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SELECTION OF AN APPROPRIATE NUMERICAL INTEGRATION METHOD FOR SOLVING THE OPTIMAL CONTROL PROBLEM OF AN INDUCTION MOTOR

Анотація. *У даній роботі описано застосування управління з прогнозуванням на основі градієнта в стратегії енергоефективного керування векторно-керованим асинхронним двигуном в перехідному режимі при зміні умов навантаження. Для моделювання асинхронного двигуна з орієнтацією по полю ротора, використовується модель в просторі станів. Завдання оптимального управління визначається як мінімізація інтеграла втрат енергії з обмеженнями. З цією метою умови оптимальності першого порядку визначаються на основі принципу максимуму Понтрягіна. Описано основний алгоритм вирішення задачі оптимального управління. Обговорюються алгоритмічні параметри управління з прогнозуванням для завдання чисельного інтегрування. Показано, що шляхом належного вибору методу чисельного інтегрування можна отримати оптимальну траєкторію струму намагнічування з більш низькими піковими значеннями в порівнянні з іншими методами.*

Ключові слова: *Векторне керування, асинхронні двигуни, управління з прогнозуванням на основі градієнта, оптимальний потік ротора, енергоефективність, інтегратори*

Аннотация. *В настоящей работе описано применение управления с прогнозированием на основе градиента в стратегии энергоэффективного управления векторно-управляемым асинхронным двигателем в переходном режиме при изменении условий нагрузки. Для моделирования асинхронного двигателя с ориентацией по полю ротора, используется модель в пространстве состояний. Задача оптимального управления определяется как минимизация интеграла потерь энергии с ограничениями. С этой целью условия оптимальности первого порядка определяются на основе принципа максимума Понтрягина. Описан основной алгоритм решения задачи оптимального управления. Обсуждаются алгоритмические настройки управления с прогнозированием для задачи численного интегрирования. Показано, что путем надлежащего выбора метода численного интегрирования можно получить оптимальную траекторию тока намагничивания с более низкими пиковыми значениями по сравнению с другими методами.*

Ключевые слова: *Векторное управление, асинхронные двигатели, управление с прогнозированием на основе градиента, оптимальный поток ротора, энергоэффективность, интеграторы*

Abstract. *The application of the gradient based model predictive control in an energy efficient control strategy of vector-controlled induction motor in transient behaviour when load conditions are changing is described in the current paper. A state-space approach is employed for the modelling a rotor-flux-oriented induction motor. The optimal control problem is defined as the minimization of the time integral of the energy losses with constraints. To this end the first-order optimality conditions are determined based on Portryagin's Maximum Principle. The basic algorithm to solve the optimal control problem is introduced. The algorithmic options of model predictive control concerning the numerical integrations are discussed. It is shown that by appropriately choosing the numerical integration method the field-generating current optimal trajectory with the lower spikes can be obtained compared to the other embedded methods.*

Index Terms: *Field-oriented control, induction motors, gradient based model predictive control, optimal rotor flux, energy efficiency, integrators*

I. Introduction

In the course of technological progress, the more acute becomes the problem of global energy conservation because of not only the increase in electricity consumption in industry and households, and the related need for the construction and commissioning of new energy capacities, but also limited world reserves of resources in nature. Since it is not a secret for anyone that electric motors consume more than 50 % of the world's electric power, most of which are asynchronous motors, the main way to solve this problem is the introduction of regulated electric drive systems in all branches of the national economy, which are recognized in the world practice as one of the most effective energy-saving and resource-saving environmentally friendly technologies.

Induction machines are the most frequently used type of asynchronous drives in many industrial applications due to their simple structure, robustness and low manufacturing cost. Along with the advances of power electronics, the task to operate the AC machine with any speed by means of separate control of magnetic flux and torque was solved with the introduction of the so-called field-oriented control about 40 years ago. Since then, field-oriented control is state-of-the-art nowadays for synchronous as well as for asynchronous machines.

