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THEORETICAL FOUNDATIONS OF POINT CLOUD COORDINATE SYSTEM TRANSFORMATION

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ТЕОРЕТИЧНІ ОСНОВИ ТРАНСФОРМАЦІЇ СИСТЕМИ КООРДИНАТ ХМАР ТОЧОК

Purpose. To provide theoretical foundations and develop mathematical models for the efficient transformation of coordinate systems for point clouds in geophysical research; the scientific analysis is aimed at developing algorithms and establishing necessary dependencies for the reliable integration of data obtained at different time points into a unified coordinate system, opening up prospects for further study and analysis of processes in geophysical research.

The methods. The calculation is carried out using the following steps. Determination of known coordinates of four points (x_1', y_1', z_1'; x_2', y_2', z_2'; x_3', y_3', z_3'; x_4', y_4', z_4') in a hypothetical coordinate system (X', Y', Z') and the coordinates of the same points (x_1, y_1, z_1; x_2, y_2, z_2; x_3, y_3, z_3; x_4, y_4, z_4) in the coordinate system (X, Y, Z) to which the point clouds need to be transformed. Determination of constants a_1, a_2, a_3, d, b_1, b_2, b_3, e, c_1, c_2, c_3, f through a system of equations. After determining the constants, the coordinates of points (x', y', z') in the hypothetical coordinate system (X', Y', Z') are calculated using equations where each equation expresses the coordinates of points (x', y', z') in terms of coordinates of points (x, y, z) in the coordinate system (X, Y, Z) and the determined constants. After performing the calculations, point clouds can be merged into a single coordinate system using the computed coordinates (x', y', z'). This methodology allows for the successful transformation of coordinate systems for point clouds in geophysical research.

Findings. Analytical regularities have been established based on known coordinates of four points in both coordinate systems, allowing for the efficient transformation of a point cloud from one coordinate system to another.

The originality. For the first time, precise analytical dependencies have been established that enable the efficient transformation of point clouds from one coordinate system to another using known coordinates of four points in both systems.

Practical implementation. The obtained dependencies enable the efficient transformation of point clouds from one coordinate system to another using known coordinates of four points in both systems.

Keywords: *coordinate system transformation, point cloud, geophysical research, analytical dependencies.*

Introduction. In the modern scientific and practical context, the issue of transforming the coordinate system of point clouds holds particular significance in geophysics. This is closely tied to the rapid advancement of geophysical research and the increasing popularity of laser scanners as the primary tool for determining object locations.

Analysis of research and scientific publications [1–4] demonstrates that coordinate system transformation remains a pertinent challenge. Existing methods require further refinement and consideration of the specific conditions and tasks at hand.

Aspects of effectively transforming the coordinates of point clouds, especially in the context of different time periods and observation conditions, remain unresolved. This article is dedicated to addressing this specific aspect within the framework of geophysical research.

Main part. In the conduct of geophysical studies for determining the location of objects at a given time, laser scanners are becoming increasingly prevalent. Typically, capturing objects with a laser scanner is done in several steps, meaning that multiple fragments are captured in a conditional coordinate system. Subsequently, it is necessary to transform these fragments from the conditional coordinate systems of each fragment into a unified (single) coordinate system. In other words, all the fragments need to be merged into one fragment.

To convert the coordinates of a point x', y', z' into the new coordinates x, y, z, one should employ the coordinate system transformation formulas [5]:

$$\begin{aligned} x &= a_1 \cdot x' + a_2 \cdot y' + a_3 \cdot z' + d \\ y &= b_1 \cdot x' + b_2 \cdot y' + b_3 \cdot z' + e \\ z &= c_1 \cdot x' + c_2 \cdot y' + c_3 \cdot z' + f \end{aligned} \quad (1)$$

where a_1, a_2, a_3, d, b_1, b_2, b_3, e, c_1, c_2, c_3, f are constants used for converting from one coordinate system to another. These 12 constants can be determined based on the values of the corresponding coordinates of the 4 points. In other words, when the coordinate values of each of the 4 points are known in both the conditional coordinate system of the fragment and the coordinate system to which the point clouds (or other objects, such as polylines or segments) of all the fragments will be transformed. This process is carried out using a laser scanner or another instrument or device.

Consolidating everything, we obtain the following system of equations:

$$\begin{aligned} a_1 \cdot x' + a_2 \cdot y' + a_3 \cdot z' + d - x &= 0 \\ b_1 \cdot x' + b_2 \cdot y' + b_3 \cdot z' + e - y &= 0 \\ c_1 \cdot x' + c_2 \cdot y' + c_3 \cdot z' + f - z &= 0 \end{aligned} \quad (2)$$

Based on the known coordinates for each of the four points (x_1, y_1, z_1; x_1'. y_1'..z_1'; x_2, y_2, z_2; x_2', y_2', z_2'; x_3, y_3, z_3; x_3'. y_3'. z_3'; x_4, y_4, z_4; x_4', y_4', z_4'), to determine the 12 unknowns a_1, a_2, a_3, d, b_1, b_2, b_3, e, c_1, c_2, c_3, f, construct the following system of 12 equations (3) … (14).

