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COMPARISON OF DIFFERENT POLYNOMIAL SYSTEMS FOR MODELED HEAVY-TAIL PROCESS PREDICTION

Introduction and the results. As is known, the fractional Gaussian noise may describe telecommunication traffic, see, for example, [1–8]. In our paper [9] we generated modeled heavy-tail data which are the smoothed fractional Gaussian noise data and may describe telecommunication traffic in stable systems with data packet transfer. In particular, in [9] we investigated the prediction of the obtained modeled data on the basis of the continuous Kolmogorov–Wiener filter based on the Walsh functions. In paper [10] we investigated the corresponding prediction on the basis of the Chebyshev polynomials of the first kind and in paper [11] we investigated the corresponding prediction on the basis of the Chebyshev polynomials of the second kind. As is shown [9–11], the results for the Chebyshev polynomials of the first kind and of the second kind are almost the same, and the prediction made on the basis of the Walsh functions gives better results than that made on the basis of the Chebyshev polynomials. But a question occurs – may another polynomial system enhance the prediction based on the orthogonal polynomials?

In this paper we investigate the corresponding prediction based on the orthogonal polynomials without weight which are introduced as follows:

$$S_n(\tau) = S'_n(\tau) \Big/ \sqrt{\int_0^T dt \left(S'_n(t)\right)^2} \tag{1}$$

where

$$S'_n(\tau) = \begin{vmatrix} \mu_0 & \mu_1 & \mu_2 & \cdots & \mu_n \\ \mu_1 & \mu_2 & \mu_3 & \cdots & \mu_{n+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mu_{n-1} & \mu_n & \mu_{n+1} & \cdots & \mu_{2n-1} \\ 1 & \tau & \tau^2 & \cdots & \tau^n \end{vmatrix}, \ \mu_n = \int_0^T x^n dx = \frac{T^{n+1}}{n+1}. \tag{2}$$

The polynomials (1) obey the following orthogonality condition:

$$\int_0^T S_n(\tau) S_k(\tau) d\tau = \delta_{nk} = \begin{cases} 1, n = k \\ 0, n \neq k \end{cases}. \tag{3}$$

The prediction algorithm is similar to that described in [10, 11]. The obtained MAPE results are given in Table 1.

Table 1. **Average MAPE for different numbers of polynomials**

n	MAPE, %	n	MAPE, %	n	MAPE, %
1	23.3	11	12.6	21	7.83
3	20.7	13	11.2	23	7.41
5	18.5	15	10.2	25	7.33
7	16.4	17	9.16	27	7.50
9	14.3	19	8.43	29	8.05

Conclusions. The obtained results are similar by the order of the value to the results for the Chebyshev polynomials of the first kind and of the second kind. So, all the considered polynomial systems give almost identical results. The Walsh functions give better prediction results than the polynomials for the problem under consideration. The fractional Gaussian noise is widely used not only on telecommunication traffic theory. For example, it is used in the econometric analysis of agriculture, see [12]. So, the obtained results may be useful not only for the telecommunication traffic theory.

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References

1. V. Paxson, "Fast, approximate synthesis of fractional Gaussian noise for generating self-similar network traffic", ACM SIGCOMM Computer Communication Review, 27, Issue 5, pp 5–18, 1997. doi: 10.1145/269790.269792.

2. T. D. Dang, B. Sonkoly and S. Molnar, "Fractal analysis and modeling of VoIP traffic", 11th International Telecommunications Network Strategy and Planning Symposium. NETWORKS 2004, 2004. doi: 10.1109/NETWKS.2004.240805.

3. S. Ledesma and D. Liu, "Synthesis of fractional gaussian noise using linear approximation for generating self-similar network traffic", ACM SIGCOMM Computer Communication Review, 30, Issue 2, pp 4-17, 2000. doi: 10.1145/505680.505682.

4. B. Canberk and S. Oktug, "Self Similarity Analysis and Modeling of VoIP Traffic under Wireless Heterogeneous Network Environment", 2009 Fifth Advanced International Conference on Telecommunications, 2009. doi: 10.1109/AICT.2009.19.

5. A. Nogueira, P. Salvador, R. Valadas and A. Pacheco, "Modeling Network Traffic with Multifractal Behavior", Telecommunication Systems, 24, pp. 339–362, 2003. doi: 10.1023/A:1026183318200.

6. J. R. Gallardo, D. Makrakis and L. Orozco-Barbosa, "Fast simulation of broadband telecommunications networks carrying long-range dependent bursty traffic", ACM Transactions on Modeling and Computer Simulation, 11, Issue 3, pp. 274–293, 2001. doi: org/10.1145/502109.502112.

7. D. I. Ostry, "Synthesis of accurate fractional Gaussian noise by filtering", IEEE Transactions on Information Theory, 52, Issue 4, pp. 1609–1623, 2006. doi: 10.1109/TIT.2006.871036.

8. S. Ma and C. Ji, "Modeling heterogeneous network traffic in wavelet domain", IEEE/ACM Transactions on Networking, 9, Issue: 5, pp. 634-649, 2001. doi: 10.1109/90.958331.

9. V. Gorev, A. Gusev, V. Korniienko and Y. Shedlovska, "On the use of the Kolmogorov–Wiener filter for heavy-tail process prediction", Journal of Cyber Security and Mobility, 12(3), pp. 315-338, 2023. doi: 10.13052/jcsm2245-1439.123.4.

10. V. Gorev, A. Gusev, V. Korniienko and Y. Shedlovska, "Polynomial solutions for the Kolmogorov–Wiener prediction of modeled smoothed heavy-tail process", Computer Science, Software Engineering and Cyber Security, in press.

11. V. Gorev, "Modeled heavy-tail process prediction based on the Chebushev polynomials of the second kind", матеріали XI Міжнародної науково-технічної конференції студентів, аспірантів та молодих вчених "Молодь: наука та інновації", Дніпро, 2023, том 2, с. 10-11.

12. M. A. Ashraf, "Climate change impact on rice productivity in Bangladesh agricultue: an econometric analysis", Daffodil International University Journal of Business and Economics, Vol. 7, No. 1, pp. 65-79, 2013.