One of the drawbacks is the fact that the power efficiency of an induction machine is lower than some other AC machine types, for example a permanent-magnet synchronous machine. Mainly because a field-generating current in addition to torque-generating current is required. Combined with the low-cost stator design it leads to higher power losses. Moreover, in part-loaded operation mode with rated flux in many applications like fans, pumps or conveyors, the efficiency will drop even more due to over-excitation and redundant copper losses. All applications stated above have in common that the machine is operated over considerable time intervals in steady state at a given speed and a given torque.

To bypass the issue of low efficiency in part-loaded mode of operation for listed types of applications a generous number of energy efficient control strategies have been developed [1-4]. The main idea of all these methods is to define the appropriate states of the machine either with the help of explicit calculation based on machine parameters or by online search algorithms or by a combination of both, so that the field-generating current and thereby the field flux is set to an optimal level. It opens an option to obtain the same torque with lower stator current and magnetic flux, resulting in reduction of ohmic and iron losses in induction machines. But such techniques frequently attenuate the problem of slow torque response under reduced magnetic flux magnitude. And, of course, the loss minimization is not achieved during transients.

Besides the above mentioned applications, there are servo motor applications. The servo motor is small and efficient, but serious to use in some applications like precise position control. They are extensively used in those applications where a particular task is to be done frequently in an exact manner. Currently in this domain the permanent-magnet synchronous machine is preferred choice, due to higher efficiency and lower moment of inertia. However, they are quite expensive because of the rare-earth materials used to manufacture the permanent magnets in the rotor. Furthermore, in order to weaken the magnetic field some additional measures are required. Therefore, the efficiency growth of induction machines in dynamics would also raise its attractivity for servo motor applications.

Nevertheless, only a comparatively small number of works addressed the minimizations of power losses for dynamic operation due to a changing motor torque. The first treatment of this problem [5] and [6] is purely offline numerical investigation on a PC, based on full knowledge of the torque and speed trajectories [5], or motion control of the application [6]. It is shown that compared to the operation under constant magnetic flux linkage, a significant improvement of reduction of power losses can be achieved. However, the offline optimization is not feasible in most applications as well as obtained optimal trajectories are valid solely for one specific application. An analytic investigation of the problem during transients is presented in [7]. It is based on the calculus of variations. The conditions of optimality use these variations to minimize the total energy losses taking into account torque-tracking constrains. But for time-varying torque the given problem can not be exactly solved. So, for such a case, authors have presented a suboptimal solution by replacing constant torque by time-varying torque in order to obtain solution for flux linkage based on the equation for the constant torque. An online implementable control scheme is proposed in [8]. It is shown in this paper that with a good approximation, it can be assumed that the optimal magnetic flux is an exponential function and that its time constant can be in this case obtained numerically. A simple solution is found using a parametrized prototype function and the parameters are evaluated by an online parameter optimization algorithm embedded in a predictive scheme where it is performed at every sampling step. Despite heuristic approximation, the solution is sufficiently accurate. Another recent work where the optimal trajectory is computed online based on an online optimization is presented in [9]. The optimal rotor flux reference is determined from the steady state power losses online and the result both for static and dynamic mode of operation is evaluated. It is shown that in order to avoid high field-generating current levels during flux transients, the flux linkage reference must be filtered. An appropriate choice of the filter time constant based on the first-order filter structure has been numerically investigated in [10] and determined as a fraction of the rotor time constant to give a user a simple design criterion. The synthesis of the filter can be pursued ever further using the methodology given in the recent paper [11].

This paper discusses another approach. In this context, we use method described in [12] with the purpose of improvement the efficiency of the induction machine in transient behaviour when load conditions are changing. This approach is appropriate for fast and high-dimensional nonlinear systems with control constraints and based on so-called model predictive control.

To illustrate this solution first the essential model equations for the induction motor as well as the relationships for the power losses will be given. Subsequently, in Section III the optimization problem is formulated and the algorithm for the solution according to [12] is described. Section IV discusses the selection of appropriate algorithmic options and presents the results obtained.