$$a_1 \cdot x_1' + a_2 \cdot y_1' + a_3 \cdot z_1' + d - x_1 = 0 \quad (3)$$
$$b_1 \cdot x_1' + b_2 \cdot y_1' + b_3 \cdot z_1' + e - y_1 = 0 \quad (4)$$
$$c_1 \cdot x_1' + c_2 \cdot y_1' + c_3 \cdot z_1' + f - z_1 = 0 \quad (5)$$
$$a_1 \cdot x_2' + a_2 \cdot y_2' + a_3 \cdot z_2' + d - x_2 = 0 \quad (6)$$
$$b_1 \cdot x_2' + b_2 \cdot y_2' + b_3 \cdot z_2' + e - y_2 = 0 \quad (7)$$
$$c_1 \cdot x_2' + c_2 \cdot y_2' + c_3 \cdot z_2' + f - z_2 = 0 \quad (8)$$
$$a_1 \cdot x_3' + a_2 \cdot y_3' + a_3 \cdot z_3' + d - x_3 = 0 \quad (9)$$
$$b_1 \cdot x_3' + b_2 \cdot y_3' + b_3 \cdot z_3' + e - y_3 = 0 \quad (10)$$

$$c_1 \cdot x_3' + c_2 \cdot y_3' + c_3 \cdot z_3' + f - z_3 = 0 \quad (11)$$
$$a_1 \cdot x_4' + a_2 \cdot y_4' + a_3 \cdot z_4' + d - x_4 = 0 \quad (12)$$
$$b_1 \cdot x_4' + b_2 \cdot y_4' + b_3 \cdot z_4' + e - y_4 = 0 \quad (13)$$
$$c_1 \cdot x_4' + c_2 \cdot y_4' + c_3 \cdot z_4' + f - z_4 = 0 \quad (14)$$

Defining the constants:

$$K_1 = - x_1; \quad (15)$$
$$K_2 = - y_1; \quad (16)$$
$$K_3 = - z_1; \quad (17)$$
$$K_4 = - x_2; \quad (18)$$
$$K_5 = - y_2; \quad (19)$$
$$K_6 = - z_2; \quad (20)$$
$$K_7 = - x_3; \quad (21)$$
$$K_8 = - y_3; \quad (22)$$
$$K_9 = - z_3; \quad (23)$$
$$K_{10} = - x_4; \quad (24)$$
$$K_{11} = - y_4; \quad (25)$$
$$K_{12} = - z_4. \quad (26)$$

After the transformation, equations 3 to 14 take the following form:

$$a_1 \cdot x_1' + a_2 \cdot y_1' + a_3 \cdot z_1' + d + K_1 = 0 ; \quad (27)$$
$$b_1 \cdot x_1' + b_2 \cdot y_1' + b_3 \cdot z_1' + e + K_2 = 0 ; \quad (28)$$
$$c_1 \cdot x_1' + c_2 \cdot y_1' + c_3 \cdot z_1' + f + K_3 = 0 ; \quad (29)$$
$$a_1 \cdot x_2' + a_2 \cdot y_2' + a_3 \cdot z_2' + d + K_4 = 0 ; \quad (30)$$
$$b_1 \cdot x_2' + b_2 \cdot y_2' + b_3 \cdot z_2' + e + K_5 = 0; \quad (31)$$
$$c_1 \cdot x_2' + c_2 \cdot y_2' + c_3 \cdot z_2' + f + K_6 = 0; \quad (32)$$
$$a_1 \cdot x_3' + a_2 \cdot y_3' + a_3 \cdot z_3' + d + K_7 = 0; \quad (33)$$
$$b_1 \cdot x_3' + b_2 \cdot y_3' + b_3 \cdot z_3' + e + K_8 = 0; \quad (34)$$
$$c_1 \cdot x_3' + c_2 \cdot y_3' + c_3 \cdot z_3' + f + K_9 = 0; \quad (35)$$
$$a_1 \cdot x_4' + a_2 \cdot y_4' + a_3 \cdot z_4' + d + K_{10} = 0; \quad (36)$$
$$b_1 \cdot x_4' + b_2 \cdot y_4' + b_3 \cdot z_4' + e + K_{11} = 0; \quad (37)$$
$$c_1 \cdot x_4' + c_2 \cdot y_4' + c_3 \cdot z_4' + f + K_{12} = 0. \quad (38)$$

From equation 27, we determine a_1:

$$a_1 = (- a_2 \cdot y_1' - a_3 \cdot z_1' - d - K_1) : x_1'; \quad (39)$$
$$a_1 = - a_2 \cdot y_1' : x_1' - a_3 \cdot z_1' : x_1' - d : x_1' - K_1 : x_1'. \quad (40)$$

Substituting the value of a_1 into formula 30:

$$(- a_2 \cdot y_1' : x_1' - a_3 \cdot z_1' : x_1' - d : x_1' - K_1 : x_1') \cdot x_2' + + a_2 \cdot y_2' + a_3 \cdot z_2' + d + K_4 = 0; \quad (41)$$
$$- a_2 \cdot y_1' \cdot x_2' : x_1' - a_3 \cdot z_1' \cdot x_2' : x_1' - d \cdot x_2' : x_1' - - K_1 \cdot x_2' : x_1'' + a_2 \cdot y_2' + a_3 \cdot z_2' + d + K_4 = 0; \quad (42)$$
$$a_2 \cdot y_2' - a_2 \cdot y_1' \cdot x_2' : x_1' + a_3 \cdot z_2' - a_3 \cdot z_1' \cdot x_2' : x_1' + + d - d \cdot x_2' : x_1' + K_4 - K_1 \cdot x_2' : x_1' = 0; \quad (43)$$
$$a_2 \cdot (y_2' - y_1' \cdot x_2' : x_1') + a_3 \cdot (z_2' - a_3 \cdot z_1' \cdot x_2' : x_1') +$$

