering the assumption of linear pressure distribution along the vertical in the interlayer, the following relation can be used

$$\frac{m_0^2 \mu n_0 \sigma}{K_0 T P_{am}} \cdot \frac{dh_j}{d\tau} = \frac{\left(P_j^{'} - \bar{P}(\tau)\right)}{h_j}, \quad (32)$$

by integrating which in a time interval ($\tau^* \le \tau \le \tau^* + \Delta\tau$), the expression for value of h_j at the end of integration step is obtained

$$h_j\left(\tau^* + \Delta\tau = \sqrt{h_j\left(\tau^*\right) + \frac{K_0 T P_{aT}}{\mu n \sigma m_0^2}(P_j - \bar{P}(\tau^*)}\right). \quad (33)$$

Calculation results of separation boundary motion in a layered seam with a well located in the center for the above geological conditions are presented in Fig. 4. These graphs compare calculations according to the first "boundary" scheme (curves $\overline{P_I}$, $r_{1,I}$, $r_{2,I}$), as well as using piecewise-constant approximation of separation boundary in the interlayer (curves $\overline{P_I}$, $r^I{}_1$, $r^I{}_2$). Comparison of calculation results for a scheme with approximation of a boundary in the interlayer and according to the second "boundary" scheme ($y\,(r,\,\tau) \equiv 0$) has the worst match, which allows considering the first "boundary" scheme more reliable and recommend it for use in estimation calculations.