II. Background

For the modelling of the induction motor, all variables are transformed from the three-phase system (abc) to an orthogonal amplitude invariant dq reference frame with a direct (d) and a quadrature (q) axis, so that the length of a pointer corresponds to amplitude of the associated sinusoidal signal. Consider Γ-inverse equivalent circuit given in Figure 1 with orientation of the rotor flux ψ_2 along the d-axis of the synchronously rotating coordinate system $\psi_{2d} = \psi_2$, $\psi_{2q} = 0$, where L_σ – stator leakage inductance, L_μ – mutual inductance.

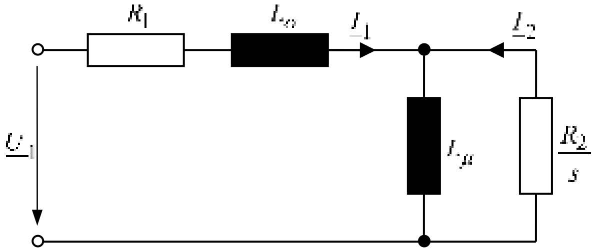

Fig. 1. Г-inverse equivalent circuit of induction machine

An induction motor in this case is characterized by the following equations in state-space, rotating with the speed ω_s:

$$\begin{aligned} \dot{I}_{1d} &= \frac{R_2}{L_\mu L_\sigma}\psi_2 - \frac{R_2}{L_\sigma}I_{1d} - \frac{R_1}{L_\sigma}I_{1d} + \omega_s I_{1q} + \frac{U_{1d}}{L_\sigma} \\ \dot{I}_{1q} &= -\frac{\omega_r}{L_\sigma}\psi_2 - \frac{R_2}{L_\sigma}I_{1q} - \frac{R_1}{L_\sigma}I_{1q} - \omega_s I_{1d} + \frac{U_{1q}}{L_\sigma} \\ \dot{\psi}_2 &= -\frac{R_2}{L_\mu}\psi_2 + R_2 I_{1d} \\ M_M &= \frac{3}{2}Z_p \psi_2 I_{1q} \end{aligned} \tag{1}$$

Assume that the speed and current regulators of field-oriented control have high enough performance to ensure the control characteristic close to perfectly rigid, that is, the dynamics of the stator currents is significantly higher than the dynamics of the magnetic flux and speed. In other words, a stepwise change in the load on the motor shaft does not lead to a significant deviation of the speed from its reference. In such a case, the dynamics of the speed and current controllers can be disregarded. The reduced motor model can be rewritten as:

$$\dot{\psi}_2 = -\frac{R_2}{L_\mu}\psi_2 + R_2 I_{1d}, \tag{2}$$

$$M_M = \frac{3}{2}Z_p \psi_2 I_{1q}. \tag{3}$$

The system of model equations in the form (2, 3) is the main mathematical object of the present study. The power losses to be considered in this paper are so-called copper losses that come from resistance of the stator winding and the rotor cage. Losses due to eddy currents and the magnetization of the plates in the stator and rotor are not taken into account. It is made for the sake of simplification, but it does not constitute any limitations for the application of the method described in this paper. For the copper losses:

$$P_V = \frac{3}{2}R_1 I_1^2 + \frac{3}{2}R_2 I_2^2 \tag{4}$$

Now rewrite equation (4) taking in account the fact that the total values of the stator current and the rotor current in the rotating dq coordinate system are calculated by Pythagoras' Theorem:

$$P_V = \frac{3}{2}R_1\left(I_{1d}^2 + I_{1q}^2\right) + \frac{3}{2}R_2\left(I_{2d}^2 + I_{2q}^2\right) \tag{5}$$

According to the equations of the magnetic system of the motor, in field oriented control we have

$$I_{2d} = -I_{1d} + \frac{1}{L_\mu}\psi_2, \tag{6}$$

$$I_{2q} = -I_{1q}. \tag{7}$$

Now (6) and (7) can be substituted in (5) resulting in:

$$\begin{aligned} P_V &= \frac{3}{2}(R_1+R_2)I_{1q}^2+\frac{3}{2}R_1 I_{1d}^2+\frac{3}{2}R_2\left(-I_{1d}+\frac{1}{L_\mu}\psi_2\right)^2 \\ &= \frac{3}{2}(R_1+R_2)I_{1d}^2+\frac{3}{2}(R_1+R_2)I_{1q}^2+\frac{3R_2}{2L_\mu^2}\psi_2^2-3\frac{R_2}{L_\mu}\psi_2 I_{1d}. \end{aligned} \tag{8}$$