$$+ d \cdot (1 - x_2' : x_1') + K_4 - K_1 \cdot x_2' : x_1' = 0. \quad (44)$$

$$(- a_2 \cdot y_1' \cdot x_2' : x_1' - a_3 \cdot z_1' \cdot x_2' : x_1' - d \cdot x_2' : x_1' - K_1 \cdot x_2' : x_1') + + a_2 \cdot y_2' + a_3 \cdot z_2' + d + K_4 = 0; \quad (45)$$

$$a_2 \cdot (y_2' - y_1' \cdot x_2' : x_1') + a_3 \cdot (z_2' — z_1' \cdot x_2' : x_1') + + d \cdot (1 - x_2' : x_1') + K_4 - K_1 \cdot x_2' : x_1' = 0. \quad (46)$$

Substituting the value of a_1 into formula 33:

$$(- a_2 \cdot y_1' \cdot x_3' : x_1' - a_3 \cdot z_1' \cdot x_3' : x_1' - d \cdot x_3' : x_1' - K_1 \cdot x_3' : x_1') + + a_2 \cdot y_3' + a_3 \cdot z_3' + d + K_7 = 0; \quad (47)$$

$$a_2 \cdot (y_3' - y_1' \cdot x_3' : x_1') + a_3 \cdot (z_3' - z_1' \cdot x_3' : x_1') + + d \cdot (1 - x_3' : x_1') + K_7 - K_1 \cdot x_3' : x_1' = 0. \quad (48)$$

Substituting the value of a_1 into formula 36:

$$(- a_2 \cdot y_1' \cdot x_4' : x_1' - a_3 \cdot z_1' \cdot x_4' : x_1' - d \cdot x_4' : x_1' - K_1 \cdot x_4' : x_1') + + a_2 \cdot y_4' + a_3 \cdot z_4' + d + K_{10} = 0; \quad (49)$$

$$a_2 \cdot (y_4' - y_1' \cdot x_4' : x_1') + a_3 \cdot (z_4' - z_1' \cdot x_4' : x_1') + + d \cdot (1 - x_4' : x_1') + K_{10} - K_1 \cdot x_4' : x_1' = 0. \quad (50)$$

Determining the constants:

$$K_{13} = x_2' : x_1'; \quad (51)$$

$$K_{14} = y_2' - y_1' \cdot K_{13}; \quad (52)$$

$$K_{15} = z_2' - z_1' \cdot K_{13}; \quad (53)$$

$$K_{16} = 1 - K_{13}; \quad (54)$$

$$K_{17} = K_4 - K_1 \cdot K_{13}; \quad (55)$$

$$K_{18} = x_3' : x_1'; \quad (56)$$

$$K_{19} = y_3' - y_1' \cdot K_{18}; \quad (57)$$

$$K_{20} = z_3' - z_1' \cdot K_{18}; \quad (58)$$

$$K_{21} = 1 - K_{18}; \quad (59)$$

$$K_{22} = K_7 - K_1 \cdot K_{18}; \quad (60)$$

$$K_{23} = x_4' : x_1'; \quad (61)$$

$$K_{24} = y_4' - y_1' \cdot K_{23}; \quad (62)$$

$$K_{25} = z_4' - z_1' \cdot K_{23}; \quad (63)$$

$$K_{26} = 1 - K_{23}; \quad (64)$$

$$K_{27} = K_{10} - K_1 \cdot K_{23}. \quad (65)$$

Substituting the constants into equations 46, 48, 50:

$$a_2 \cdot K_{14} + a_3 \cdot K_{15} + d \cdot K_{16} + K_{17} = 0; \quad (66)$$

$$a_2 \cdot K_{19} + a_3 \cdot K_{20} + d \cdot K_{21} + K_{22} = 0; \quad (67)$$

$$a_2 \cdot K_{24} + a_3 \cdot K_{25} + d \cdot K_{26} + K_{27} = 0. \quad (68)$$

As a result, we obtain a system of eleven equations:

$$b_1 \cdot x_1' + b_2 \cdot y_1' + b_3 \cdot z_1' + e + K_2 = 0 \quad (69)$$

$$c_1 \cdot x_1' + c_2 \cdot y_1' + c_3 \cdot z_1' + f + K_3 = 0 \quad (70)$$

$$a_2 \cdot K_{14} + a_3 \cdot K_{15} + d \cdot K_{16} + K_{17} = 0 \quad (71)$$

$$b_1 \cdot x_2' + b_2 \cdot y_2' + b_3 \cdot z_2' + e + K_5 = 0 \quad (72)$$

$$c_1 \cdot x_2' + c_2 \cdot y_2' + c_3 \cdot z_2' + f + K_6 = 0 \quad (73)$$
$$a_2 \cdot K_{19} + a_3 \cdot K_{20} + d \cdot K_{21} + K_{22} = 0. \quad (74)$$
$$b_1 \cdot x_3' + b_2 \cdot y_3' + b_3 \cdot z_3' + e + K_8 = 0 \quad (75)$$
$$c_1 \cdot x_3' + c_2 \cdot y_3' + c_3 \cdot z_3' + f + K_9 = 0 \quad (76)$$
$$a_2 \cdot K_{24} + a_3 \cdot K_{25} + d \cdot K_{26} + K_{27} = 0 \quad (77)$$
$$b_1 \cdot x_4' + b_2 \cdot y_4' + b_3 \cdot z_4' + e + K_{11} = 0 \quad (78)$$
$$c_1 \cdot x_4' + c_2 \cdot y_4' + c_3 \cdot z_4' + f + K_{12} = 0 \quad (79)$$

