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ON THE DISCRETE KOLMOGOROV–WIENER FILTER FOR THE ONE-POINT PREDICTION OF EXPONENTIALLY SMOOTHED HEAVY-TAIL PROCESSES

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The prediction of telecommunication traffic is an important problem for telecommunications and cyber security, see a detailed description in [1]. There are a plenty of different (and rather sophisticated) approaches to traffic prediction, see [1]. The telecommunication traffic is considered to be stationary random process in a couple of models, and, as is known, such a simple algorithm as the Kolmogorov–Wiener filter may be applied to prediction of stationary processes. So, it is of interest to investigate the possibility of the Kolmogorov–Wiener filter application to heavy-tail process prediction, because traffic in telecommunication systems with data packet transfer in considered to be a heavy-tail random process, see [2,3]. Out previous paper [4] is devoted to the corresponding problem.

In [4] we generate the heavy-tail process data X_t on the basis of the symmetric moving average approach. The Hurst parameter of the process is close to 0.8, the average value is close to 0 and the variance close to 1. In paper [4] only the linear smoothing based on the arithmetic mean is used. So it is interesting enough to investigate the exponentially smoothed process which is defined by (1):

$$\tilde{X}_t = \frac{1-\lambda}{1-\lambda^t} \sum_{k=0}^{t-1} \lambda^k X_{t-k} \quad (1)$$

where $\lambda \in (0,1)$ is a constant. The modeled non-negative traffic data $X'_t \geq 0$ and the corresponding centralized process XC_t are given by formulas (2):

$$X'_t = \tilde{X}_t - \min\left(\tilde{X}_t\right) + 10^{-3}, \; XC_t = X'_t - \langle X' \rangle \quad (2)$$

where a small summand 10^{-3} is added in order to avoid an infinite mean absolute percentage error. The algorithm is as follows. The weight coefficients are calculated as follows, see (3)

$$h = \begin{pmatrix} h_0 & h_1 & h_2 & \cdots & h_T \end{pmatrix}^T = A^{-1} B \quad (3)$$

where the components of matrices A and B are given by (4):

$$A_{ij} = R\left(|i-j|\right), \; B_i = R\left(i+1\right), \; R(\tau) = \langle XC_t XC_{t+\tau} \rangle, \quad (4)$$

$R(\tau)$ is the correlation function of the process XC_t, $T+1$ is the number of points on the basis of which the forecast is made. The superscript T in (3) denotes the matrix transposition rather than the number of points.

First of all we take $T+1$ points of the process XC_t and calculate the prediction for XC_{T+2}, then we take the points with the numbers from 2 to $T+2$ and calculate the prediction for XC_{T+3}, and so on, see (5):

$$XC_{T+2}^p = \sum_{\tau=0}^{T} h_\tau XC_{T+1-\tau}, \; XC_{T+3}^p = \sum_{\tau=0}^{T} h_\tau XC_{T+2-\tau}, \ldots, \; XC_{T+k+1}^p = \sum_{\tau=0}^{T} h_\tau XC_{T+k-\tau}, \ldots, \quad (5)$$

the superscript p denotes the predicted value. At each iteration we calculate the mean absolute error (MAE) as (6):

$$\text{MAE}_k = \left| XC_{T+1+k}^p + \langle X' \rangle - X'_{T+1+k} \right|. \quad (6)$$

The results for the average MAE over the whole array for different λ are given in the Table 1, the value $T=100$ is chosen:

Table 1

λ	$\langle \text{MAE} \rangle$	$\langle X' \rangle$
0.9	0.0697	1.91
0.8	0.139	2.22
0.7	0.209	2.54
0.6	0.279	2.73
0.5	0.349	2.87
0.4	0.418	3.03
0.3	0.488	3.21
0.2	0.558	3.38
0.1	0.627	3.61

The values for the average MAE in the table are rounded off to 3 significant digits. So, one can see that for rather large values of λ the corresponding prediction based on the Kolmogorov–Wiener filter leads to reliable results.

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2. V. Gorev et al, “Kolmogorov–Wiener Filter Weight Function for Stationary Traffic Forecasting: Polynomial and Trigonometric Solutions”, in P. Vorobiyenko, M. Ilchenko, I. Strelkovska (Eds.), Lecture Notes in Networks and Systems, vol 212, Springer, 2021, pp. 111–129, doi:10.1007/978-3-030-76343-5_7.

3. V.N. Gorev, A.Yu. Gusev, V.I. Korniienko, “Kolmogorov–Wiener filter for continuous traffic prediction in the GFSD model”, Radio Electronics, Computer Science, Control, 3, p. 31–37, 2022. doi: 10.15588/1607-3274-2022-3-3.

4. V. Gorev, A. Gusev and V. Korniienko, “The use of the Kolmogorov–Wiener filter for prediction of heavy-tail stationary processes”, CEUR Workshop Proceedings, 3156, pp. 150-159, 2022. Available at: http://ceur-ws.org/Vol-3156/paper9.pdf