The equations (1) and (6) give the following relationship:

$$R_2 I_{2d}=\frac{R_2}{L_\mu}\psi_2-R_2 I_{1d}=-\dot{\psi}_2 \tag{9}$$

Thus, the expression for power losses in steady state mode:

$$P_{V,stationary}=\frac{3}{2}R_1 I_{1d}^2+\frac{3}{2}(R_1+R_2)I_{1q}^2. \tag{10}$$

III. Optimization problem

This article takes its point of departure in the following considerations:

Field-oriented control scheme is used to control induction machine with orientation of the rotor flux ψ_2 along the d-axis of the synchronously rotating coordinate system $\psi_{2d}=\psi_2,\ \psi_{2q}=0$ at a specific rotational speed and a given torque. At a certain point in time, the torque setpoint is changed. What must be done is to adjust motor parameters in such a manner that the energy loss during transient process is as low as possible. It is assumed that the dynamics of current regulators can be neglected as stated above. This neglect is justified in so far as their time constants in practice lie in a range up to one millisecond. That is why in this paper for the sake of simplicity a decision was made to proceed with this move. But the time constant for the transient process after torque setpoint change is comparable to the rotor time constant $T_2=L_\mu/R_2$ and thus around 50 ms. The rotational speed reference is ramp-shaped which is customary in practice. A great emphasis will be placed on the field-generating current regulator.

When vector control is used, the magnitude of the torque-generating current I_{1q} is determined by the torque on the motor shaft and magnetic flux linkage. Taking into account that in steady state the motor develops a torque equal to the load torque, from the equation for the output torque (3) the steady state value of I_{1q} can be written

$$I_{1q}=\frac{2M_M}{3Z_p\psi_2}. \tag{11}$$

After substitution of (11) in (8), the power loss P_V for a constant M_M depends only on the state variable ψ_2 and manipulated field-generating current I_{1d}. At time $t=0$ a torque setpoint change from M_{M0} to M_{M1} occurs when motor operated in steady state. Thus, the optimal control problem consists in the minimization of the time integral of power losses, that is, the energy loss:

$$J=\int_0^T\left(\frac{3}{2}(R_1+R_2)I_{1d}^2+\frac{2}{3}(R_1+R_2)\frac{M_M^2}{Z_p^2\psi_2^2}+\frac{3}{2}\frac{R_2}{L_\mu^2}\psi_2^2-3\frac{R_2}{L_\mu}\psi_2 I_{1d}\right)dt \tag{12}$$

under the constraint (2) with boundary conditions $\psi_2(0)=\psi_{20}$ and $\psi_2(T)=\psi_{21}$. According to the task description the motor operates in steady state before torque setpoint change and afterwards when transient process is finished. Therefore, in steady-state at a given motor torque the optimal rotor flux linkage values ψ_{20} and ψ_{21} can be calculated by the following expression:

$$\psi_{2,stationary} = \sqrt{\frac{2}{3}\frac{M_M L_\mu}{Z_p}\sqrt{\frac{R_1+R_2}{R_1}}}. \quad (13)$$