From equation 69, we determine b_1:

$$b_1 = - b_2 \cdot y_1' : x_1' - b_3 \cdot z_1' : x_1' - e : x_1' - K_2 : x_1'. \quad (80)$$

We substitute the value of b_1 into formula 72:

$$(- b_2 \cdot y_1' : x_1' - b_3 \cdot z_1' : x_1' - e : x_1' - K_2 : x_1') \cdot x_2' + + b_2 \cdot y_2' + b_3 \cdot z_2' + e + K_5 = 0; \quad (81)$$
$$b_2 \cdot (y_2' - y_1' \cdot x_2' : x_1') + b_3 \cdot (z_2' - z_1' \cdot x_2' : x_1') + + e \cdot (1 - x_2' : x_1') + K_5 - K_2 \cdot x_2' : x_1' = 0. \quad (82)$$

Substitute the value of b_1 into formula 75:

$$(- b_2 \cdot y_1' : x_1' - b_3 \cdot z_1' : x_1' - e : x_1' - K_2 : x_1') \cdot x_3' + + b_2 \cdot y_3' + b_3 \cdot z_3' + e + K_8 = 0; \quad (83)$$
$$b_2 \cdot (y_3' - y_1' \cdot x_3' : x_1') + b_3 \cdot (z_3' - z_1' \cdot x_3' : x_1') + + e \cdot (1 - x3' : x_1') + K_8 - K_2 \cdot x_3' : x_1' = 0. \quad (84)$$

Substitute the value of b_1 into formula 78:

$$(- b_2 \cdot y_1' : x_1' - b_3 \cdot z_1' : x_1' - e : x_1' - K_2 : x_1') \cdot x_4' + + b_2 \cdot y_4' + b_3 \cdot z_4' + e + K_{11} = 0; \quad (85)$$
$$b_2 \cdot (y_4' - y_1' \cdot x_4' : x_1') + b_3 \cdot (z_4' - z_1' \cdot x_4' : x_1') + + e \cdot (1 - x_4' : x_1') + K_{11} - K_2 \cdot x_4' : x_1' = 0; \quad (86)$$

In formula 82, we substitute the constants K_{13}, K_{14}, K_{15}, K_{16}:

$$b_2 \cdot K_{14} + b_3 \cdot K_{15} + e \cdot K_{16} + K_5 - K_2 \cdot K_{13} = 0. \quad (87)$$

In formula 84, we substitute the constants K_{18}, K_{19}, K_{20}, K_{21}:

$$b_2 \cdot K_{19} + b_3 \cdot K_{20} + e \cdot K_{21} + K_8 - K_2 \cdot K_{18} = 0. \quad (88)$$

In formula 86, we substitute the constants K_{23}, K_{24}, K_{25}, K_{26}:

$$b_2 \cdot K_{24} + b_3 \cdot K_{25} + e \cdot K_{26} + K_{11} - K_2 \cdot K_{23} = 0; \quad (89)$$

Determining the constants:

$$K_{28} = K_5 - K_2 \cdot K_{13}; \quad (90)$$
$$K_{29} = K_8 - K_2 \cdot K_{18}; \quad (91)$$
$$K_{30} = K_{11} - K_2 \cdot K_{23}. \quad (92)$$

Substituting constants K_{28}...K_{30} into formulas 87, 88, 89:

$$b_2 \cdot K_{14} + b_3 \cdot K_{15} + e \cdot K_{16} + K_{28} = 0; \quad (93)$$
$$b_2 \cdot K_{19} + b_3 \cdot K_{20} + e \cdot K_{21} + K_{29} = 0; \quad (94)$$

$$b_2 \cdot K_{24} + b_3 \cdot K_{25} + e \cdot K_{26} + K_{30} = 0. \quad (95)$$

After transforming formulas 70...79, they take the following form:

$$c_1 \cdot x_1' + c_2 \cdot y_1' + c_3 \cdot z_1' + f + K_3 = 0; \quad (96)$$
$$a_2 \cdot K_{14} + a_3 \cdot K_{15} + d \cdot K_{16} + K_{17} = 0; \quad (97)$$
$$b_2 \cdot K_{14} + b_3 \cdot K_{15} + e \cdot K_{16} + K_{28} = 0; \quad (98)$$
$$c_1 \cdot x_2' + c_2 \cdot y_2' + c_3 \cdot z_2' + f + K_6 = 0; \quad (99)$$
$$a_2 \cdot K_{19} + a_3 \cdot K_{20} + d \cdot K_{21} + K_{22} = 0; \quad (100)$$
$$b_2 \cdot K_{19} + b_3 \cdot K_{20} + e \cdot K_{21} + K_{29} = 0; \quad (101)$$
$$c_1 \cdot x_3' + c_2 \cdot y_3' + c_3 \cdot z_3' + f + K_9 = 0 ; \quad (102)$$
$$a_2 \cdot K_{24} + a_3 \cdot K_{25} + d \cdot K_{26} + K_{27} = 0; \quad (103)$$
$$b_2 \cdot K_{24} + b_3 \cdot K_{25} + e \cdot K_{26} + K_{30} = 0; \quad (104)$$
$$c_1 \cdot x_4' + c_2 \cdot y_4' + c_3 \cdot z_4' + f + K_{12} = 0. \quad (105)$$