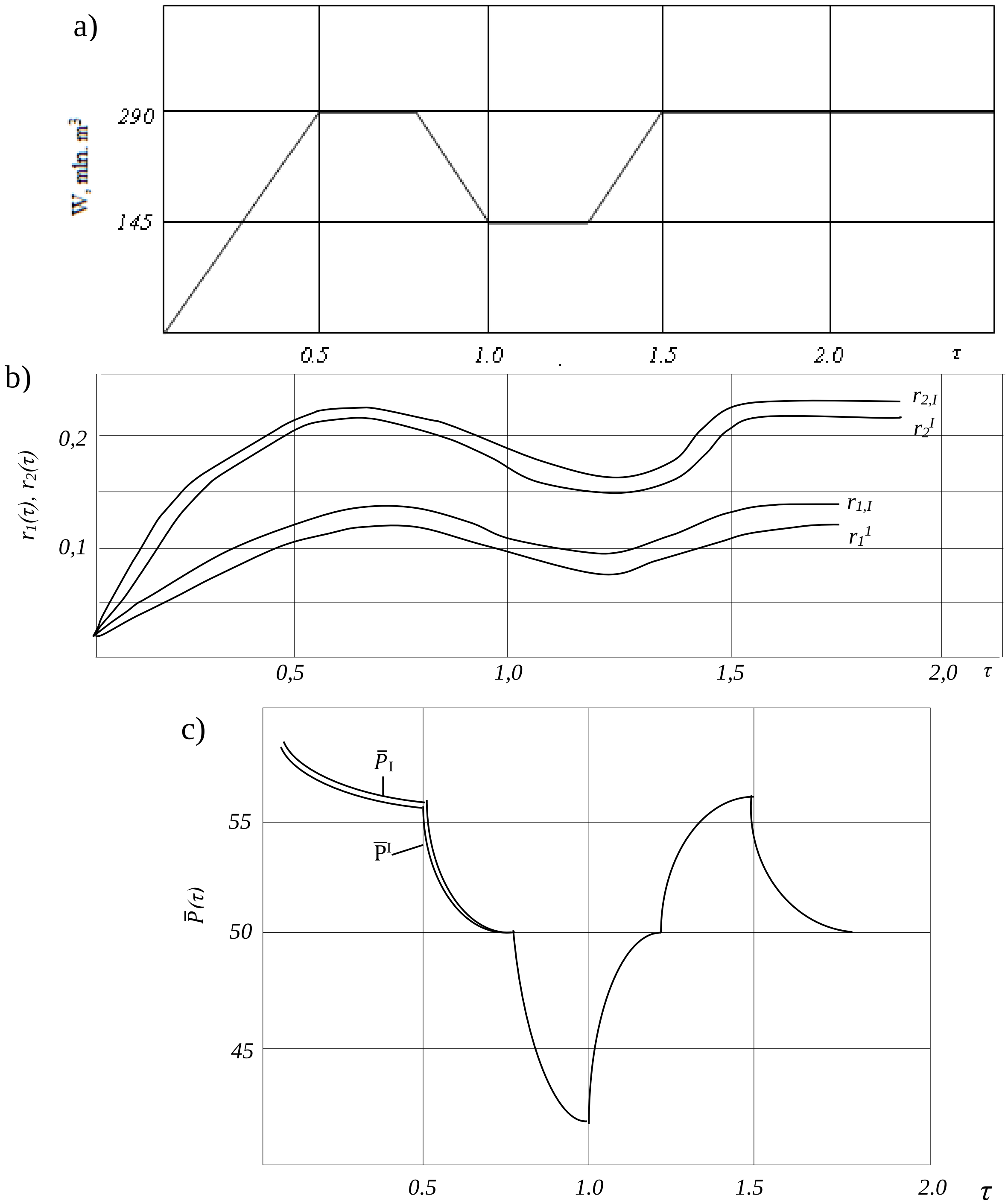

Fig. 4. Calculation results according to the first boundary scheme and a scheme with approximation of an unknown boundary

Conclusions. The suggested gas-hydrodynamic model of underground gas storage created in an inhomogeneous aquifer allows calculating its cyclic operation in a three-layer seam considering cross-flows through a low permeability interlayer. A new method of linearization of a system of differential equations for determining the pressures in a collecting seam is obtained in this research, and it is a generalization of previously used methods with the introduction of "boundary schemes". Calculation results indicate a significant influence of characteristics of a layered porous medium on an advance of gas-water contact along individual layers. The results can be used in estimation calculations at a design stage of gas storage facilities in aquifers.

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АНОТАЦІЯ

Метою даної роботи є розробка та апробація математичної моделі сховища газу в водоносному шаруватому пласті з слабопроникним пропластком для випадку плоскопаралельної і вісесиметричної фільтрації.

Методологія. Однією з найпоширеніших моделей анізотропії гірських порід є модель шаруватого пласта, що пояснюється геологічними умовами осадоутворення, які призводять до нашарування пластів з різними колекторськими властивостями. У практиці підземного зберігання газу розгляд такої моделі має особливе значення, так як внаслідок зміни положення газової зони просування межі газо-водяного контакту по пропласткам з різними фільтраційними характеристиками може значним чином відрізнятися. Застосовано комплексний підхід, що включає збір, систематизацію та аналіз фактичних даних про фільтраційні та фізико-механічні властивості вміщуючих порід, що впливають на формування природно-техногенних родовищ, а також аналітичні та чисельні методи рішення рівнянь просування газоводяного контакту в різних умовах.

Результати. Обґрунтована газогідродинамічна модель підземного газосховища в неоднорідному водоносному горизонті, для розрахунку його циклічної роботи в тришаровому пласті з урахуванням перетоків через слабопроникну перемичку. Отримані результати можуть бути використані при проведенні оціночних розрахунків на стадії проектування сховищ газу в водоносних пластах.

Наукова новизна. В роботі розроблена та апробована математична модель з слабопроникним пропластком для випадку плоскопаралельної і вісесиметричної фільтрації. Запропонована модель підземного сховища газу, створеного в неоднорідному водоносному горизонті, дозволяє розраховувати його циклічну роботу в тришаровому пласті з урахуванням перетоків через слабопроникну перемичку. Отриманий в роботі новий спосіб лінеаризації системи диференціальних рівнянь для визначення тисків в пласті-колекторі є узагальненням використовуваних раніше способів, з введенням «граничних схем».

Практичне значення. Результати проведених розрахунків показують істотний вплив характеристик шаруватого пористого середовища на просування газоводяного контакту за окремими шарами. Отримані результати можуть бути використані при проведенні оціночних розрахунків на стадії проектування сховищ газу в водоносних пластах.

Ключові слова: *водоносний пласт, сховище газу, фільтрація, газоводяний контакт, неоднорідність.*