To find a solution of optimal control problem the Hamiltonian has to be defined:

$$H(I_{1d},\psi_2,\lambda) = \frac{3}{2}(R_1+R_2)I_{1d}^2 + \frac{2}{3}(R_1+R_2)\frac{M_M^2}{Z_p^2\psi_2^2} + \frac{3}{2}\frac{R_2}{L_\mu^2}\psi_2^2 - 3\frac{R_2}{L_\mu}\psi_2 I_{1d} + \lambda\left(-\frac{R_2}{L_\mu}\psi_2 + R_2 I_{1d}\right) \quad (14)$$

with the next first-order optimality conditions for the optimal trajectories that follow from Pontryagin's Maximum Principle:

$$\frac{\partial H}{\partial I_{1d}} = 3(R_1+R_2)I_{1d} - 3\frac{R_2}{L_\mu}\psi_2 + \lambda R_2 = 0 \quad (15)$$

$$\frac{\partial H}{\partial \psi_2} = -\frac{4}{3}(R_1+R_2)\frac{M_M^2}{Z_p^2\psi_2^3} + 3\frac{R_2}{L_\mu^2}\psi_2 - 3\frac{R_2}{L_\mu}I_{1d} - \lambda\frac{R_2}{L_\mu} = -\dot{\lambda} \quad (16)$$

$$\frac{\partial H}{\partial \lambda} = -\frac{R_2}{L_\mu}\psi_2 + R_2 I_{1d} = \dot{\psi} \quad (17)$$

The solution of the optimization problem can be obtained from the differential equation system above. However, only a numerical solution is possible. For this purpose, the model-predictive method based on a projected gradient search algorithm GRAMPC from [12] is used in this article, the principles of which are briefly described below.

GRAMPC is a software package that implements a model predictive control scheme for nonlinear systems by solving an optimal control problem with the prediction horizon $T_P > 0$. In order to maintain real-time feasibility of the overall MPC algorithm as well as limit the computational time a fixed number of gradient iterations $N_{I\max}$ is used in each new MPC step t_k and the current solution is used as new initialization (warmstart) to successively refine the predicted MPC trajectories over the single MPC steps. The result is a time-discrete trajectory for the control variable. In the present task, this would be a sequence of setpoint values for field-generating current I_{1d}. For the problem introduced in this paper, the basic algorithm to find a solution looks like as follows:

- Initialization of input trajectory $I_{1d}(t)$ for $t =\in [t_k \; t_k + T_P]$ and calculation of resulting value for $\psi_2(t)$ using expression (2).
- Iterations to solve the optimization problem:
  1) The adjoint state λ is calculated by backward time integration of (14).
  2) Taking into account the result of the previous step and expression (15), the search direction of the line search problem is defined for the iterative improvement of the solution. An improved trajectory $I_{1d}(t)$ is defined with a likewise calculated step size. Then a limitation of the control variable can be taken into account using projection function.
  3) With the improved trajectory $I_{1d}(t)$ a new prediction for $\psi_2(t)$ is carried out.
  4) Once the number of desired iterations is reached, the new control trajectory is calculated. $I_{1d}(t_k)$ is used as a manipulated variable for the process in this algorithm. Then the method starts again for $t_k + 1$. Otherwise the next iteration starts with the second item.

A detailed description of the gradient algorithm of GRAMPC, individual steps and the discussion of the background can be found in documentation [12] (see algorithm in Table 3.1).

IV. The method application

The algorithmic options of model predictive control concerning the numerical integrations in the basic procedure are in the foreground in this section. An algorithmic option named integrator specifies which integration scheme is used for the forward and backward integration of the system and adjoint dynamics. So far, the following integration methods are implemented within GRAMPC: Euler method (Option value: euler), modified Euler method (Option value: modeuler), Heun method (Option value: heun) and a Runge-Kutta method of 4th order (option value: ruku45). The first mentioned integrators use a fixed step size, whereas the Runge-Kutta method is a variable step size integrator. The solution of the ordinary differential equations, first of all, comes down to the choice of the order of the numerical integration method. The order of the numerical method is not related to the order of the differential equation. These methods are characterized by such attributes as the rate of convergence, convenience, quality of the result, and type of step size.