Determining c_1 from equation 96:

$$c_1 = - c_2 \cdot y_1':x_1' - c_3 \cdot z_1':x_1' - f:x_1' - K_3:x_1'. \quad (106)$$

Substituting the value of c_1 into formulas 99, 102, 105:

$$(- c_2 \cdot y_1':x_1' - c_3 \cdot z_1':x_1' - f:x_1' - K_3:x_1') \cdot x_2' + + c_2 \cdot y_2' + c_3 \cdot z_2' + f + K_6 = 0; \quad (107)$$
$$c_2 \cdot (y_2' - y_1' \cdot x_2':x_1') + c_3 \cdot (z_2' - z_1' \cdot x_2':x_1') + + f \cdot (1 - x_2':x_1') + K_6 - K_3 \cdot x_2':x_1' = 0; \quad (108)$$
$$(- c_2 \cdot y_1':x_1' - c_3 \cdot z_1':x_1' - f:x_1' - K_3:x_1') \cdot x_3' + + c_2 \cdot y_3' + c_3 \cdot z_3' + f + K_9 = 0 ; \quad (109)$$
$$c_2 \cdot (y_3' - y_1' \cdot x_3':x_1') + c_3 \cdot (z_3' - z_1' \cdot x_3':x_1') + + f \cdot (1 - x_3':x_1') + K_9 - K_3 \cdot x_3':x_1' = 0 ; \quad (110)$$
$$(- c_2 \cdot y_1':x_1' - c_3 \cdot z_1':x_1' - f:x_1' - K_3:x_1') \cdot x_4' + + c_2 \cdot y_4' + c_3 \cdot z_4' + f + K_{12} = 0; \quad (111)$$
$$c_2 \cdot (y_4' - y_1' \cdot x_4':x_1') + c_3 \cdot (z_4' - z_1' \cdot x_4':x_1') + + f \cdot (1 - x_4':x_1') + K_{12} - K_3 \cdot x_4':x_1' = 0. \quad (112)$$

In formula 108, the constants K_{13}, K_{14}, K_{15}, K_{16} are substituted:

$$c_2 \cdot K_{14} + c_3 \cdot K_{15} + f \cdot K_{16} + K_6 - K_3 \cdot K_{13} = 0. \quad (113)$$

In formula 110, the constants K_{18}, K_{19}, K_{20}, K_{21} are substituted:

$$c_2 \cdot K_{19} + c_3 \cdot K_{20} + f \cdot K_{21} + K_9 - K_3 \cdot K_{18} = 0. \quad (114)$$

In formula 112, the constants K_{23}, K_{24}, K_{25}, K_{26} are substituted:

$$c_2 \cdot K_{24} + c_3 \cdot K_{25} + f \cdot K_{26} + K_{12} - K_3 \cdot K_{23} = 0. \quad (115)$$

Determining the constants:

$$K_{31} = K_6 - K_3 \cdot K_{13}; \quad (116)$$
$$K_{32} = K_9 - K_3 \cdot K_{18}; \quad (117)$$
$$K_{33} = K_{12} - K_3 \cdot K_{23}. \quad (118)$$

Substituting the constant K_{31} into formula 113:

$$c_2 \cdot K_{14} + c_3 \cdot K_{15} + f \cdot K_{16} + K_{31} = 0. \quad (119)$$

Substituting the constant K_{32} into formula 114:

$$c_2 \cdot K_{19} + c_3 \cdot K_{20} + f \cdot K_{21} + K_{32} = 0\ . \quad (120)$$

Substituting the constant K_{33} into formula 115:

$$c_2 \cdot K_{24} + c_3 \cdot K_{25} + f \cdot K_{26} + K_{33} = 0. \quad (121)$$

After transforming formulas 97...105, they take the following form:

$$a_2 \cdot K_{14} + a_3 \cdot K_{15} + d \cdot K_{16} + K_{17} = 0; \quad (122)$$
$$b_2 \cdot K_{14} + b_3 \cdot K_{15} + e \cdot K_{16} + K_{28} = 0; \quad (123)$$
$$c_2 \cdot K_{14} + c_3 \cdot K_{15} + f \cdot K_{16} + K_{31} = 0; \quad (124)$$
$$a_2 \cdot K_{19} + a_3 \cdot K_{20} + d \cdot K_{21} + K_{22} = 0; \quad (125)$$
$$b_2 \cdot K_{19} + b_3 \cdot K_{20} + e \cdot K_{21} + K_{29} = 0; \quad (126)$$
$$c_2 \cdot K_{19} + c_3 \cdot K_{20} + f \cdot K_{21} + K_{32} = 0\ ; \quad (127)$$
$$a_2 \cdot K_{24} + a_3 \cdot K_{25} + d \cdot K_{26} + K_{27} = 0; \quad (128)$$
$$b_2 \cdot K_{24} + b_3 \cdot K_{25} + e \cdot K_{26} + K_{30} = 0; \quad (129)$$
$$c_2 \cdot K_{24} + c_3 \cdot K_{25} + f \cdot K_{26} + K_{33} = 0. \quad (130)$$

Determining a_2 from equation 122:

$$a_2 = - a_3 \cdot K_{15} : K_{14} - d \cdot K_{16} : K_{14} - K_{17} : K_{14}. \quad (131)$$

Substituting the value of a_2 into formula 125:

$$(- a_3 \cdot K_{15} : K_{14} - d \cdot K_{16} : K_{14} - K_{17} : K_{14}) \cdot K_{19} + + a_3 \cdot K_{20} + d \cdot K_{21} + K_{22} = 0; \quad (132)$$

$$a_3 \cdot (K_{20} - K_{15} \cdot K_{19} : K_{14}) + d \cdot (K_{21} - K_{16} \cdot K_{19} : K_{14}) + + K_{22} - K_{17} \cdot K_{19} : K_{14} = 0. \quad (133)$$

Substituting the value of a_2 into formula 128 :

$$(- a_3 \cdot K_{15} : K_{14} - d \cdot K_{16} : K_{14} - K_{17} : K_{14}) \cdot K_{24} + + a_3 \cdot K_{25} + d \cdot K_{26} + K_{27} = 0; \quad (134)$$

$$a_3 \cdot (K_{25} - K_{15} \cdot K_{24} : K_{14}) + d \cdot (K_{26} - K_{16} \cdot K_{24} : K_{14}) + + K_{27} - K_{17} \cdot K_{24} : K_{14} = 0. \quad (135)$$

Determining the constants:

$$K_{34} = K_{19} : K_{14}; \quad (136)$$
$$K_{35} = K_{20} - K_{15} \cdot K_{34}; \quad (137)$$
$$K_{36} = K_{21} - K_{16} \cdot K_{34}; \quad (138)$$
$$K_{37} = K_{22} - K_{17} \cdot K_{34}; \quad (139)$$
$$K_{38} = K_{24} : K_{14}; \quad (140)$$
$$K_{39} = K_{25} - K_{15} \cdot K_{38}; \quad (141)$$
$$K_{40} = K_{26} - K_{16} \cdot K_{38}; \quad (142)$$
$$K_{41} = K_{27} - K_{17} \cdot K_{38}. \quad (143)$$

Substituting constants K_{35}, K_{36}, K_{37} into formulas 133:

$$a_3 \cdot K_{35} + d \cdot K_{36} + K_{37} = 0; \quad 144)$$

Substituting constants K_{39}, K_{40}, K_{41} into formulas 135:

$$a_3 \cdot K_{39} + d \cdot K_{40} + K_{41} = 0. \quad (145)$$

After transforming formulas 123...130, they take the following form

$$b_2 \cdot K_{14} + b_3 \cdot K_{15} + e \cdot K_{16} + K_{28} = 0; \quad (146)$$
$$c_2 \cdot K_{14} + c_3 \cdot K_{15} + f \cdot K_{16} + K_{31} = 0; \quad (147)$$
$$a_3 \cdot K_{35} + d \cdot K_{36} + K_{37} = 0; \quad (148)$$
$$b_2 \cdot K_{19} + b_3 \cdot K_{20} + e \cdot K_{21} + K_{29} = 0; \quad (149)$$
$$c_2 \cdot K_{19} + c_3 \cdot K_{20} + f \cdot K_{21} + K_{32} = 0 ; \quad (150)$$
$$a_3 \cdot K_{39} + d \cdot K_{40} + K_{41} = 0; \quad (151)$$
$$b_2 \cdot K_{24} + b_3 \cdot K_{25} + e \cdot K_{26} + K_{30} = 0; \quad (152)$$
$$c_2 \cdot K_{24} + c_3 \cdot K_{25} + f \cdot K_{26} + K_{33} = 0. \quad (153)$$

Determining b_2 from equation 146:

$$b_2 = - b_3 \cdot K_{15} : K_{14} - e \cdot K_{16} : K_{14} - K_{28} : K_{14}. \quad (154)$$

We substitute the value of b_2 into formula 149:

$$(- b_3 \cdot K_{15} : K_{14} - e \cdot K_{16} : K_{14} - K_{28} : K_{14}) \cdot K_{19} + \\ + b_3 \cdot K_{20} + e \cdot K_{21} + K_{29} = 0; \quad (155)$$
$$b_3 \cdot (K_{20} - K_{15} \cdot K_{19} : K_{14}) + e \cdot (K_{21} - K_{16} \cdot K_{19} : K_{14}) + \\ + K_{29} - K_{28} \cdot K_{19} : K_{14} = 0. \quad (156)$$

We substitute the value of b_2 into formula 152:

$$(- b_3 \cdot K_{15} : K_{14} - e \cdot K_{16} : K_{14} - K_{28} : K_{14}) \cdot K_{24} + \\ + b_3 \cdot K_{25} + e \cdot K_{26} + K_{30} = 0; \quad (157)$$
$$b_3 \cdot (K_{25} - K_{15} \cdot K_{24} : K_{14}) + e \cdot (K_{26} - K_{16} \cdot K_{24} : K_{14}) + \\ + K_{30} - K_{28} \cdot K_{24} : K_{14} = 0. \quad (158)$$

Substituting constants K_{35} and K_{36} into formula 156:

$$b_3 \cdot K_{35} + e \cdot K_{36} + K_{29} - K_{28} \cdot K_{19} : K_{14} = 0. \quad (159)$$

Substituting constants K_{39} and K_{40} into formula 158:

$$b_3 \cdot K_{39} + e \cdot K_{40} + K_{30} - K_{28} \cdot K_{24} : K_{14} = 0. \quad (160)$$

Determining the constants:

$$K_{42} = K_{29} - K_{28} \cdot K_{34}; \quad (161)$$
$$K_{43} = K_{30} - K_{28} \cdot K_{38}; \quad (162)$$