In mathematics and computational science, the Euler method (also called forward Euler method) is a first-order numerical procedure for solving ordinary differential equations with a given initial value. It is the most basic explicit method for numerical integration of ordinary differential equations and also efficient enough to yield fairly accurate approximations of the actual solutions. Explicit methods calculate the state of the system at a later time from the state of the system at the current time without the need to solve algebraic equations. For the forward method, first a step size or h is chosen. The size of h determines the accuracy of the approximate solutions, meaning that as the step size decreases the error between the actual and the approximation reduces as well. This method produces a series of line segments, which thereby approximates the solution curve. Let x_k, $k = 0, 1, 2, \ldots$ be a sequence in time with

$$x_{k+1} = x_k + h. \tag{18}$$

Let us denote y_k and Y_k as the exact and the approximate solution at $x = x_k$, respectively. To get Y_{k+1} from (x_k, Y_k), the differential equation is used. Since the slope of the solution to the equation $y_t = f(x, y)$ at the point (x_k, y_k) is $f(x_k, y_k)$, the Euler method determines the point (x_{k+1}, Y_{k+1}) assuming that it lies on the line through the point (x_k, Y_k) with the slope $f(x_k, Y_k)$. From this reasoning, the formula for the slope of a line looks like as follows

$$Y_{k+1} = Y_k + f(x_k, Y_k)h\,. \tag{19}$$

Thus, knowing a value of the function $y_t = f(x, y)$ at the initial point (x_0, y_0) allows us with the help of the formula above find the approximate solution $y_1 = y_0 + f_0 h$. The Euler method is more accurate if the step size is smaller. Roughly speaking, the error is halved by halving the step size. However, it doubles the amount of computation.

The next method that is to be described is still the same first-order method, however, in the middle of the step, a "primary" solution is found, and then its refinement occurs. This allows us to raise the order of the convergence rate to two. It is called Modified Euler method is another popular method of numerical analysis for the integration of initial value problem. It solves ordinary differential equations by approximating in an interval with slope as an arithmetic average. This method is a simple improvement on Euler’s method in function evaluation per step. The calculation formula for the Euler method can be obtained using the expansion of the function $y(x)$ into Taylor series in a neighborhood of some point x_k:

$$y(x_k + h) = y(x_k) + h\frac{dy(x_k)}{dx} + \frac{h^2}{2!}\frac{d^2 y(x_k)}{dx^2} + \frac{h^3}{3!}\frac{d^3 y(x_k)}{dx^3} + \ldots \tag{20}$$

Then define the second derivative, approximating it to a finite difference:

$$\frac{d^2 y(x_k)}{dx^2} = \frac{d}{dx}\left(\frac{dy(x_k)}{dx}\right) \approx \frac{\Delta y'}{\Delta x} = \frac{y'(x_k + h) - y'(x_k)}{h}. \tag{21}$$

Substituting this expression in (20) and discarding the terms of series beginning with the ones containing h^3

$$y(x_k + h) = y(x_k) + \frac{h}{2}\left[\frac{dy(x_k)}{dx} + \frac{dy(x_k + h)}{dx}\right]. \tag{22}$$

Replacing in the last expression the derivatives and using the abbreviated notation, we obtain the calculation formula of the modified Euler method:

$$y_{k+1} = y_k + \frac{h}{2}\left[f(x_k, y_k) + f(x_{k+1}, y_{k+1})\right]. \tag{23}$$

The relation (23) gives a solution for y_{k+1} in an implicit form, since y_{k+1} is present simultaneously in the left and right parts. It should be noted, that the use of implicit methods is justified by the fact that as a rule they are more stable than explicit ones. The modified Euler method provides the second order of accuracy. The error at each step is proportional to h^3. Increase in accuracy can be achieved due to additional computer time in the calculation of each step.

A direct substitution of (19) into right-hand side of (23) gives the calculated ratio for Heun's method. Heun's method may refer to the improved or modified Euler's method, or a similar two-stage Runge-Kutta method. According to Heun's method, first the intermediate value $\tilde{y}_{k+1}$ to be calculated and then the final approximation y_{k+1} at the next integration point.

$$\begin{aligned} &\tilde{y}_{k+1} = y_k + hf(x_k, y_k), \\ &y_{k+1} = y_k + \frac{h}{2}\left[f(x_k, y_k) + f(x_{k+1}, \tilde{y}_{k+1})\right]. \end{aligned} \tag{24}$$

In the modified Euler's method, to obtain a second derivative $d^2y(x_k)/dx^2$ a finite-difference formula (21) is used, which includes the values of the first derivative $y'(x_k)$ and $y'(x_k + h)$ at the initial and final points of the step. If we calculate the third derivative in a similar way, having pre-calculated the second derivative at two points of the step, then (20) can be used to construct the calculated formula of the third-order accuracy method. For this purpose, the definition of the first derivative $y'(x)$ at the additional intermediate point between x_k and $x_k + h$.