Substituting the constant K_{42} into formula 159:

$$b_3 \cdot K_{35} + e \cdot K_{36} + K_{42} = 0. \quad (163)$$

Substituting the constant K_{43} into formula 160:

$$b_3 \cdot K_{39} + e \cdot K_{40} + K_{43} = 0. \quad (164)$$

After transforming formulas 147...153, they take the following form:

$$c_2 \cdot K_{14} + c_3 \cdot K_{15} + f \cdot K_{16} + K_{31} = 0; \quad (165)$$

$$a_3 \cdot K_{35} + d \cdot K_{36} + K_{37} = 0; \quad (166)$$
$$b_3 \cdot K_{35} + e \cdot K_{36} + K_{42} = 0; \quad (167)$$
$$c_2 \cdot K_{19} + c_3 \cdot K_{20} + f \cdot K_{21} + K_{32} = 0 ; \quad (168)$$
$$a_3 \cdot K_{39} + d \cdot K_{40} + K_{41} = 0; \quad (169)$$
$$b_3 \cdot K_{39} + e \cdot K_{40} + K_{43} = 0; \quad (170)$$
$$c_2 \cdot K_{24} + c_3 \cdot K_{25} + f \cdot K_{26} + K_{33} = 0; \quad (171)$$

Determining c_2 from equation 165:

$$c_2 = - c_3 \cdot K_{15} : K_{14} - f \cdot K_{16} : K_{14} - K_{31} : K_{14}. \quad (172)$$

Substituting the value of c_2 into formula 168:

$$(- c_3 \cdot K_{15} : K_{14} - f \cdot K_{16} : K_{14} - K_{31} : K_{14}) \cdot K_{19} + + c_3 \cdot K_{20} + f \cdot K_{21} + K_{32} = 0 ; \quad (173)$$
$$c_3 \cdot (K_{20} - K_{15} \cdot K_{19} : K_{14}) + f \cdot (K_{21} - K_{16} \cdot K_{19} : K_{14}) + + K_{32} - K_{31} \cdot K_{19} : K_{14} = 0. \quad (174)$$

Substituting the value of c_2 into formula 171:

$$(- c_3 \cdot K_{15} : K_{14} - f \cdot K_{16} : K_{14} - K_{31} : K_{14}) \cdot K_{24} + + c_3 \cdot K_{25} + f \cdot K_{26} + K_{33} = 0; \quad (175)$$
$$c_3 \cdot (K_{25} - K_{15} \cdot K_{24} : K_{14}) + f \cdot (K_{26} - K_{16} \cdot K_{24} : K_{14}) + K_{33} - K_{31} \cdot K_{24} : K_{14} = 0. \quad (176)$$

Substituting constants K_{34}, K_{35}, K_{36} into formula 174:

$$c_3 \cdot K_{35} + f \cdot K_{36} + K_{32} - K_{31} \cdot K_{34} = 0. \quad (177)$$

Substituting constants K_{38}, K_{39}, K_{40} into formula 176:

$$c_3 \cdot K_{39} + f \cdot K_{40} + K_{33} - K_{31} \cdot K_{38} = 0. \quad (178)$$

Determining the constants:

$$K_{44} = K_{32} - K_{31} \cdot K_{34}; \quad (179)$$
$$K_{45} = K_{33} - K_{31} \cdot K_{38}. \quad (180)$$

Substituting constants K_{44} and K_{45} into formulas 179, 180:

$$c_3 \cdot K_{35} + f \cdot K_{36} + K_{44} = 0 ; \quad (181)$$
$$c_3 \cdot K_{39} + f \cdot K_{40} + K_{45} = 0. \quad (182)$$

After transforming formulas 166 … 171, they take the following form:

$$a_3 \cdot K_{35} + d \cdot K_{36} + K_{37} = 0; \quad (183)$$
$$b_3 \cdot K_{35} + e \cdot K_{36} + K_{42} = 0; \quad (184)$$
$$c_3 \cdot K_{35} + f \cdot K_{36} + K_{44} = 0 ; \quad (185)$$
$$a_3 \cdot K_{39} + d \cdot K_{40} + K_{41} = 0; \quad (186)$$
$$b_3 \cdot K_{39} + e \cdot K_{40} + K_{43} = 0; \quad (187)$$
$$c_3 \cdot K_{39} + f \cdot K_{40} + K_{45} = 0. \quad (188)$$

Determining a_3 from equation 183:

$$a_3 = - d \cdot K_{36} : K_{35} - K_{37} : K_{35}. \quad (189)$$

Substitute the value of a_3 into formula 186:

$$(- d \cdot K_{36} : K_{35} - K_{37} : K_{35}) \cdot K_{39} + d \cdot K_{40} + K_{41} = 0; \quad (190)$$

$$d \cdot (K_{40} - K_{36} \cdot K_{39} : K_{35}) + K_{41} - K_{37} \cdot K_{39} : K_{35} = 0; \quad (191)$$

$$d \cdot (K_{40} - K_{36} \cdot K_{39} : K_{35}) + K_{41} - K_{37} \cdot K_{39} : K_{35} = 0. \quad (192)$$

Determining the constants:

$$K_{46} = K_{39} : K_{35}; \quad (193)$$

$$K_{47} = K_{40} - K_{36} \cdot K_{46}; \quad (194)$$

$$K_{48} = K_{41} - K_{37} \cdot K_{46}. \quad (195)$$

Substituting constants K_{46}, K_{47},K_{48} into formula 192:

$$d \cdot K_{47} + K_{48} = 0. \quad (196)$$

Determining d from equation 196:

$$d = (- K_{48}) : K_{47}. \quad (197)$$

Determining b_3 from equation 184:

$$b_3 = - e \cdot K_{36} : K_{35} - K_{42} : K_{35}. \quad (198)$$

Substitute the value of b_3 into formula 187:

$$e \cdot (K_{40} - K_{36} \cdot K_{39} : K_{35}) + K_{43} - K_{42} \cdot K_{39} : K_{35} = 0. \quad (199)$$

Determine the constant:

$$K_{49} = K_{43} - K_{42} \cdot K_{46}. \quad (200)$$

Substituting constants K_{47} and K_{49} into formula 199:

$$e \cdot K_{47} + K_{49} = 0. \quad (201)$$

Finding the value of e from equation 201:

$$e = (- K_{49}) : K_{47}. \quad (202)$$

Finding the value of c_3 from equation 185:

$$c_3 = - f \cdot K_{36} : K_{35} - K_{44} : K_{35}. \quad (203)$$

Substituting the value of c_3 into equation 188:

$$(- f \cdot K_{36} : K_{35} - K_{44} : K_{35}) \cdot K_{39} + f \cdot K_{40} + K_{45} = 0; \quad (204)$$

$$f \cdot (K_{40} - K_{36} \cdot K_{39} : K_{35}) + K_{45} - K_{44} \cdot K_{39} : K_{35} = 0. \quad (205)$$

Determine the constant:

$$K_{50} = K_{45} - K_{44} \cdot K_{46}; \quad (206)$$

Substituting constants K_{47} and K_{50} into formula 205:

$$f \cdot K_{47} + K_{50} = 0. \quad (207)$$

From equation 207, find the value f:

$$f = (- K_{50}) : K_{47}. \quad (208)$$

Conclusions. The article provides the theoretical foundations for the transformation (conversion) of point cloud coordinate systems in the case where there are 4 points with known coordinates in two coordinate systems: the conditional system and the one to which the coordinates of the point cloud, known in the conditional coordinate system, need to be transformed. Provided that for each of the 4 points, the coordinate values (x_1'. y_1'..z_1'; x_2', y_2', z_2'; x_3'. y_3'. z_3'; x_4', y_4', z_4') are known in the conditional coordinate system (X', Y', Z'), and the coordinate values (x_1, y_1, z_1; x_2, y_2, z_2; x_3, y_3, z_3; x_4, y_4, z_4) are known in the coordinate system X, Y, Z to which the point clouds will be transformed, the calculation of 12 coefficients (a_1, a_2, a_3, d, b_1,b_2, b_3, e, c_1, c_2, c_3, f) is performed using the following formulas in the specified order: (51), (52), (53), (54), (55), (56), (57), (58), (59), (60), (61), (62), (63), (64), (65), (90), (91), (92), (116), (117), (118), (136), (137), (138), (139), (140), (141), (142), (143), (161), (162), (179), (180), (193), (194), (195), (197), (200), (202), (206), (208), (203), (198), (189), (172), (154), (131), (106), (80), (40).

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АНОТАЦІЯ

Мета. Надати теоретичні основи та розробити математичні моделі для ефективної трансформації системи координат хмар точок у геофізичних дослідженнях; науковий аналіз спрямований на розробку алгоритмів та встановлення необхідних залежностей для надійного об'єднання даних, отриманих у різні моменти часу, в єдину систему координат, що відкриває перспективи для подальшого вивчення та аналізу процесів у геофізичних дослідженнях.

Методика. Розрахунок здійснюється за допомогою наступних кроків. Визначення відомих координат чотирьох точок (x_1', y_1', z_1'; x_2', y_2', z_2'; x_3', y_3', z_3'; x_4', y_4', z_4') в умовній системі координат (X', Y', Z') і координат цих же точок (x_1, y_1, z_1; x_2, y_2, z_2; x_3, y_3, z_3; x_4, y_4, z_4) в системі координат (X, Y, Z), до якої потрібно привести хмари точок. Визначення констант a_1, a_2, a_3, d, b_1, b_2, b_3, e, c_1, c_2, c_3, f за допомогою системи рівнянь. Після визначення констант обчислення координат точок (x', y', z') в умовній системі координат (X', Y', Z'), яке відбувається за допомогою рівнянь, де кожне рівняння виражає координати точок (x', y', z') через координати точок (x, y, z) в системі координат (X, Y, Z) та визначені константи. Після виконання

розрахунків, можна об'єднати хмари точок у єдину систему координат, використовуючи обчислені координати (x', y', z'). Ця методика дозволяє успішно виконувати трансформацію системи координат для хмар точок у геофізичних дослідженнях.

Результати. За допомогою відомих координат чотирьох точок у обох системах координат встановлено аналітичні закономірності, які дозволяють ефективно перетворити хмару точок з однієї системи координат до іншої.

Наукова новизна. Вперше встановленні точні аналітичні залежності, що дозволяють ефективно трансформувати хмару точок з однієї системи координат до іншої, використовуючи відомі координати чотирьох точок у обох системах.

Практична значимість. Отримані залежності дозволяють ефективно трансформувати хмару точок з однієї системи координат до іншої, використовуючи відомі координати чотирьох точок у обох системах.

Ключові слова: *трансформація системи координат, хмара точок, геофізичні дослідження, аналітичні залежності.*