Similar reasoning allows us to derive calculation formulas for higher-order methods that provide a noticeable reduction in the error of the solution. However, in practice, their implementation requires a significant increase in the amount of computation using additional intermediate points at each step.

There are other ways of constructing numerical methods with a high order of accuracy. One of them is used in the construction of the Runge-Kutta group of methods. It consists in approximating the solution of the differential equation by the sum

$$y(x_k + h) \approx \xi(x_k, h) = y(x_k) + \sum_{n=1}^{p} A_n k_n(h), \tag{25}$$

where A_n - coefficient of expansion, k_n - sequence of functions.

One of the most well-known version of the Runge-Kutta method corresponds to $p = 4$. This is a fourth-order accuracy method for which the error in the step is of order h^5. Its calculated formulas have the following form:

$$y_{k+1} = y_k + \frac{k_1 + 2k_2 + 3k_3 + k_4}{6}, \tag{26}$$

where

$$\begin{aligned} &k_1 = hf(x_k, y_k), && k_2 = hf\left(x_k + \frac{h}{2}, y_k + \frac{k_1}{2}\right), \\ &k_3 = hf\left(x_k + \frac{h}{2}, y_k + \frac{k_2}{2}\right), && k_4 = hf(x_k + h, y_k + k_3). \end{aligned}$$

The Euler method and its modification considered above are essentially Runge-Kutta methods of the first and second order, respectively. Despite the increase in the volume of calculations, the fourth-order method has an advantage over the methods of the first and second orders, since it provides a small local error. The Runge-Kutta method is a variable step size integrator and, consequently, the calculation time can be shortened.

The figure below represents the result of solving the optimal control problem with the help of numerical integration methods described above.

The curves presented are the optimal trajectories for the field-generating current $I_{1d}(t)$.The given investigation is conducted using parameters of induction motor with rated power 370W. The main inductance is assumed to be constant. The test was performed for the case of load step change from 25% up to 100%. This simulation illustrates the fact that for solving the energy efficiency optimization problem for induction motor torque steps, the most convenient choice is Euler's integration method, due to

1) better optimal trajectory with lower current spikes that lead to additional losses;

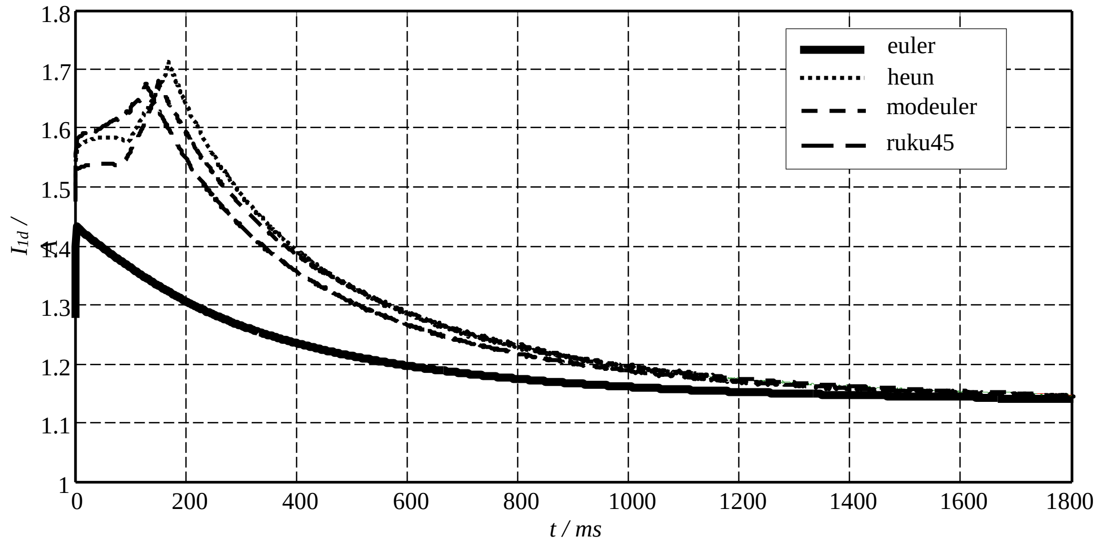

Fig. 2. Optimal control problem solution

2) higher computational speed which is very important for the task of real-time implementation as the model predictive approach requires high computational expenses for microprocessor technologies.

V. Conclusion

This paper has shown how a control method with a predictive model can be used to optimize the energy efficiency of an induction motor in dynamic mode. So, firstly the expression of power losses when considering only copper losses in dynamic mode was obtained. Assuming that the optimal control problem and first-order optimality conditions were formulated, the predictive model parameter concerning the numerical integrations was considered and it was shown that the Euler's integration method suits the best for solving the highlighted optimization problem.

Various additional issues will be addressed in the future including optimal choice of algorithmic parameters like prediction horizon, the maximum number of iterations to improve the solution of the optimization problem and number of data points for the control trajectory.

References

1. Bazzi, A. M. and Krein , P. T., Review of Methods for the Real-Time Loss Minimization in Induction Machines, *IEEE Transactions on Industry Applications*, vol. 46, no. 6, pp. 2319-2328, 2010.
2. Blanusa, B., New Trends in Efficiency Optimization of Induction Motor Drives, New Trends in Technologies: Devices, Computer, Communication and Industrial Systems, 2010.
3. Raj C. T., Srivastava S. P., and Agarwal P., Energy Efficient Control of Three-Phase Induction Motor – A Review, *International Journal of Computer and Electrical Engineering*, vol. 1, no. 1, pp. 61-70, 2009.
4. Abrahamsen, F., Energy Optimal Control of Induction Motor Drives. Dissertation, Aalborg: Institut for Energiteknik, Aalborg Universitet, 2000.
5. Lorenz, R. D. and Yang, S. M., Efficiency-Optimized Flux Trajectories for Closed-Cycle Operation of Field-Orientation Induction Machine Drives, *IEEE Transactions on Industry Applications*, vol. 28, no. 3, pp. 574-580, 1992.
6. Klenke, F. and Hofmann, W., Energy-Efficient Control of Induction Motor Servo Drives With Optimized Motion and Flux Trajectories, *Proceedings of the 14th European Conference on Power Electronics and Applications*, pp. 1-7, 2011.
7. Canudas de Wit, C. and Ramirez, J., Optimal Torque Control for Current-Fed Induction Motors, *IEEE Transactions on Automatic Control*, vol. 44, no. 5, 1084-1089, 1999.
8. Stumper, J.-F., Dötlinger, A. and Kennel, R., Loss Minimization of Induction Machines in Dynamic Operation, *IEEE Transactions On Energy Conversion*, vol. 28 no. 3, pp. 726–735, 2013.
9. Ranta, M., Qu, Z., Hinkkanen, M., and Luomi, J., Loss-Minimizing Flux Level Control of Induction Motor Drives, *IEEE Transactions on Industry Applications*, vol. 48, no. 3, pp. 952-961, 2012.
10. Diachenko, G. and Schullerus, G., Simple dynamic energy efficient field oriented control in induction motors, *Proceeding of the 18th International Symposium on Power Electronics*, Novi Sad, Republic of Serbia, 28-30 Octob. 2015.
11. Aziukovskyi, O.O. and Bakutin, A.V., Design of Optimal High-Order Speed Controller, *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, no. 1, pp. 71-76, 2014.
12. Käpernick, B., Graichen, K., U., GRAMPC documentation, Institute of Measurement, Control, and Microtechnology, University of Ulm, 2014.

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